Spring Term FDPE 2007 Hannu Vartiainen

Problem Set 4

1. (LA Confidential) Two agents 1 and 2 may have committed a crime. An impartial third agent, "the wittness", may have observed the incidence. With the true state being $\theta \in \{\text{Innocent, Guilty}\}$, the wittness has observed θ with probability $\rho > 1/2$. When in court, if both 1 and 2 confess, then they are convicted to jail for 10 years. If both of them deny the crime, they are set free. All parties (i.e. 1, 2 and the wittness) submit their messages to the court simultanously. Payoffs of the agents 1 and 2 are:

Wittness: Guilty		Confess	Deny
	Confess	-10,-10	x,y
	Deny	y,x	0,0
			,
Wittness: Innocent		Confess	Deny
	Confess	-10,-10	$^{\mathrm{z,w}}$
	D		0.0

Can you find parameters x, y, z, w such that it is a dominant strategy for both agents 1 and 2 to confess when θ = Guilty and deny when θ = Innocent? What about for *strictly* dominant strategies? Discuss whether truthful information revelation can be obtained in strictly dominant strategies if the 10 year penalty is replaced by the life in prison.

- 2. Consider the nonlinear pricing problem studied in the classes.
 - (a) Identify the second best transfer scheme t^{**} .
 - (b) What is the associated price per unit?
 - (c) Does the price per unit increase or decrease in the size of the bought bundle a^{**} . Can you give an interpretation?
- 3. Consider the bilateral trade setting. Assume the types are distributed uniformly on [0, 1].
 - (a) What is the expected joint payoff from an efficient trade?
 - (b) Compute the size of the subsidy that an efficient trade would need.
- 4. Let in the bilateral trade setting the probability measure of the buyer (seller) denote the mass of buyers (sellers), each with unit demand (supply) of the product and reservation valuation θ .

- (a) Construct a choice rule f that matches buyers and sellers in an efficient way.
- (b) Show that this rule is incentive compatible and individually rational.
- 5. A principal hires an agent whose productivity is θ , and the agent chooses effort level e. The problem for the principal is that only the sum

$$x = \theta + e$$

is observable, but not θ and e separately.

At the moment of signing the contract, the agent knows her own type. The wage offer w may only depend on x. Assume that by rejecting the contract, the agent gets an outside utility of 0. Assume also that $\theta \in \{\theta_L, \theta_H\}$, where $\theta_L < \theta_H$ and $\Pr\{\theta = \theta_L\} = \mu$. Agent's utility is given by:

$$u(w, e, \theta) = w - c(e) = w - \frac{1}{2}e^{2}$$

The utility of the principal is:

$$v\left(w,x\right) = x - w.$$

- (a) Assuming full information, solve for the optimal wages for each type of agent.
- (b) Is the solution in part a incentive compatible if the principal does not know the true type θ ?
- (c) Formulate the adverse selection problem where the menu $\{(x_L, w_L), (x_H, w_H)\}$ is offered to the agent. Show first which constraints in the problem must be binding in equilibrium and sole for the optimal contract.