Spring Term
FDPE 2007
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Problem Set 3

1. Let players A and B negotiate on the division of cake of size 1 (linear payoffs from consumption) through alternating offers bargaining game: Player A begins by making an offer of the division. If B accepts the offer, then the offer is implemented. Otherwise the play moves to next period with reversed roles, etc.. The common discount factor is $\delta < 1$. Let the maximum number of periods be T. Both players get zero payoff if the game ends without an agreement. Solve the subgame perfect Nash equilibrium division of the cake if:

   (a) $T = 1$.
   (b) $T = 2$.
   (c) $T = 3$.
   (d) Discuss the case $T \to \infty$.

2. Consider the Coordinated Attack game discussed in the lecture notes. Suppose that any reply-message is not sent back automatically but through a concious decision by a speculant.

   (a) Construct an equilibrium where coordinated attack becomes possible.
   (b) Discuss whether the players can agree on such equilibrium through e-mail messages.
   (c) Can you find the equilibrium that is most profitable to the speculants?

3. (The wallet game) Consider the following common values auction. There are two bidders whose types θ_i are independently drawn from a uniform distribution $[0, 100]$. The value of the object to both bidders is the sum of the types, i.e. $\theta_i + \theta_j$. The object is offered for sale in a first price auction. Hence the payoffs depend on the bids b_i and types as follows (again we are ignoring ties for convenience):

$$u_i(b_i, b_j, \theta_i, \theta_j) = \begin{cases} (\theta_i + \theta_j - b_i, & \text{if } b_i > b_j, \\ 0 \text{ otherwise.} \end{cases}$$

(a) Show that the linear strategies where $b_i = \theta_i$ for $i = 1, 2$ form a Bayes Nash equilibrium in this game.

(b) If $\theta_i = 1$, the optimal bid is 1, but it might seem that the expected value of the object is $1+51 = 51$. Why doesn't the bidder behave more aggressively?

4. A principal hires an agent whose productivity is θ, and the agent chooses effort level e. The problem for the principal is that only the sum

$$x = \theta + e.$$

is observable, but not θ and e separately.

At the moment of signing the contract, the agent knows θ. The wage offer w may only depend on x. Assume that by rejecting the contract, the agent gets an outside utility of 0. Assume also that $\theta \in \{\theta^L, \theta^H\}$, where $\theta^L < \theta^H$ and $\Pr\{\theta = \theta L\} = \mu$. Agent's utility is given by:

$$u(w, e, \theta) = w - \frac{1}{2}e^2.$$

The utility of the principal is:

$$v(w, x) = x - w.$$

(a) Assuming full information, solve for the optimal wages for each type of agent.

(b) Is the solution in part a) incentive compatible if the principal does not know the true type θ?

(c) Formulate the adverse selection problem where the menu $\{(x^L, w^L), (x^H, w^H)\}$ is offered to the agent. Show first which constraints in the problem must be binding in equilibrium and solve for the optimal contract.

5. Consider an adverse selection model where a monopolist banker is providing a loan for a privately informed entrepreneur. Assume that the opportunity cost of funds for the monopolist is R per unit of capital and hence the cost of lending k units is Rk. The entrepreneur uses the capital in a production process with a production function $y = \theta f(k)$. Assume that $\theta \in \{\theta^H, \theta^L\}$, where $\theta^H > \theta^L$. The banker sets a schedule of repayments, m_i and amounts to be lent k_i for $i \in \{H, L\}$. The profit of the bank is then $m_i - Rk_i$ from contract i and the payoff to the

entrepreneur is $\theta^i f(k_i) - m_i$. Assume that the production function is concave and satisfies the Inada conditions, i.e. $\lim_{k\to 0} f'(k) = \infty$ and $\lim_{k\to\infty} f'(k) = 0$.

(a) Assume first that the type θ_i is observable to the banker and solve for the optimal contracts to offer.

(b) Assume next that θ_i is private information, and write the profit maximization problem for the banker. To do this, let p denote the probability that $\theta = \theta_i$ and use the revelation principle to formulate the problem as one where at most two capital-repayment combinations are offered. Write the incentive compatibility constraints and the individual rationality (i.e. participation) constraints of the two types.

(c) Argue which constraints must be binding, and solve for the optimal capital-repayment pairs.

6. In the standard mechanism design framework we assume the following time line: (1) the agents are informed of their types, (2) the agents voluntarily participate the mechanism, (3) the agents play a perfect Bayesian equilibrium of the mechanism. By the revelation principle, any induced outcome function is captured by an incentive compatible and interim individually rational direct mechanism. Do you need to modify the revelation principle to capture the following changes to the time line? (If yes, write down the appropriate formulation.)

   (a) Assume *ex ante* individual rationality, i.e. reverse the order of items (1) and (2).

   (b) Assume *ex post* individual rationality, i.e. reverse the order of items (2) and (3).