Spring Term
FDPE 2007
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Problem Set 2

1. Consider the following variant of the alternating offers bargaining game. Three players are bargaining over 1. The bargaining protocol is as follows: In the first round, player 1 suggests split (x_1, x_2, x_3) such that $\Sigma_i x_i = 1$. (I.e. if accepted, player i would get x_i euros). If either player 2 or 3 accepts, these shares are realized. If both reject, then player 2 suggests (y_1, y_2, y_3) such that $\Sigma_i y_i = 1$. If either 1 or 3 accepts, the shares are implemented, otherwise play moves to third round where player 3 suggests (z_1, z_2, z_3) such that $\Sigma_i z_i = 1$. If either 1 or 2 accepts, the shares are implemented, otherwise the game ends and all players receive payoffs of 0. Let δ_1, δ_2 and δ_3 be the discount factors of the three players. Analyse the game by backwards induction. What happens to the payoffs when δ_1, δ_2 and δ_3 tend to one?

2. Find all pure and mixed strategy Nash equilibria in the following game:

	L	R
L	$6, 6$	$1, 7$
R	$7, 1$	$0, 0$

3. Suppose that the game in Exercise 2 is played twice and the payoff criterion is the sum of stage game payoffs. Show that there are no pure strategy SPNE where (L, L) is played in the first period. Show that if the game is played at least 3 times, then there is a pure strategy SPNE where the first period action profile is (L, L).

4. Consider the following model of intergenerational giving (Becker, 1974). A child takes and action $a \in \mathbb{R}$ to produce wealth $w(a)$. The child gets share $\lambda w(a)$ of the wealth and the parent gets share $(1-\lambda)w(a)$, $\lambda \in (0,1)$. The cost of action a to the child is ca, $c \geq 0$. After the child has chosen a, the parent chooses a bequest b to leave for the child. Child's utility from action a and bequest b is

$$u(\lambda w(a) + b) - ca.$$

Parent's utility from action a and bequest b is

$$v((1-\lambda)w(a) - b) + u(\lambda w(a) + b).$$

(a) Suppose that $c = 0$, and that $w(\cdot)$, $u(\cdot)$ and $v(\cdot)$ are increasing and (weakly) concave functions. For each level of a, find the first order condition for the optimal bequest b for the parent. Using this condition, show that the child's optimal a maximizes $w(a)$.

(b) Assume that $c > 0$, $w(a) = a$, and that $u(\cdot) = v(\cdot) = \ln(\cdot)$. Solve this game by backward induction and compare the solution with that in part a.

(c) Comment on the differences in the solutions and comment of the efficiency properties of the solution in both cases. (For this recall the definition of Pareto efficiency).

5. Consider three voters 1,2, 3 and three alternatives a, b, and c. Suppose that voters' preferences form a "Condorcet cycle".

Pref. rank	1	2	3
1.	a	b	c
2.	b	c	a
3.	c	a	b

The majority rule over two alternatives chooses the alternative with more support.

(a) Identify the (weakly) dominant strategy in the majority contest between a and b?

(b) Consider the following elimination tree. First a is majority voted against b, and then the winner against c. The winner of the latter majority contest is implemented. Analyse the game by backwards indiction (assume players use weakly undominated strategies).