FDPE Microeconomic Theory I
Fall 2007

Problem Set 5

1. An investor has initial wealth w_0 that she must divide between two assets. Denote the total investment into asset i by w_i for $i \in \{1, 2\}$. There are two possible states H and L for the economy. Assume that the economy will be in state H with probability p and in L with probability $(1-p)$ .The first asset pays a gross return R^H if the state of the economy is H and a gross return 0 if the state is L. I.e. in state H, an investment of w_1 in asset 1 results in a return $w_1 R^H$ and in state L , the return is 0. The second asset pays R in both states. Denote the fraction of initial wealth that is invested in asset i by α_i for $i \in \{1, 2\}$ so that $w_i = \alpha_i w_0$.

   (a) For an arbitrary investment portfolio $\{\alpha_1, \alpha_2\}$, determine the final wealth of the investor.

   (b) Assume the investor has a Bernoulli utility function $u(w)$ where w denotes the final wealth. Assume that $u'(w) > 0$ and $u''(w) < 0$. Write down the expected utility resulting from an arbitrary portfolio $\{\alpha_1, \alpha_2\}$ and characterize the first order conditions for the optimal portfolio. Argue also that second order conditions are also satisfied at the point that satisfies the first order conditions.

   (c) Consider now a third asset that pays 0 in H and R^L per unit invested in L. Assume further that the utility function takes the form $u(w) = \ln w$.For what values of p, R^H, R^L, R will the investor invest in assets 1 and 3 only?

   (d) (Slightly harder) Can you generalize your answer in c. to the case where there are N states and N risky assets such that asset i pays a return of R^i in state i and nothing in the other states.

2. (Edgeworth Boxes) Consider a two person, two good exchange economy. The utility functions, u^h, and initial endowments, ω^h are as specified below. For each case, find the set of Pareto optimal allocations and the Walrasian equlibria and illustrate them in an Edgeworth box.

(a) $u^1(x^1, y^1) = \ln x^1 + \ln y^1$; $u^2(x^2, y^2) = x^2 y^2$, $\omega^1 = \omega^2 = (.5, .5)$.

(b) $u^h(x^h, y^h) = \max\{x^h, y^h\}$, $\omega^h = (1, 1)$ for $h = 1, 2$.

3. It is impossible to consume an ounce of a chili without the accompaniment of between1/2 and 2 pints of beer. The exact quantity consumed is a matter of individual taste. Assume that chili can be consumed in any nonnegative amount

   (a) Draw this individual's consumption set.

   (b) Consider a two-person exchange economy in which each of the agents has-consumption sets as in part (a). Draw the Edgeworth box for this economy.

   (c) Show by example that there are economies of the sort in part (b) in which utility is strictly increasing yet there are equilibria in which one of the goods has price zero. What must happen for this to be true?

   (d) Can the result in (c) occur with the 'normal' consumption set?

4. (Gorman aggregation) Consider an exchange economy with H households having the same differentiable, strictly increasing and strictly quasi-concave utility functions but (possibly) different endowments. Assume that the common utility function is homogeneous of degree 1 and that the aggregate endowment of all goods is positive.

   (a) Find the core this economy.

   (b) Find the Walrasian equilibria for this economy.

   (c) How would your answer to questions (a) and (b) change if u was homogeneous of degree $k > 0$? (Ignore corner solutions for this problem!)