FDPE Microeconomic Theory I

Fall 2007

Problem Set 4

1. A consumer has a Bernoulli utility function of the form $u(x) = \frac{-1}{x}$ for $x \geq 0$. Suppose she is given a bet with a possible gain x_1 and a possible loss of x_2 with probabilities p and $(1-p)$ respectively.

   (a) At what initial wealth level x_0 is she indifferent between accepting the bet or not.

   (b) Suppose that $x_1 = x_2 < x_0$. For each level of initial wealth, calculate the probability with which the individual accepts the bet. Based on this evidence, would you guess that the individual has increasing or decreasing absolute risk aversion?

   (c) Verify or disprove your guess in b. by computing the coefficient of absolute risk aversion.

2. Prove that if a risk averse decision maker rejects a fixed favorable bet at all levels of wealth, then the Bernoulli utility of the decision maker is bounded from above.

3. Consider an agent living two periods, $t = 1, 2$. At $t = 1$ the agent owns an asset of size $s_1 > 0$. The size of this asset is fully known at $t = 1$ and it is the only source of income (consumption) for this agent. Let c_t denote consumption, $t = 1, 2$. Then, the asset available for consumption at $t = 2$ is what is saved from $t = 1$ plus a random term z, that is, $c_2 = s_1 - c_1 + z \geq 0$. Random term z is distributed according to some cumulative distribution function $F(z)$ on $[-k, k]$, where $s_1 \geq k$, and has mean zero. That is, $E\{z\} = \int_{-k}^{k} z dF(z) = 0$. The consumer's Bernoulli utility function $u(c_t)$ for consumption per period is increasing, strictly concave, and three times differentiable. That is, the consumer gets utility $u(c_1)$ from the first period and $u(c_2)$ from the second period (no discounting). The decision problem is to maximize expected utility

achievable from the initial asset s_1 over the two periods. Assume that $u'(c_t) \to \infty$ as $c_t \to 0$, so that optimal consumptions will be positive for both periods.

(a) Write the consumer's expected utility as a function of her consumption at $t = 1$.

(b) Derive the first-order condition for optimal c_1.

(c) Consider then another agent who is facing exactly the same problem but with a different Bernoulli utility function. Fix c_1 for both agents and consider the following statement: the second agent is willing to trade her remaining random asset for a nonrandom asset of a given size, whereas our original agent does not accept this offer. What can you tell about the agents' attitudes to risk? There is also a link between their utility functions. What is it?

4. Consider an economy with one representative agent and two dates, $t = 0, 1$. The economy has an exogenous consumption process in the following sense: in the absence of savings, period $t = 1$ consumption, $\tilde{c}_1$, is distributed according to some cumulative distribution function $F(c)$. Thus, $E\{\tilde{c}_1\} = \int c dF(c)$. Period $t = 0$ consumption c_0 is given. The agent has separate attitudes towards time and risk, so we seek to formulate preferences such that the two can disentangled. The utility over time is given by

$$U(c_0, \tilde{c}_1) = u(c_0) + \beta u(c(v, F))$$

where $\beta \in (0, 1)$ is the discount factor, u is a weakly concave function, and $c(v, F)$ is the certainty equivalent consumption for period $t = 1$ using another weakly concave function v. That is, $v(c(v, F)) = E\{v(\tilde{c}_1)\}$. Note first that if $u = v$ we have the usual time-separable expected-utility objective. Second, given $c(v, F)$, all uncertainty has been removed from calculations using $U(c_0, \tilde{c}_1)$, so the concavity of u relates to consumption smoothing only. Third, concavity of v measures risk aversion only.

(a) Suppose the agent can invest and sacrify consumption at $t = 0$ to achieve a sure benefit $(1 + r)$ per unit invested at $t = 1$. Now, if you can find a rate of return r that makes the agent just indifferent between investing and not investing, given c_0 and the expected $\tilde{c}_1$, you have found the socially efficient discount rate for this economy. Do this and discuss how it depends on consumption smoothing and risk aversion.

(b) Consider now the effect of increasing uncertainty on the discount rate. To obtain a benchmark compute the socially efficient discount rate under the assumption that $E\{\tilde{c}_1\}$ is the period $t = 1$ consumption for sure. Denote this by r^c. Then, calculate the true socially efficient discount rate under the assumption that $\tilde{c}_1$ is uncertain. Does the uncertainty reduce the discount rate? Assume time separable preferences, that is, $v = u$.

(c) The same problem as above, but assume now $v \neq u$. Show that the socially efficient discount rate falls with uncertainty about future income levels if the agent is decreasingly absolute risk averse.

5. Consider an economy where all agents face an independent risk to lose 100 with probability p. N agents decide to create a mutual agreement where the aggregate loss in the pool is equally split among its members.

   (a) Describe the change in the lotteries facing individuals in the pool when N is cahnged from 2 to 3.

   (b) Show that the risk with $N = 3$ is smaller in the sense of second order stochastic dominance that the risk with $N = 2$.