FDPE Microeconomic Theory !
Fall 2007

Problem Set 3

1. A consumer is interested in buying apples x and oranges y. The prices are given by p_x and p_y respectively when the fruits are purchased separately. Suppose that the fruits can also be purchased in bundles where one apple and one orange are sold at a joint price p. Suppose furthermore that $p > \max\{p_x, p_y\}$ and $p < p_x + p_y$ so that the pair is cheaper than its individual components.

   (a) Suppose that the consumer has welath w available. Draw the budget set in the (x, y) plane.

   (b) Assume tha the consumer has a diffenrentiable strictly increasing and quasiconcave utility function $u(x, y)$. Give the first order conditions for the optimal consumption bundle and argue that they are also sufficient conditions.

   (c) For the case of $u(x, y) = x^{.3}y^{.7}$, find the Walrasian demand.

2. The government finances public expenditure of magnitude g by collecting taxes. In this question, you are invited to think about the optimal ways of collecting taxes.

   (a) Suppose that there are goods x and y. The government can finance g by choosing either a tax on income t_w or by taxing consumption of good x by rate t_x. The government budget constraint for the two cases reads: $t_w w = g$ and $t_x x(p_x, p_y, t_x) = g$. Show that the consumer prefers an income tax in this case.

   (b) Suppose now that there is no exogenous income in the model and good y is now interpreted as leisure. Assume that he consumer has an initial endowment y^e of leisure that she may sell to buy the other good. Hence the budget constraint is now

   $$p_x x(p_x, p_y) = p_y (y^e - y(p_x, p_y)), \text{ or}$$

   $$p_x x(p) + p_y y(p) = p_y y^e.$$

   This last equation gives a way in which all problems with income resulting from sales of endowments should be thought of. First sell the endowment at market prices and then purchase the desired amounts of the goods wit the proceeds. Compare now the effect of taxes on x and y as in the previous part.

3. Suppose that the expenditure function of a consumer is of Gorman polar form:

$$e(p,u) = a(p) + ub(p).$$

Derive the demands for each good and calculate also the income shares that each good receives. Can you find an economic interpretation for your results.

4. Preferences are said to be additively separable if they can be represented by a utility function of the form;

$$u(x) = \Sigma_{i=1}^{L} u_i(x_i).$$

(a) If $u_i''(x_i < 0)$ for all i and for all $x_i \geq 0$, show that all goods are normal.

(b) Show also that

$$\frac{\partial x_i(p,w)/\partial p_k}{\partial x_j(p,w)/\partial p_k} = \frac{\partial x_i(p,w)/\partial w}{\partial x_j(p,w)/\partial w}.$$