FDPE Microeconomic Theory I

Problem Set

1. Throughout this exercise we consider for a given choice set X the following binary relations on $X \times X$:

$$xRy \text{ is } " \ x \succeq y \ "$$
$$xPy \text{ is } " \ x \succ y \ "$$
$$xIy \text{ is } " \ x \sim y \ ".$$

If we want to say "not xRy ", we write $x\tilde{R}y$.

Asymmetry: For no x and y, we have both xPy and yPx.

Negative Transitivity: the following holds for all $y \in X$,

$$(xPz) \Longrightarrow (xPy \text{ or } yPz). \tag{1}$$

A binary relation P on a set X is called a preference relation if it is asymmetric and negatively transitive. Explain the sense in which this formalization is the same as R.

2. Consider preference relation R. Let $I(x) \equiv \{y \,|y \in X, yIx\}$. Show that the set $\{I(x) \,|x \in X\}$ is a partition of X, i.e.,

   (a) $\forall x, y$, either $I(x) = I(y)$ or $I(x) \cap I(y) = \varnothing$.

   (b) For every $x \in X$, $\exists y \in X$ such that $x \in I(y)$.

3. Assume that $X = \mathbb{R}^n$ and that P is a preference relation. Suppose that P satisfies the Weak Monotonicity Axiom:

$$(x_i \geq y_i, \forall i) \Longrightarrow (xRy).$$

Moreover, suppose that P satisfies the Local Non-Satiation Axiom: $\forall x$ and scalars $\delta > 0$, $\exists y$ such that

$$1. \ \|y - x\| < \delta \text{ and}$$

$$2. \qquad yPx. \tag{2}$$

Your job is to show that for all $(x, z) \in X$,

$$(z_i > x_i, \forall i) \Longrightarrow (zPx).$$

4. Consider the Weak Axiom (WA) as defined in MWG. Show that the WA is equivalent to the following property:

$$[B \in \mathcal{B}, B' \in \mathcal{B}, (x, y) \in B, (x, y) \in B', x \in C(B), y \in C(B')]$$
$$\Longrightarrow$$
$$[x \in C(B') \textit{ and } y \in C(B)]$$

(You need to demonstrate that WA implies the property and that the property implies WA.)

5. Consider the following choice function induced by a complete and transitive preference relation $\succeq$ on X.

$$\text{For all } A \subset X,\ x \in C^{-}(A; \succeq) \Longleftrightarrow\ y \succeq x \text{ for all } y \in A.$$

Show that $C^{-}(A; \succeq)$ is a legitimate choice function satisfying the Weak Axiom. Consider the welfare interpretation of this choice function.

6. Let P be a preference relation defined on a finite choice set X with N elements.

   (a) Show that the elements of X can be ranked in order of preference.

   (b) By induction on N, show that there exists a utility function that represents P. (the induction hypothesis is that the representation exists for sets of size $N-1$. Then, consider putting back the eliminated element and look if you can attach a utility number to it. Go through all the cases and complete the induction step).

7. Consider the following model of decision making. Let $X, Y \subset \mathbb{R}$ and suppose that there is are two well defined (sub-utility) functions

$u, v : (X, Y) \to \mathbb{R}$. The preference relation of the decision maker is parametrized by a real number $\sigma > 0$ and given by:

$$i)\ (x,y) \succ (x',y') \text{ if } u(x,y) > u(x',y') + \sigma, \text{ or}$$
$$|u(x,y) - u(x',y')| < \sigma \text{ and } v(x,y) > v(x',y')$$
$$ii)\ (x,y) \sim (x',y') \text{ otherwise.}$$

In other words, the decision maker uses u as the primary criterion in her decision making, but cannot distinguish between outcomes that yield utilities that are less than σ apart from each other. Is the above preference relation complete and transitive?

8. Consider the following statements and determine if they are true or false.

   (a) If $\succsim$ is represented by a continuous function, then $\succsim$ is continuous.

   (b) A continuous preference can be represented by a noncontinuous function.

   (c) Let $X = \mathbb{R}$ and $U(x) =$ [the largest integer n such that $x \geq n$]. The underlying preference relation is continuous.

   (d) If both U and V represent $\succsim$, then there is a strictly monotonic function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x) = f(U(x))$