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# Collective choice with endogenous reference outcome

Hannu Vartiainen

*Yrjö Jahnsson Foundation, Ludviginkatu 3-5, 00130 Helsinki, Finland*

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**Abstract**

A *collective choice* problem—essentially a bargaining problem without disagreement outcome—is studied. An extended solution, which determines a solution and a reference point simultaneously, is characterized. The unique extended solution that meets the extended versions of Pareto-optimality, independence of irrelevant alternatives, symmetry, and scale invariance maximizes the Nash product with respect to *both* the solution *and* the reference point.


*JEL classification:* C71; C78

*Keywords:* No disagreement; Bargaining theory; Axioms

## 1. Introduction

We analyze a two-player collective choice problem, represented by a two dimensional, convex, and compact utility possibility set $U$. Set $U$ is the only primitive of the model, there is *no* disagreement outcome á la Nash (1950).

In many bargaining scenarios disagreement has a well defined meaning and clear welfare implications. In such cases bargaining theory is the natural framework for analyzing collective decision making. However, in other scenarios it may not be clear what disagreement means: disagreement payoffs may not be part of the data that describes the problem.

Imagine two roommates occupying the only available apartment in town, and agreeing on the rules of the household: when and by whom to clean, cook, listen to music, invite friends, etc. On the one hand, the agreed rule must be self-enforcing, i.e. a (correlated) equilibrium of the underlying game. On the other hand, any such rule can be agreed upon. But even if an agreement

*E-mail address:* hannu.vartiainen@yjs.fi.

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is not reached, the roommates still have to live together and play the underlying game. Hence, *some* equilibrium will be played even in the absence of an agreement.

To apply bargaining theory to the roommates' negotiation problem, we must know the disagreement payoffs. But if no agreement is reached, then we only know that *some* equilibrium is played. Hence, bargaining theory is of limited use. A theory which does not rely on the existence of a disagreement outcome, or generates one *endogenously,* would solve the problem. Coming up with such a theory is the aim of this paper.

In the arbitration literature, which sees a bargaining solution as a scheme implemented by an impartial and "fair" arbitrator (see e.g. Luce and Raiffa, 1992, and Border and Segal, 1997), the question of how to select the disagreement point is central. Since a typical solution is sensitive to the position of the disagreement point, the success of the arbitration process depends on how this question is solved.[1] However, in this literature, no explanation is typically given as to *why* the chosen disagreement point should be implemented in the absence of agreement.[2] In our view, this calls for an axiomatic treatment.

We analyze the collective choice through an *extended solution* $F$ which specifies, under any collective choice problem $U$, a nonempty, closed, irreflexive subset of $U \times U$. An element of the extended solution $F(U)$, say $(s, r)$, is an ordered pair of utility vectors where the first entry, $s$, is referred to as the *solution* and the second, $r$, as the *reference outcome*.

The solution should be viewed as an acceptable compromise given that in the absence of cooperation players would end up to the reference outcome. Negotiation, in our framework, could be thought as a process that reduces the size of the collective choice problem to a form where an indisputable decision can be made. The binary choice problem between the solution and the reference outcome could be viewed as the final stage of the negotiation process.

Technically, why should we not focus directly on the solution since that is what we are primarily interested in? The problem is that reasonable axiomatizations tend to be too strong to allow any solution—as has become painfully clear in the social choice literature—or too loose to pin down a well defined solution. A reference outcome gives a necessary degree of freedom that permits us to circumvent this problem. The key property of a reference outcome is that it can be used to compare utilities across outcomes. Hence, with a reference outcome the collective choice problem moves from the preference aggregation domain to the utility comparison domain. As argued by Conley et al. (1997, 2000) and Thomson (1981), this difference is crucial (see Section 5 for further discussion).[3]

The axioms we impose on the extended solution are analogues to those of Nash (1950), i.e. *Pareto-optimality, symmetry*, *independence of irrelevant alternatives* (IIA), and *scale invariance*. As there is no fixed disagreement point, also the axioms on the extended solution need to be extended. The extended axioms now assume that the selection process of the *combination* of a solution and a reference point is governed by unified principles.

Our result is the following: The unique extended solution meeting the axioms contains $(s, r)$ under $U$ if and only if $(s, r)$ maximizes the Nash product on $U$ with respect to *$s$ and $r$*. Such

[1] A related question is analyzed by Border and Segal (1997). They keep the disagreement point fixed but axiomatize the preferences of an arbitrator over possible bargaining solutions. Their analysis gives support to the Nash solution.

[2] In particular, Kalai and Rosenthal (1978) (see also Rosenthal, 1978) allow players to choose the disagreement point noncooperatively. They show that, for a fixed, well behaved arbitration scheme, the induced game has a unique equilibrium outcome.

[3] Conley et al. (2000) (see also Sen, 1982, and Myerson, 1978) show that no social choice rule satisfies the standard axioms of bargaining theory.

solution, $N$, is called the *extended Nash solution*. Note that $N$ is single-valued whenever $U$ is strictly convex. The key feature of the extended Nash solution is that it generates the reference point endogenously.

To sketch the main argument, suppose that $U$ is strictly convex. Then $N(U) = \{(s, r)\}$. Note that $s$ is the Nash solution of the bargaining problem $(U, r)$ where $r$ is the disagreement point, and $-r$ is the Nash solution of the bargaining problem $(-U, -s)$. Thus $s$ and $r$ lie in the boundary of $U$, and the slope of the tangents at *both* of these points is $(s_1 - r_1)/(s_2 - r_2)$. Now we can use Nash's geometric argument to both sides of the problem *simultaneously*.

Section 2 specifies the model and introduces the axioms. Section 3 defines the extended Nash solution, and establishes the characterization. Section 4 contains some discussion on the axioms and the concluding section relates the paper to the literature.

## 2. The model

Let

$$\mathcal{U} = \{U \subset \mathbb{R}^2\colon\ U \text{ is compact, convex, and contains } u, v \text{ such that } u > v\}$$

be the domain of utility possibility sets.[4] Correspondence $F : \mathcal{U} \to \mathbb{R}^2 \times \mathbb{R}^2$ is an *extended solution* if $F(U)$ is a nonempty, closed, and irreflexive[5] subset of $U \times U$, for all $U \in \mathcal{U}$. For any ordered pair $(s, r) \in F(U)$, the first entry, $s$, is called a *solution* and the second entry, $r$, a *reference point*.

The axioms we impose on the extended solution are direct analogues to those of Nash (1950). Use the notation $au + b = (a_1u_1 + b_1, a_2u_2 + b_2)$, for $u = (u_1, u_2) \in \mathbb{R}^2$, $a = (a_1, a_2) \in \mathbb{R}^2$, and $b = (b_1, b_2) \in \mathbb{R}^2$. When both coordinates of $u$ are multiplied by a scalar $\lambda$, write $\lambda u$. Also write $aX + b = \{au + b\colon\ u \in U\}$, for any $U \subset \mathbb{R}^2$. In particular, write $-1U = -U$.

EINV (Extended Scale Invariance): $F(aU + b) = aF(U) + b$, for all $a \in \mathbb{R}^2_{++}$, $b \in \mathbb{R}^2$, and for all $U \in \mathcal{U}$.

EIIA (Extended Independence of Irrelevant Alternatives): $F(V) \cap (U \times U) \neq \emptyset$ and $U \subseteq V$ imply $F(V) \cap (U \times U) = F(U)$, for all $U, V \in \mathcal{U}$.

Thus if the intersection of a smaller problem with the solution of the bigger problem is nonempty, then the solution under the smaller problem should be equal to the intersection. An analogous independence axiom is used by Mariotti (1999).

Set $U$ is *symmetric* if $(u_1, u_2) \in U$ implies $(u_2, u_1) \in U$.

ESYM (Extended Symmetry): If $U$ is symmetric, then $(s_1, r_1) = (s_2, r_2)$ for some $(s, r) \in F(U)$.

Denote the Pareto frontier by $P(U) = \{u \in U\colon\ u \not\leqslant v, \text{ for all } v \in U\}$.

PAR (Pareto-optimality) $s \in P(U)$, for all $(s, r) \in F(U)$, for all $U \in \mathcal{U}$.

[4] Vector inequalities: for $u, v \in \mathbb{R}^2$, $u \geqq v$ means $u - v \in \mathbb{R}^2_+$; $u \geqslant v$ means $u \geqq v \neq u$; $u > v$ means $u - v \in \mathbb{R}^2_{++}$.

[5] I.e. $(u, v) \in F(U)$ implies $u \neq v$.

## 3. The characterization of the extended Nash solution

**Definition 1.** The *extended Nash solution* is the correspondence $N:\mathcal{U} \to \mathbb{R}^2 \times \mathbb{R}^2$ such that, for all $U \in \mathcal{U}$,

$$N(U) = \underset{(u,v)\in U\times U,\, u\geqq v}{\arg\max} \quad (u_1 - v_1)(u_2 - v_2). \tag{1}$$

Correspondence $N$ is indeed an extended solution, i.e. nonempty, closed, and $(u, v) \in N(U)$ implies $u > v$. Note that $N(U)$ is single valued if $U$ is strictly convex. The difference between the extended and the standard Nash solution is that the former allows $r$ to be determined *endogenously,* and simultaneously with $s$, for any given $U$.[6]

The (topological) *boundary* of $U$ is denoted by $\bar{U}$. Denote by $[u, v]$ the closed line segment between points $u, v \in \mathbb{R}^2$.

The next lemma is an important and simple consequence of EIIA and EINV. It says that if $(s, r)$ belongs to $F(U)$, then $r$ must be an end point of any line segment in $U$ which contains $s$ and $r$.

**Lemma 2.** *Let $F$ meet EIIA and EINV. Take any $U \in \mathcal{U}$ and $(s, r) \in F(U)$. If $r \in [s, t]$, for any $t \in U$, then $r = t$.*

**Proof.** Take $(s, r) \in F(U)$. Adopt normalization $s = \mathbf{0} := (0, 0)$. Since $F(U)$ is irreflexive, $r \neq \mathbf{0}$. Let $[\mathbf{0}, t]$ be the maximal line segment $[\mathbf{0}, t'] \subseteq U$ such that $r \in [\mathbf{0}, t']$. Then there is a unique $\theta \in (0, 1]$ such that $r = \theta t$. It suffices to show that $\theta = 1$ (see Fig. 1).

Since $F$ is closed, there is a unique $\lambda > 0$ and $(\mathbf{0}, \bar{r}) \in F(U)$ such that $\bar{r} = \lambda t$, and such that $r' = \lambda' t$ and $(\mathbf{0}, r') \in F(U)$ implies $\lambda' \geqslant \lambda$. Observe that $\lambda \leqslant 1$. By EINV, then, $(\mathbf{0}, \lambda\bar{r}) \in F(\lambda U)$. By construction, $(\mathbf{0}, \bar{r}) \in F(U)$, $(\mathbf{0}, \bar{r}) \in \lambda U \times \lambda U$, and $\lambda U \subset U$. Hence, by EIIA, $(\mathbf{0}, \lambda\bar{r}) \in$

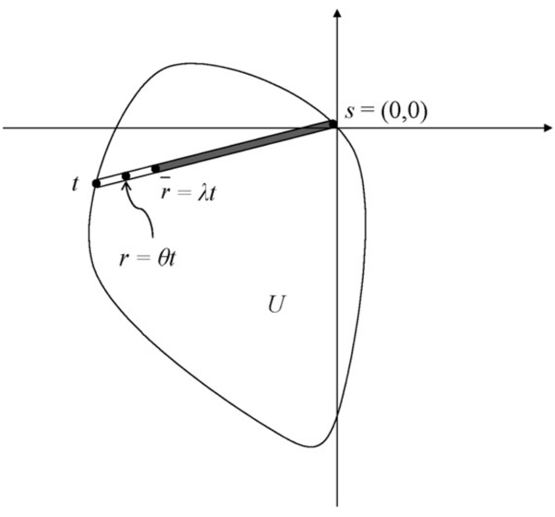


Fig. 1.

[6] As pointed out by the Referee, $N$ is upper semi-continuous but not continuous (in the Hausdorff metric).

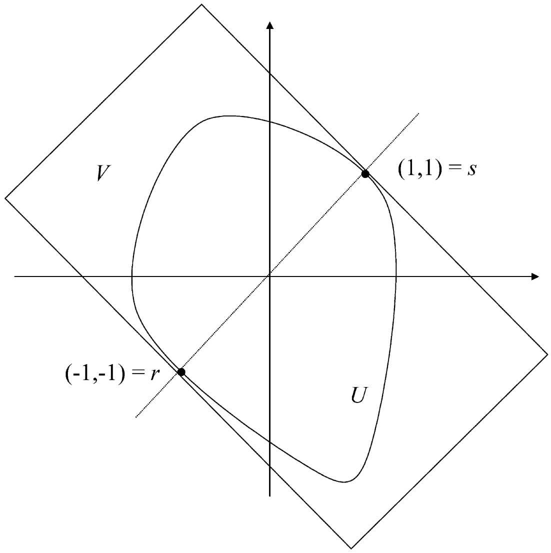


Fig. 2.

$F(U)$. Now $\lambda\bar{r} = \lambda(\lambda t) = \lambda^2 t$. By the definition of $\bar{r}$, $\lambda^2 \geqslant \lambda$. Thus $\lambda \geqslant 1$. Since $\theta \geqslant \lambda$, we have $\theta = 1$, as desired. □

In particular, Lemma 2 implies that $r$ is in the boundary. Note that the result does *not* use PAR. Thus, with PAR both $s$ and $r$ must be in the boundary. The additional implication of Lemma 2—that $r$ cannot be in the middle of a flat part in the boundary—is needed in the main characterization when the utility set is not strictly convex.

It can now be seen that $F$ meets the axioms if and only if $N = F$ (see below for a formal proof). To see why $F$ contains $N$, take $(s, r) \in N(U)$. By EINV, we can assume the normalization $r = -s$. Now, if $V$ is the smallest *symmetric* rectangle that contains $U$, then $(s, -s) \in V$ (see Fig. 2, where normalization $s = (1, 1)$ is used). Since $F(V)$ contains elements in the boundary, it follows by ESYM that $(s, -s) \in F(V)$. By EIIA, then, $(s, -s) \in F(U)$.

The point is that one can find a symmetric rectangle whose sides are parallel to the tangents of $U$ at *both* $s$ and $r$ if and only if $(s, r) \in N(U)$. This permits us to use the geometric argument of Nash to determine $s$ and $r$ simultaneously.

Conversely, to see why $N$ contains $F$, take a strictly convex $U$ and suppose that $(s, r)$ is contained in $F(U)$ but not in $N(U)$. Construct then a set $U^\varepsilon$, the convex hull of $\{s, r, u', v'\}$ where $(u', v')$ is in the interior of $U \times U$, $\varepsilon$-close to $(u, v) \in N(U)$. Then $U^\varepsilon$ is a subset of $U$ and, since $u'$ and $v'$ are in the interior of $U$, $U^\varepsilon$ intersects with the boundary of $U$ only in $\{s, r\}$ (see Fig. 3). Using EIIA, $F(U^\varepsilon) = \{(s, r)\}$. But then the fact that $N(U^\varepsilon)$ is contained in $F(U^\varepsilon)$ contradicts the assumption that $N(U^\varepsilon)$ maximizes the Nash product in $U^\varepsilon$, for small enough $\varepsilon$.

**Theorem 3.** *Extended solution $F$ satisfies PAR, EINV, ESYM, and EIIA on $\mathcal{U}$ if and only if $F = N$.*

**Proof.** Checking that $N$ satisfies PAR, EINV, ESYM, and EIIA is routine. We show that if $F$ satisfies PAR, EINV, ESYM, and EIIA, then $N = F$.

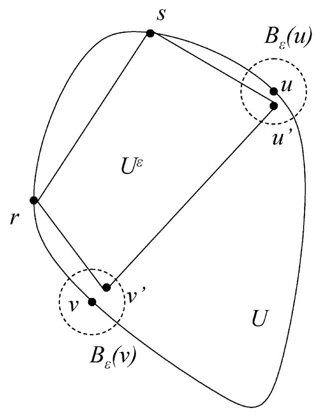


Fig. 3.

**Claim I.** *$N(U) \subseteq F(U)$, for all $U \in \mathcal{U}$.*

*Step* 1: Take any $(s,r) \in N(U)$. Since $s \neq r$, there are $a$, $b$ and $T = aU + b$ such that

$$a_i := \frac{2}{s_i - r_i}, \qquad b_i := -\frac{s_i + r_i}{s_i - r_i}, \quad \text{for } i = 1, 2.$$

Then, $(s,r)a + b = (\mathbf{1}, -\mathbf{1})$, for $\mathbf{1} := (1,1)$. By EINV, $(s,r) \in F(U)$ if and only if $(\mathbf{1}, -\mathbf{1}) \in F(T)$.

*Step* 2: Identify a symmetric rectangle $V$ such that $\{\mathbf{1}, -\mathbf{1}\} \subset \bar{V}$ and $T \subset V$. By the convexity and boundedness of $T$ and the fact that $(\mathbf{1}, -\mathbf{1}) \in N(T)$, such $V$ exists.

*Step* 3: By ESYM, PAR, and Lemma 2, $(\mathbf{1}, -\mathbf{1}) \in F(V)$. Since $T \subseteq V$ and $(\mathbf{1}, -\mathbf{1}) \in T \times T$, it follows by EIIA that $(\mathbf{1}, -\mathbf{1}) \in F(T)$. Thus $(s,r) \in F(U)$.

**Claim II.** *$F(U) \subseteq N(U)$, for all $U \in \mathcal{U}$.*[7]

Suppose that $(s,r) \in F(U) \setminus N(U)$. By PAR, $s \not\leqslant r$. *If* $s \geqslant r$, then

$$(u_1 - v_1)(u_2 - v_2) > (s_1 - r_1)(s_2 - r_2), \quad \text{for some } (u,v) \in N(U). \tag{2}$$

By Lemma 2, since $(u,v) \in \bar{U} \times \bar{U}$ and $(s,r) \neq (u,v)$, it can*not* be that $s, r \in [u,v]$. Since $s, r, u, v \in U$, and $U$ is convex, it contains a nonempty interior. Moreover, there are $\varepsilon$-neighborhoods $B_\varepsilon(u)$ and $B_\varepsilon(v)$ of $u$ and $v$, respectively, such that $(s,r) \notin B_\varepsilon(u) \times B_\varepsilon(v)$. One can find $u'$ and $v'$ in the interior of $U$ such that $u' \in B_\varepsilon(u)$ and $v' \in B_\varepsilon(v)$. Construct set $U^\varepsilon$, the convex hull of $\{u', v', s, r\} \subset U$. Since $u', v'$ are in the interior of $U$, $U^\varepsilon \subset U$ and $U^\varepsilon \in \mathcal{U}$. By construction, $\bar{U} \cap U^\varepsilon \subseteq [s,r]$. Since, by Lemma 2, $F(U) \subset \bar{U} \times \bar{U}$, necessarily $F(U) \cap (U^\varepsilon \times U^\varepsilon) \subseteq [s,r] \times [s,r]$. By EIIA, $F(U^\varepsilon) \subseteq [s,r] \times [s,r]$.

By Claim I, $N(U^\varepsilon) \subseteq F(U^\varepsilon)$, thus $N(U^\varepsilon) \subseteq [s,r] \times [s,r]$. This implies that either $s > r$ or $s < r$. Since $s \not\leqslant r$, indeed $s > r$. Therefore

$$(s_1 - r_1)(s_2 - r_2) \geqslant (s_1' - r_1')(s_2' - r_2'), \quad \text{for all } (s', r') \in F(U^\varepsilon)$$

and, consequently, $(s,r) \in N(U^\varepsilon)$. But for small enough $\varepsilon > 0$, by (2),

$$(u_1' - v_1')(u_2' - v_2') > (s_1 - r_1)(s_2 - r_2),$$

a contradiction. □

[7] I thank the Associate Editor for pointing out a flaw in the previous argument.

To see that the axioms are independent, let us relax them one at a time. Of course, $N$ remains a solution with a less restrictive set of axioms. We give an example of another solution that becomes admissible, which proves the independency.[8]

- Without PAR, the *converse* extended Nash solution $-N(-U)$, for all $U$, becomes admissible since we can now restrict the second coordinate, rather than the first one, by the Pareto-condition.
- For any $\alpha \in (0, 1)$, define an *asymmetric* extended Nash solution:

  $$N^{\alpha}(U) = \underset{(u,v)\in U\times U,\, u\geqq v}{\arg\max} \; (u_1 - v_1)^{\alpha}(u_2 - v_2)^{1-\alpha}, \quad \text{for all } U \in \mathcal{U}.$$

  Checking that $N^{\alpha}$ meets PAR and EINV is routine. EIIA follows from the convexity of $U$. Thus, without ESYM any $N^{\alpha}$ becomes admissible.[9]
- Construct the *extended Kalai–Smorodinsky* solution $KS(U) = (s, r)(U)$, for all $U$, as follows: $s(U)$ and $r(U)$ are the maximal point and the minimal point, respectively, in $U$ on the line segment joining the *utopia* point $\bar{u}(U)$ and the *anti-utopia* point $\underline{v}(U)$, where $\bar{u}_i(U) = \max\{u_i\colon u \in U\}$ and $\underline{v}_i(U) = \{u_i\colon u \in U\}$, for each $i$. If $U$ is symmetric, then $s(U)$ and $r(U)$ lie on the 45°-line. By construction, $s(U) \in P(U)$. Hence $KS$ meets EINV, PAR, and ESYM, and becomes an admissible solution when EIIA is removed.
- To see the independency of EINV, define the *extended utilitarian* solution

  $$\underset{(u,v)\in U\times U,\, u\geqq v}{\arg\max} \; \big[(u_1 + u_2) - (v_1 + v_2)\big], \quad \text{for all } U \in \mathcal{U}.$$

  The solution to this program coincides with $\{(u, v) \in (U \times U) \cap (\bar{V} \times \bar{V})\colon u \geqslant v\}$, where $\bar{V}$ is the boundary of the smallest *symmetric* rectangle that contains $U$. The extended utilitarian solution clearly satisfies ESYM, PAR, and EIIA.

## 4. Discussion

PAR and the irreflexivity of $F$ (i.e. $(u, u) \notin F(U)$) can be replaced in the characterization with a single condition of *Pareto-rankability*, i.e. $s \geqslant r$ for all $(s, r) \in F(U)$. To see this, note that Lemma 2, which only needs the irreflexivity of $F$, can be applied to establish that $s$ also has to be in the boundary. Theorem 3 uses PAR two times, first to guarantee that $s$ is in the boundary and to Pareto-rank $s$ and $r$ when $V$ is a symmetric rectangle, and second to guarantee that $s \nleqslant r$. These as well as the irreflexivity of $F$ are implied by the *assumption* $s \geqslant r$. In other words, $s$ need not be *assumed* to Pareto-dominate any other outcome than the reference outcome.

That PAR can be replaced with Pareto-rankability in the characterization of the extended Nash bargaining solution is analogous to the observation of Roth (1977) that PAR can be replaced with strict individual rationality in the characterization of the Nash bargaining solution. The condition of Pareto-rankability also matches neatly with the story in the introduction that was used to motivate the extended solution: players seek to simplify the problem until it reaches a binary form where a unanimous decision can be made.

An earlier version of this paper (Vartiainen, 2004) uses a weaker version of EIIA: $(s, r) \in F(V) \cap (U \times U)$ and $U \subseteq V$ implies $(s, r) \in F(U)$. Under the assumption that $F$ is convex

[8] Which by no means does mean that the examples are the only additional admissible solutions.

[9] However, also other extended solutions become available.

valued, the result of this paper can be recovered. However, Lemma 2 would no longer be true, and one needs to prove that $r$ is in the boundary in a roundabout way.

## 5. Relations to the literature

This paper argues that there is a natural way to extend the bargaining framework to the environments without a prespecified disagreement outcome. Our approach is based on endogenous reference point determination. We show that an extended solution, a combination of a solution and a reference point, is uniquely characterized by the appropriately adjusted axiomatization of the standard Nash solution.

Our approach is closely related to Conley et al. (1997, 2000). They argue that the existence of a reference point is the key differentiating element between the bargaining theory and the social choice theory. A crucial feature of the reference point is that it allows comparison of utilities across outcomes. Conley et al. point out that *any* scale invariant reference function permits comparison axioms (such as the IIA) which, in turn, make characterization results possible. Following Thomson (1981), they specify conditions for a reference function that together with an appropriate axiomatization give rise to a unique solution. The induced solution is then dependent on the axioms that define it *and* on the reference function. However, since there are many possible reference functions, any given axiomatization is consistent with a family of solutions. This raises the question of how to choose the reference function, for any given axiomatization, since this choice ultimately affects the solution.

Another strand of literature asks the solution to be *robust* against perturbations of the disagreement point. Dagan et al. (2002) show that IIA can be replaced with three conditions, independence of non-individually rational alternatives (INIR), twisting (a monotonicity requirement), and disagreement point convexity (DPC) in the characterization of the Nash solution. Chun and Thomson (1990) replace IIA with INIR, DPC and a continuity condition. Peters and van Damme (1991) assume individual rationality, DPC and a starshaped inverse condition (in the language of Thomson, 1994). However, this literature does not address the question of how to choose the disagreement point.

Our approach leans heavily on the symmetry properties of the (extended) Nash bargaining solution. Mariotti (2000) approaches this feature of the Nash solution from another angle. He defines the axiom of maximal symmetry (if a relabeling of players does not shift the outcome selected by the solution outside the original utility space, then the outcome should belong to the solution) and shows that this axiom together with strong individual rationality can be used in place of IIA to characterize the Nash solution in the class of multivalued solutions.

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