Microeconomic Theory

Lecture 2

Consumer behavior under WA

- Use choice theory to study consumer theory.
- Exogenous: prices and income
- Endogenous: consumption choices.
- Endogenous variables change in response to exogenous variables.
- The key question: *How do consumption choices, i.e. demand, respond to change in prices?*

- How far can we get by applealing to WA?

Interpretation of the framework

- X the choice set is now interpret as the consumption set.
- We take $X = \mathbb{R}_{+}^{L}$, where $L \in \mathbb{N}$.
- $x = (x_1, ..., x_L)$, where each $x_l \in \mathbb{R}_+$ for each $l \in \{1, ..., L\}$.
- Goods are divisible, the choice set is convex.

- Conventions on matrices and operators: Let $x, y \in \mathbb{R}^L_+$ and $f : \mathbb{R}^L_+ \to \mathbb{R}^L_+$. Then

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_L \end{bmatrix}, \; Df(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_L} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_L}{\partial x_1} & \cdots & \frac{\partial f_L}{\partial x_L} \end{bmatrix}.$$

Let $y \in \mathbb{R}^L_+$, and let X be an $L \times L$ matrix. Then

$$y^T x = y \cdot x = \sum_{l=1}^{L} y_l x_l.$$

Feasible set B

- $B \in \mathcal{B}$ defines the *budget set* of a consumer

- The feasible budget is defined by *prices* p and *disposable income* or wealth w.

- $\mathcal{B}$ gives the set of all possible budget situations, i.e. all possible prices and incomes

- A budget feasible consumption is one that can be purchased with the disposable income.

- In classical consumer theory (and in this lecture), we assume that prices are *linear*:
  - The price of an additional unit of good l is independent of the amount of good l purchased.
  - The price of an additional unit of good k is independent of consumptions of goods $l \neq k$.
  - Rules out quantity discounts and offers like 'Buy CPU from us and get printer for 50% off'.
  - Rules out progressive taxes, exemptions etc.
  - $p \in \mathbb{R}^L_+,\ w \in \mathbb{R}_+$.

- Walrasian budget set:

$$\begin{aligned} B &= \{x \in X : p \cdot x \leq w\} \\ &= \left\{ x \in X : \sum_{l=1}^{L} p_l x_l \leq w \right\} \end{aligned}$$

- Since B is determined by p and w, we write $B(p, w)$.

- Note: $B(p, w)$ rules out nonlinearities, indivisibilities, uncertainties, and interdependencies between individuals

The choice rule

- Walrasian demand correspondence $x\left(p,w\right)$: consumption choice given the budget set $B(p,w)$.

- Specifies the consumption of each commodity $l = 1, ..., L$.

- Is defined for all p and w. Hence

$$x : \mathbb{R}_+^L \times \mathbb{R}_+ \rightarrow \mathbb{R}_+^L.$$

- We *assume* that $x\left(p,w\right)$ is single valued function.

- Two additional assumptions on $x(p, w)$:

- Nothing's wasted.

Assumption 1 The Walras' Law: $p \cdot x(p, w) = w,$ for all p, w.

- Since Walras' Law holds for all p and w, it holds as an *identity.*

- Only consumption matters.

- Since $B(p, w) = B(\lambda p, \lambda w)$:

Assumption 2 Homogeneity: $x(\lambda p, \lambda w) = x(p, w)$, for all $\lambda > 0$ and all p, w.

- That is, $x(p, w)$ is homegenous of degree 0 in (p, w).

- No money illusion.

- The effect of units on the consumer's perception of opportunities.

- Notice that $p \cdot x(p, w)$ is a function of $L + 1$ variables, i.e. $p \cdot x(p, w) : \mathbb{R}_{+}^{L+1} \longrightarrow \mathbb{R}_{+}$.

- Denote by $D_p x(p, w)$ the derivative of $x(p, w)$ with respect to p and by $D_w x(p, w)$ the derivative with respect to w (assume derivatives exist). Then

$$
\begin{aligned}
D_p x(p, w) &: \mathbb{R}_{+}^{L+1} \longrightarrow \mathbb{R}_{+}^{L \times L}, \\
D_w x(p, w) &: \mathbb{R}_{+}^{L+1} \longrightarrow \mathbb{R}_{+}^{L}.
\end{aligned}
$$

- Implications of Walras' law (which is an identity):
  - Engel aggregation: increased wealth is consumed

$$p \cdot D_w x(p,w) = 1. \tag{1}$$

  - Cournot aggregation: total expedinture independent of prices

$$p \cdot D_p x(p,w) + x\left(p,w\right) = 0. \tag{2}$$

- Note that, by homogeneity, for all $l = 1, ..., L$,

$$\begin{aligned} \left.\frac{d}{d\lambda} x_l\left(\lambda p, \lambda w\right)\right|_{\lambda=1} &= \\ D_p x_l\left(p,w\right) \cdot p + D_w x_l\left(p,w\right) w &= 0. \end{aligned} \tag{3}$$

- Denote the budget share of l by

$$b_l(p,w) = \frac{p_l x_l(p,w)}{w}, \text{ for all } l = 1, ..., L.$$

Denote the price and income elasticities by

$$\begin{aligned} \varepsilon_{lk}(p,w) &= \frac{\partial x_l(p,w)}{\partial p_k}\frac{p_k}{x_l(p,w)}, \text{ for all } l = 1, ..., L, \\ \varepsilon_{lw}(p,w) &= \frac{\partial x_l(p,w)}{\partial w}\frac{w}{x_l(p,w)}. \end{aligned}$$

Then, by the Cournot and Engel aggregation rules,

$$\begin{aligned} \sum_{l=1}^{L} b_l(p,w)\,\varepsilon_{lk}(p,w) + b_k(p,w) &= 0, \\ \sum_{l=1}^{L} b_l(p,w)\,\varepsilon_{lw}(p,w) &= 1. \end{aligned}$$

By homogeneity, for all $l = 1, ..., L$,

$$\sum_{k=1}^{L} \varepsilon_{lk}(p, w) + \varepsilon_{lw}(p, w) = 0.$$

What is implied by WA, Walras Law, and homogeneity?

- Recall: WA if $x, y \in B$ and $x \in c(x)$, then $x, y \in B'$ and $y \in c(B')$ implies $x \in c(B')$.

- In the context of Walrasian budget sets, this has the form:

Axiom 1 $x\,(p, w)$ *satisfies WA if, for any two budget situations* (p, w) *and* (p', w'),

$$p \cdot x(p', w') \leq w \text{ and } x(p, w) \neq x(p', w'),$$

implies $p' \cdot x(p, w) > w'$.

- Recall that x is assumed to be single valued.

Implications of WA

- Recall the *Law of Demand*: $x\left(p, w\right)$ and p move in opposite directions.

- Intuition: The more something costs, the less one can afford it.

- However, not obvious: An increase in p_l changes relative prices (slope of the budget line) and effective wealth (i.e. is not feasible with new prices).

- *Wealth effect* not necessarily positive which implies overall ambiguity.

- To isolate the *substitution effect* we consider *compensated* price changes.

- Idea: Look at the effects of relative price changes by offsetting the associated wealth change, i.e. forcing the original consumption point to lie on the new budget line.

- Formally, (p', w') is a *compensated price change* from (p, w) if

$$p' \cdot x(p, w) = w'.$$

By Walras Law,

$$(p' - p) \cdot x(p, w) = w' - w,$$

or, in case of differential price changes, $dp \cdot x(p, w) = dw$.

Proposition 2 *Suppose $x(p, w)$ satisfies Assumptions 1-2. Then $x(p, w)$ satisfies WA if and only if, for any compensated price change (p', w'),*

$$(p' - p) \cdot [x(p', w') - x(p, w)] \leq 0 \tag{4}$$

where the inequality is strict whenever $x(p', w') \neq x(p, w)$.

- This might be called as the *compensated law of demand:* compensated demand and price move to opposite directions.

- Define the *substitution* or the *Slutsky matrix* $S(p, w)$ of $x(p, w)$:

$$S(p, w) = [D_p x(p, w) + D_w x(p, w)\, x(p, w)^T],$$

whose lk-component

$$\frac{\partial x_l(p, w)}{\partial p_k} + \frac{\partial x_l(p, w)}{\partial w} x_k(p, w)$$

specifies the pure substitution effect on l's demand from the p_k's price change, i.e. the total effect less the income effect.

- The compensated law of demand has implications for $S(p, w)$. Totally differentiating $x(p, w)$,

$$dx = D_p x(p, w) dp + D_w x(p, w) dw.$$

Compensating the consumer the amount $dw = x(p, w) \cdot dp$,

$$dx = D_p x(p, w) dp + D_w x(p, w) [x(p, w) \cdot dp].$$

Replacing for dx in the differential analog $dp \cdot dx \leq 0$ of (4), we have

$$dp \cdot [D_p x(p, w) + D_w x(p, w) x(p, w)^T] dp$$

or

$$dp \cdot S(p, w) dp \leq 0.$$

- This says that the $L \times L$ matrix $S(p, w)$ is *negative semidefinite*.

- Hence the diagonal element

$$\frac{\partial x_k(p,w)}{\partial p_k} + \frac{\partial x_k(p,w)}{\partial w} x_k(p,w)$$

must be nonpositive, i.e. the *substitution effect of a good to its own price is always nonpositive*.

Proposition 3 *Suppose* $x(p,w)$ *satisfies Assumptions 1-2 and the WA. Then, at any* (p,w) *the Slutsky matrix* $S(p,w)$ *is negative semidefinite.*

- Giffen goods.

- Does negative semidefinite $S(p, w)$ generated by $x(p, w)$ satisfying the Walras Law and Homogeneity restiction imply that WA is also satisfied?

- $S(p, w)$ is *not* symmetric in general.