Introduction to QCD

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The Standard Model

SU(3) x $SU(2)_L$ x U(1)

QCD

Electroweak

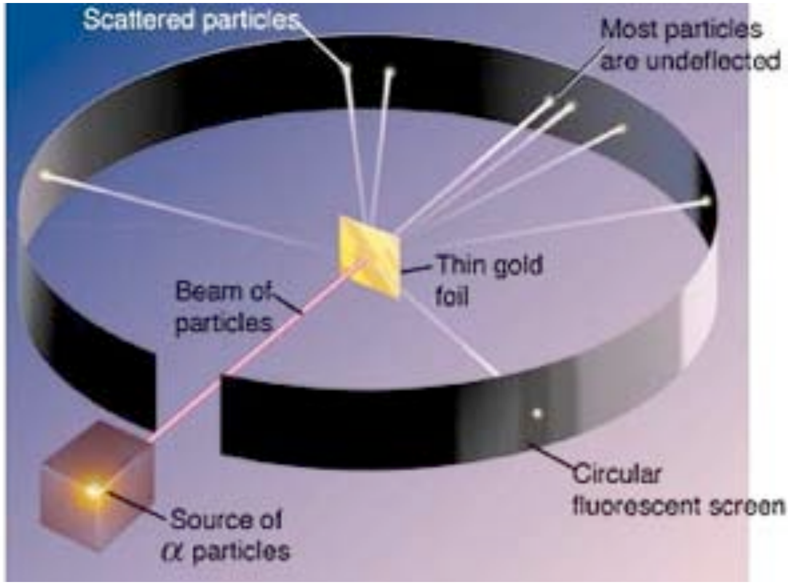
As simple as 1-2-3?

Not exactly: The strong, electromagnetic and weak interactions manifest themselves very differently in Nature

$SU(2)_L$ x U(1): Where does the lagrangian come from?

SU(3): What does the lagrangian do?

The Discovery of the Strong Interaction

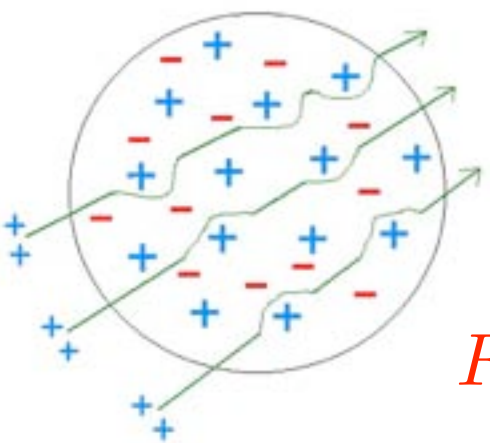


Rutherford's experiment 1911:

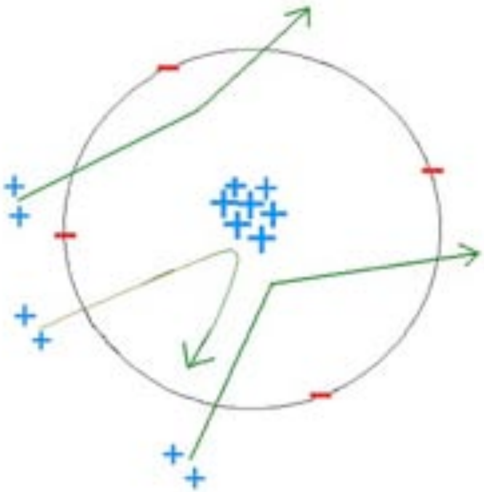
The positive charges in matter are located in a tiny nucleus, whose radius is ~ 10^{-5} of the atomic size.

⇒ There must be a strong, short-ranged force to counteract the Coulomb repulsion

Thomson's atom:

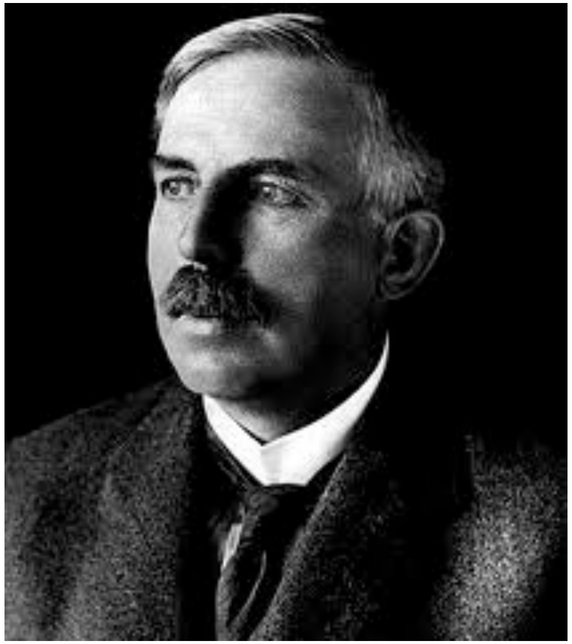


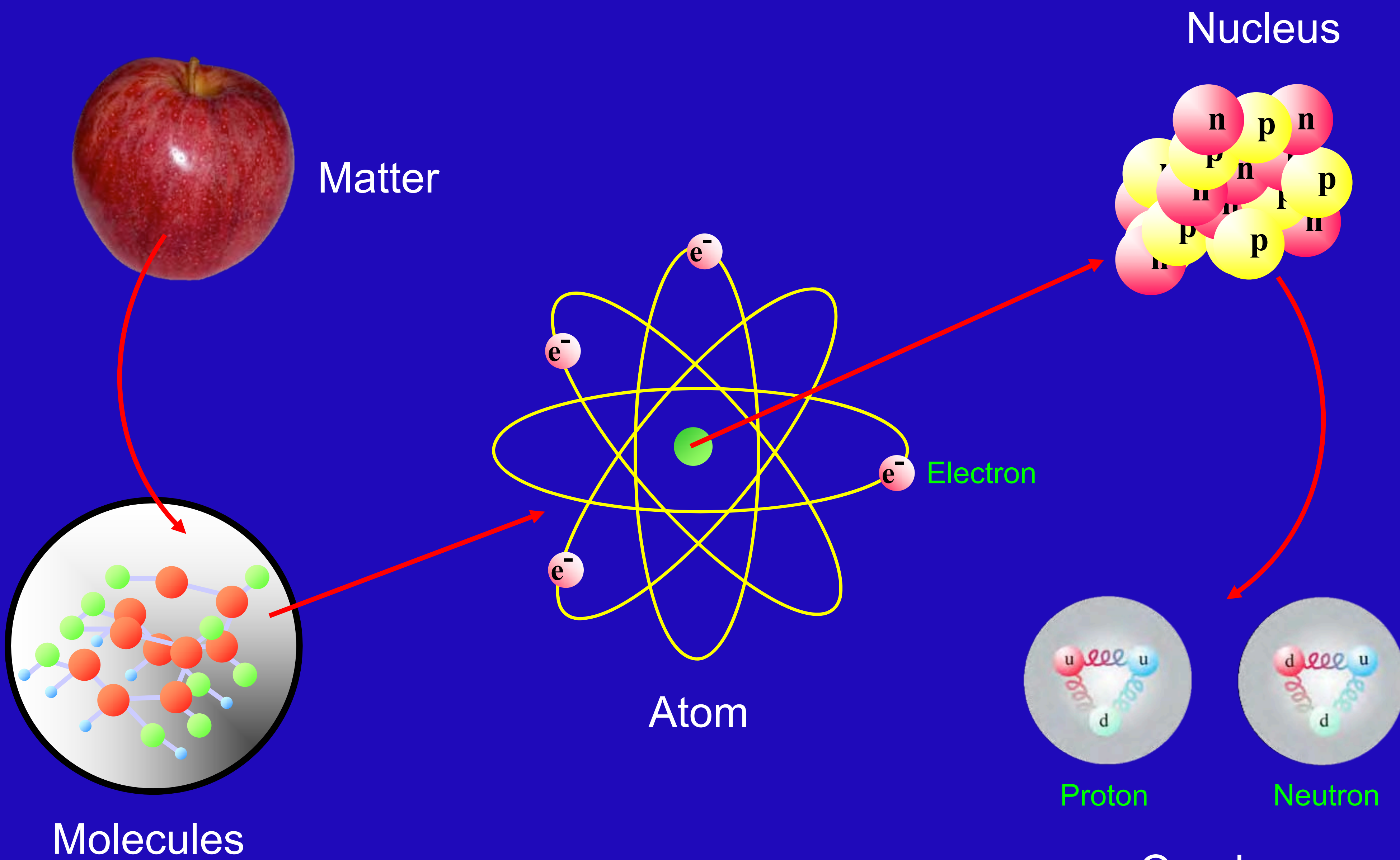
Rutherford's atom:



$$F = \frac{\alpha}{r^2}$$

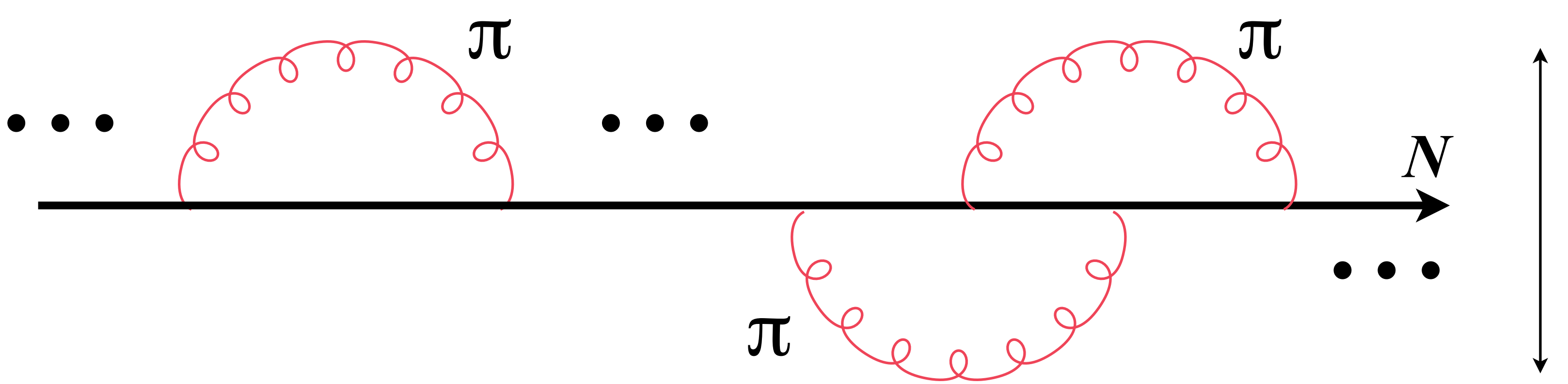
$$\frac{R_{nucleus}}{R_{atom}} \simeq 10^{-5}$$




Nucleus
Matter
Electron
Atom
Proton
Neutron
Molecules
Quarks

The Pion as a carrier of the strong force

In 1935 Hideki Yukawa suggested the existence of a new, strongly interacting particle "*U*" (later named the pion). The strength and range of the strong interaction could be understood as arising from pion exchange.



$R \lesssim 1/M_\pi$

Postulating a new particle was considered very bold in those days, when only a handful of elementary particles were known: photon, proton, neutron, electron, (neutrino).

It has been suggested that "social pressure" may have kept physicists in the West from a similar proposal.

Even today in QCD, the pion is special: It is a Goldstone boson.

Birth of Yang-Mills Theory (1954)

Quantum ElectroDynamics (QED) is invariant under local (space and time - dependent) gauge transformations:

Electron field: $\psi(x) \to e^{ie\Lambda(x)}\psi(x)$

$\Lambda(x)$ may be any regular function

Photon field: $A_\mu(x) \to A_\mu(x) - \partial_\mu \Lambda(x)$ $\Leftarrow$ As in classical ED!

In 1954, Yang and Mills generalized this local U(1) gauge symmetry to the SU(2) group of isospin, with the proton and neutron forming an SU(2) isospin doublet just as in Yukawa's theory: $\begin{pmatrix} p \\ n \end{pmatrix}$

This established the structure of non-abelian gauge symmetry.
Nature found, however, different uses of YM theories.

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

C. N. Yang † and R. L. Mills
Brookhaven National Laboratory, Upton, New York

(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistant with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a **b** field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The **b** field satisfies nonlinear differential equations. The quanta of the **b** field are particles with spin unity, isotopic spin unity, and electric charge $\pm e$ or zero.

Physical Review **96** (1954) 191

For a local gauge transformation defined by an SU(2) matrix $U(x)$, Yang and Mills found that the theory is symmetric provided the fields transform as:

Matter field: $\psi(x) \to U(x)\psi(x)$

Gauge field: $A_\mu(x) \to U(x)A_\mu(x)U^\dagger(x) - \frac{i}{g}U(x)\partial_\mu U^\dagger(x)$

g is the same for all ψ!

The same rule holds for any group, such as SU(3). In QED,

$U(x) = e^{ie\Lambda(x)}$ Then $U_1U_2 = U_2U_1$, i.e., all group elements commute, hence the $U(1)$ gauge symmetry is said to be abelian.

SU(2)

2×2 matrices: $\mathbf{U}\,\mathbf{U}^\dagger = \mathbf{U}^\dagger\,\mathbf{U} = \mathbf{1}$; $\det \mathbf{U} = 1$

$$\mathbf{U} = \exp\left\{ i\, \mathbf{T}^a \theta_a \right\} \quad ; \quad \mathbf{T}^a = \mathbf{T}^{a\dagger} \quad ; \quad \mathrm{Tr}\,(\mathbf{T}^a) = 0 \quad ; \quad a = 1, \cdots, 3$$

Commutation Relation: $\left[\mathbf{T}^a, \mathbf{T}^b\right] = i\,\varepsilon^{abc}\,\mathbf{T}^c$

Fundamental Representation: $\mathbf{T}_F^a = \frac{1}{2}\sigma^a$ **Pauli**

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad ; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

SU(3)

$$[\mathbf{T}^a, \mathbf{T}^b] = i\, f^{abc}\, \mathbf{T}^c$$

Fundamental Representation:

$$\mathbf{T}_F^a = \frac{1}{2}\lambda^a$$

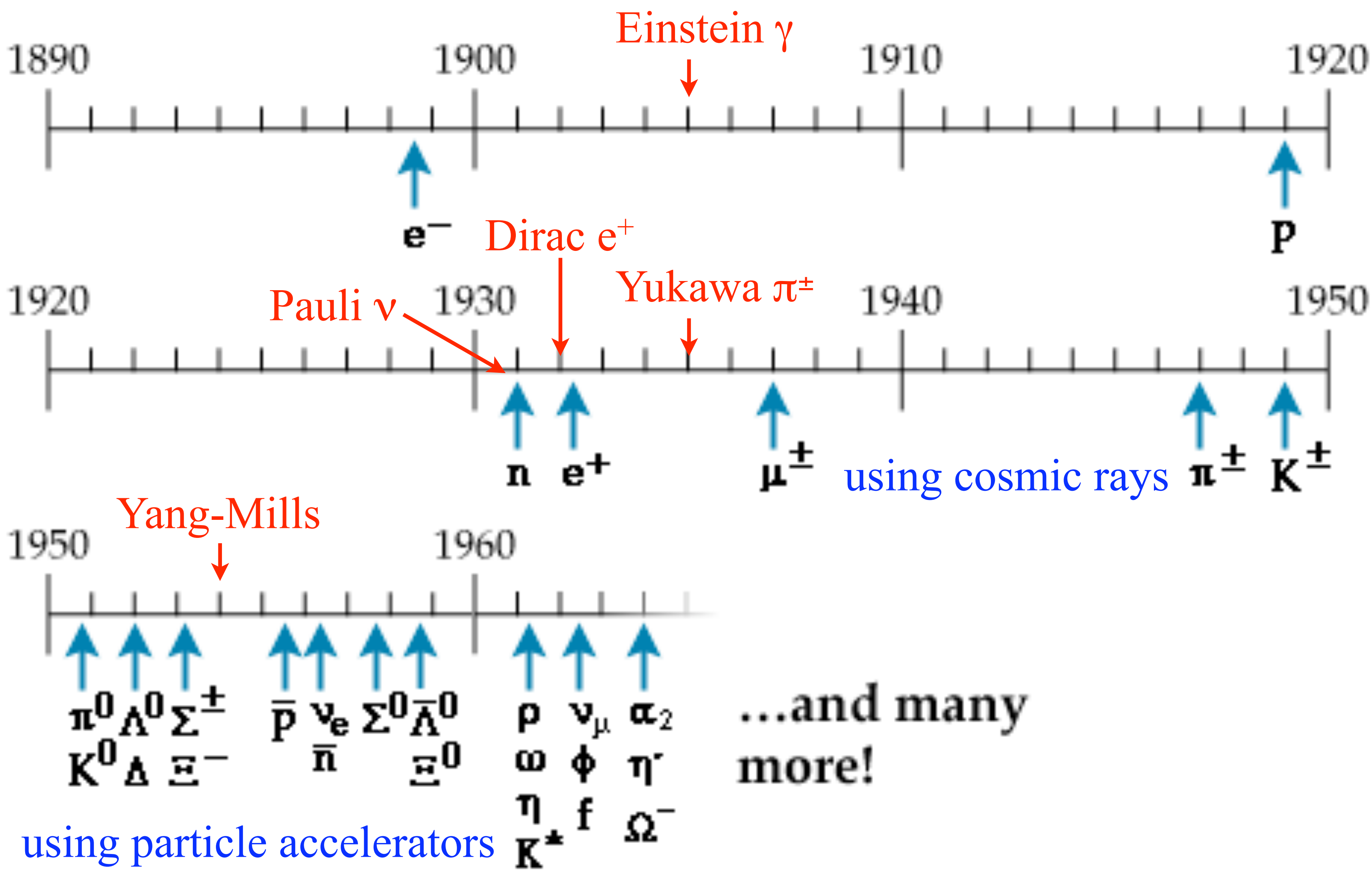
Gell-Mann

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \; ; \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \; ; \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \; ; \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

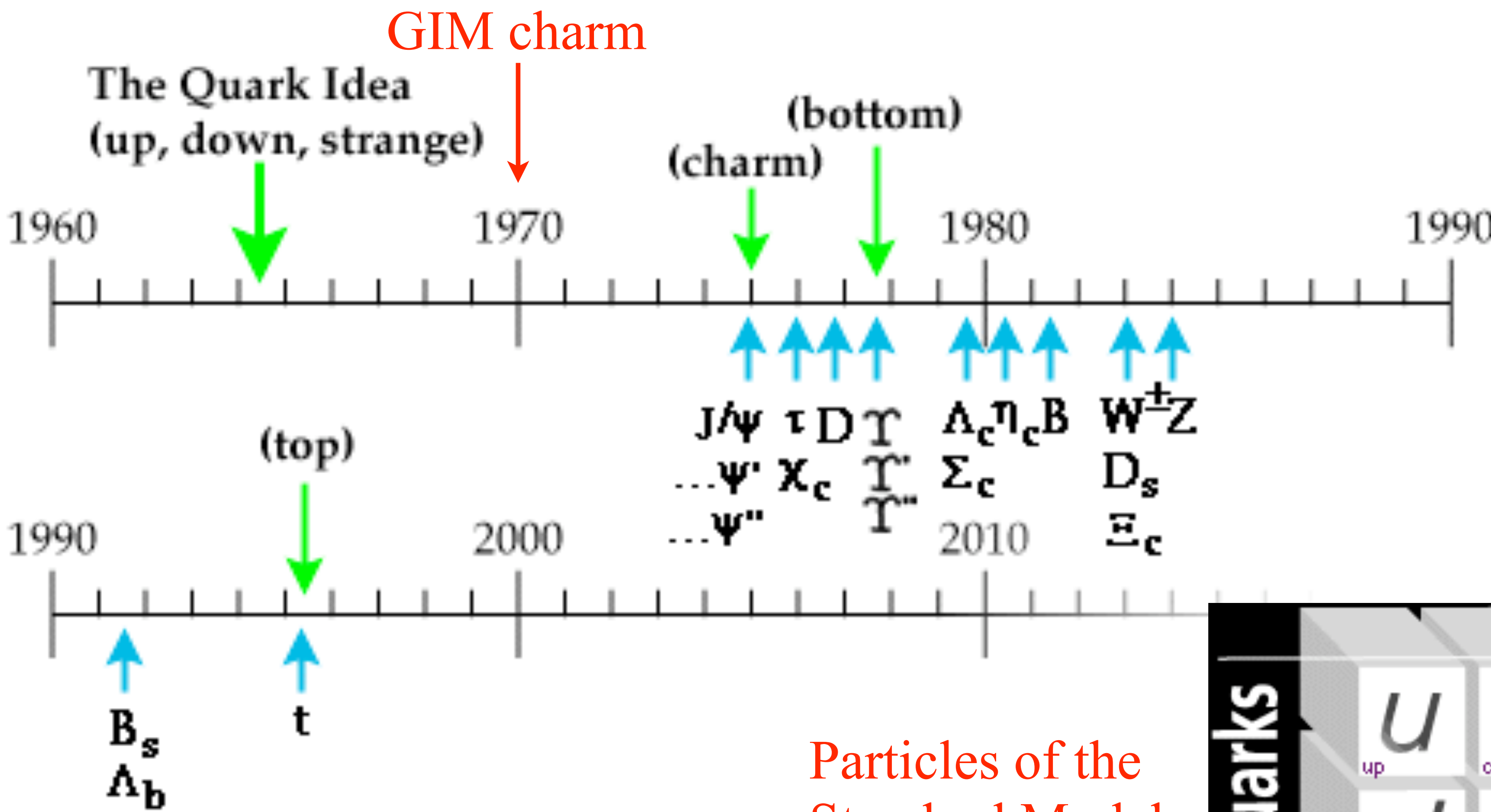
$$\frac{1}{2} f^{123} = f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{\sqrt{3}} f^{458} = \frac{1}{\sqrt{3}} f^{678} = \frac{1}{2}$$

Particles found in Experiments

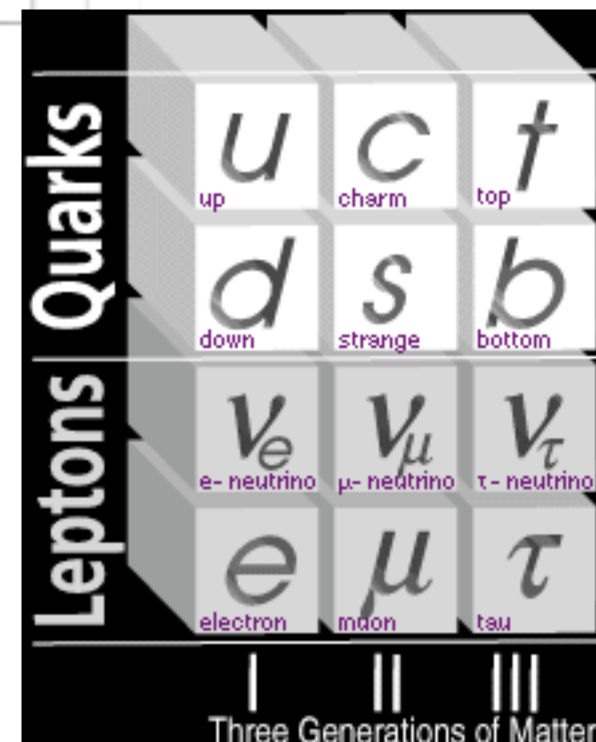


http://fafnir.phyast.pitt.edu/particles/

Hadron spectrum ⇒ Quarks



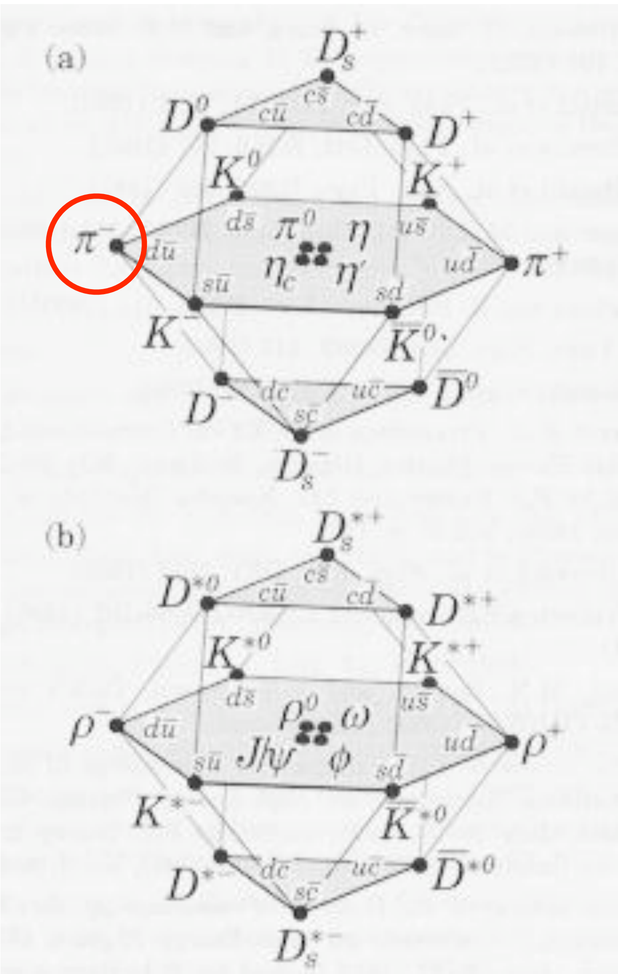
Particles of the Standard Model



All strongly interacting particles found in experiments (hadrons) have quantum numbers consistent with the Quark Model

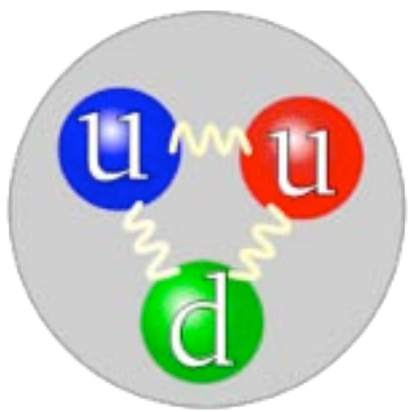
Quark Model classification of Hadrons

Mesons $q\bar{q}$



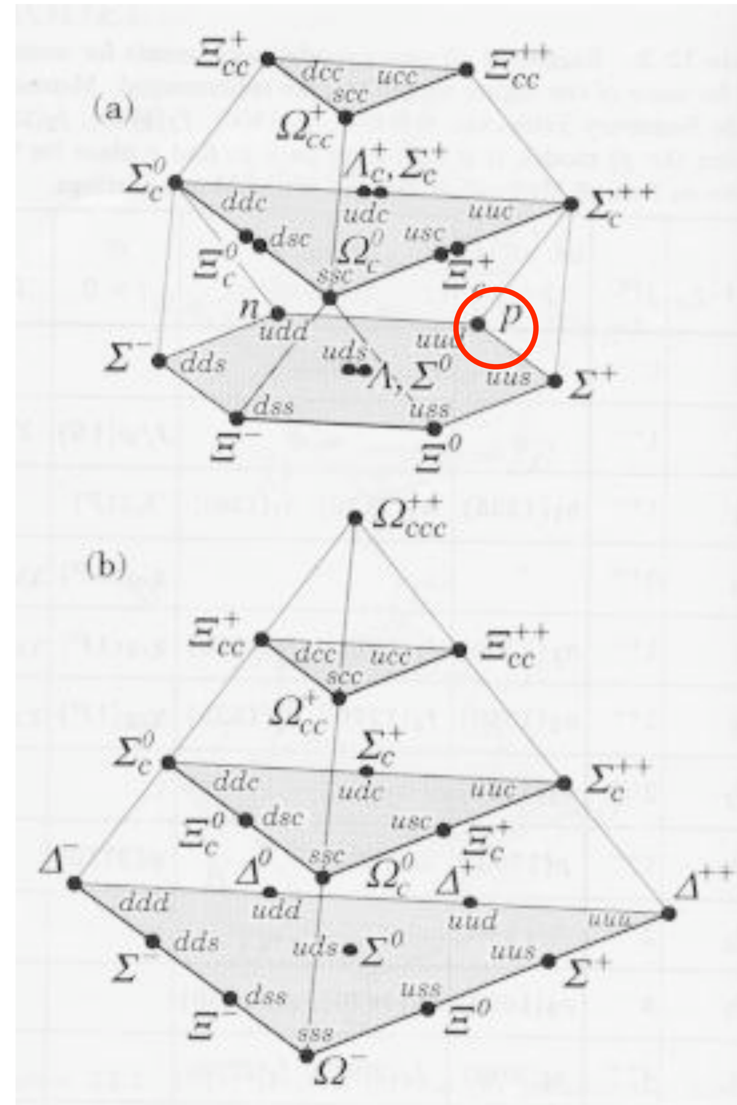
Hadrons may be formed with any combination of
$q = u,d,s,c,b,t$

π^+



p

Baryons qqq



Free quarks have never been seen:

Were quarks mere mathematical rules, or true particles? (Gell-Mann)

Quarks have Color

Three quark colors were introduced by O.W. Greenberg in 1964, to make the quark model of the proton compatible with the Pauli exclusion principle:

Baryon wave functions must be antisymmetric under the interchange of quarks. In the Quark Model, the space – spin wf is symmetric. The color wf is antisymmetric, rescuing the Pauli principle.

The antisymmetric color wave function (A,B,C = red, blue, yellow) means that the proton is a color singlet (does not change under gauge transformations).

$$|p\rangle = \sum_{A,B,C} \varepsilon_{ABC}\, q^A q^B q^C$$

$$U|p\rangle = |p\rangle$$

Mesons are also color singlets:

$$|\pi\rangle = \sum_{A} q^A \bar{q}^A \qquad\qquad U|\pi\rangle = |\pi\rangle$$

Quarks are for real: Pointlike scattering of electrons

High energy electrons scatter from pointlike quarks inside the proton:

$e + q \rightarrow e + q$ (in analogy to Rutherford's experiment)

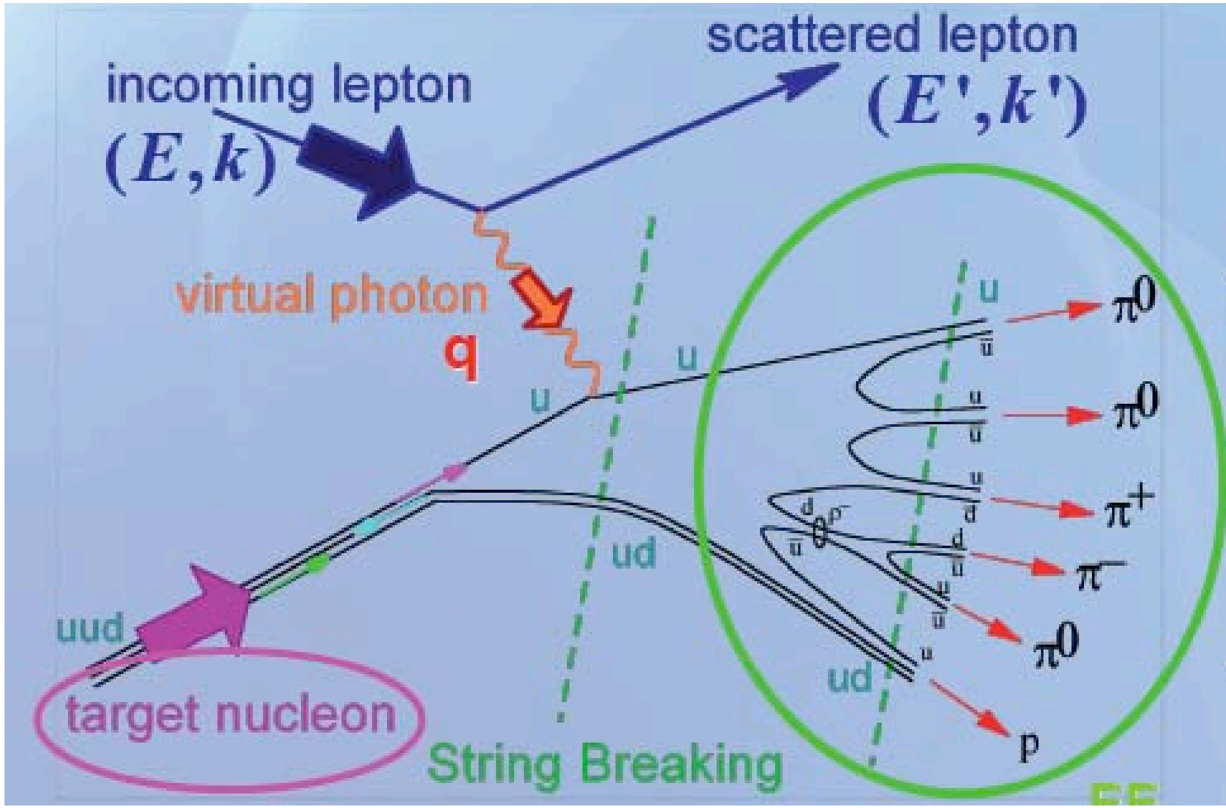
The struck quark flies out of the proton and "hadronizes" into a spray (jet) of hadrons (mostly pions).

At relativistic energies quark-antiquark pairs are created to ensure that all quarks end up as constituents of mesons or baryons.

Deep Inelastic Scattering (DIS):

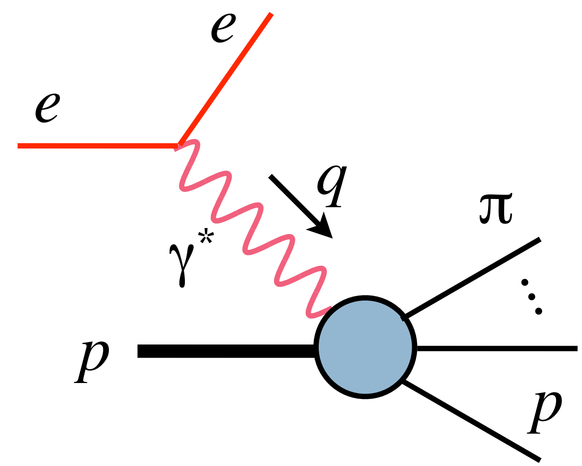
$$e + p \rightarrow e + anything$$

SLAC 1969: $E_e = 20$ GeV



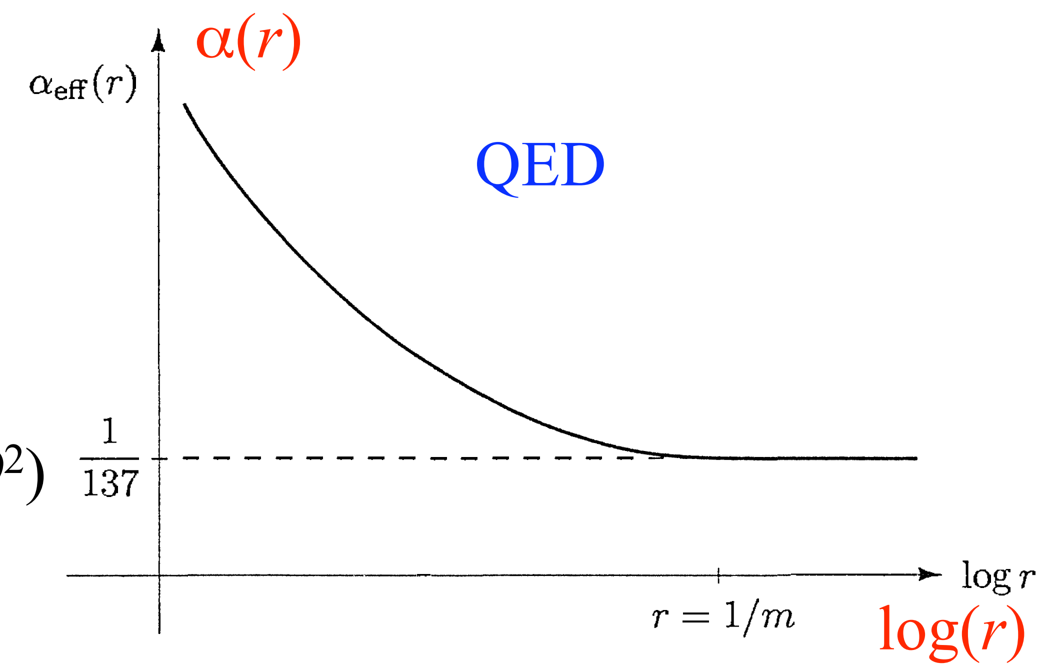
The search for asymptotic freedom

The SLAC data (1969) showed that the proton charge is located in apparently free, pointlike constituents, presumably the quarks proposed earlier based on the hadron spectrum.



This raised the question:
"How can the quarks be nearly free inside the proton, yet bind so strongly together that they do not exist as free particles?"

It was known that the coupling strength of QED "runs", ie., $\alpha = \alpha(Q^2)$ increases as the distance scale $r = 1/Q$ decreases.



If there were theories where $\alpha = \alpha(Q^2)$ decreases with r this might explain the SLAC data.

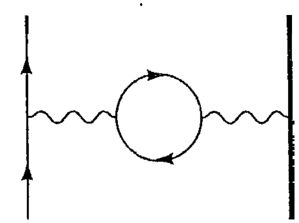
The running of α in QED (I)

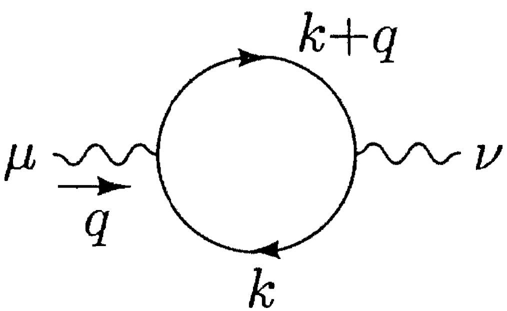
Following Pesking and Schroeder: *An Introduction to Quantum Field Theory*

The "bare" coupling e_0 in the QED lagrangian

$$\mathcal{L}_{QED} = \bar{\psi}(i\partial\!\!\!/ - e_0 A\!\!\!/ - m)\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

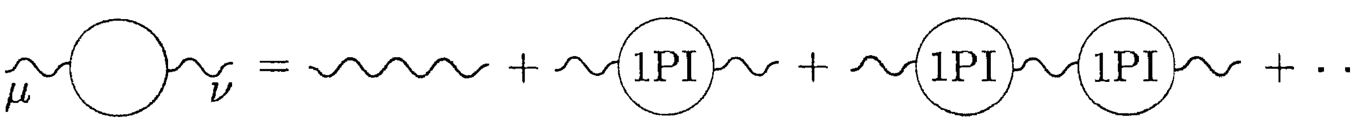
receives an infinite correction from the loop diagram





$$= (-ie)^2(-1)\int \frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\left[\gamma^\mu \frac{i}{k\!\!\!/ - m}\gamma^\nu \frac{i}{k\!\!\!/ + q\!\!\!/ - m}\right]$$
$$\equiv i\Pi_2^{\mu\nu}(q). \qquad (7.71$$

Removing the divergence by subtracting $\Pi_2(0)$ and summing the geometric series

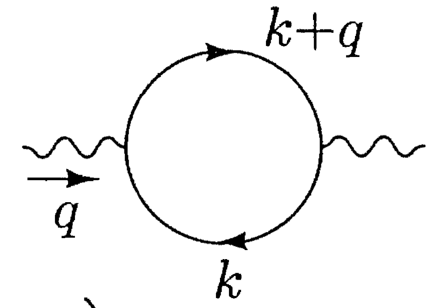


one finds:

$$\alpha_0 \to \alpha_{\text{eff}}(q^2) = \frac{e_0^2/4\pi}{1-\Pi(q^2)} \underset{\mathcal{O}(\alpha)}{=} \frac{\alpha}{1-[\Pi_2(q^2)-\Pi_2(0)]}$$

The running of α in QED (II)

Evaluation of the loop integral gives

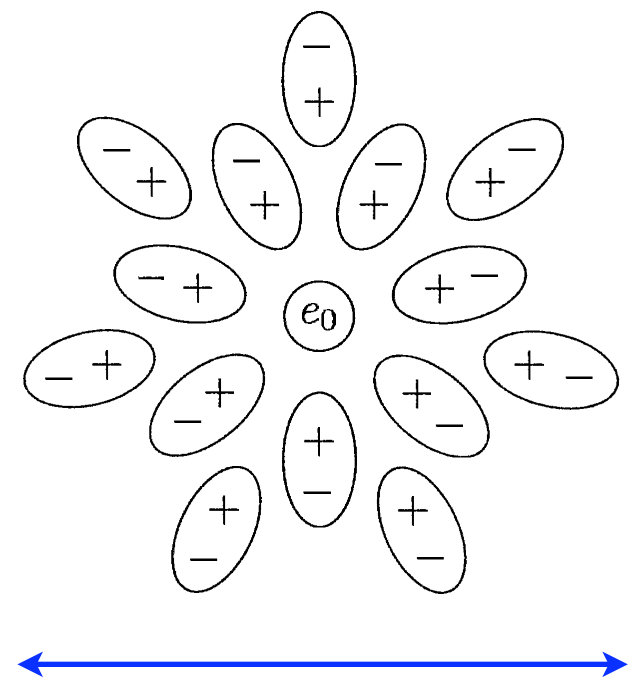


$$\widehat{\Pi}_2(q^2) \equiv \Pi_2(q^2) - \Pi_2(0) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\,\log\left(\frac{m^2}{m^2 - x(1-x)q^2}\right)$$

At large distances $r >> 1/m$, $\alpha_{\rm eff}(q^2)$ affects the Coulomb potential as follows:

$$V(r) = -\frac{\alpha}{r}\left(1 + \frac{\alpha}{4\sqrt{\pi}}\frac{e^{-2mr}}{(mr)^{3/2}} + \cdots\right) \qquad (r >> 1/m)$$

The correction decreases with r, and can be interpreted as a screening of the (infinite) bare charge e_0 by a polarization of the vacuum.



$r \approx 1/m_e$

The running of α in QED (III)

At short distances, $-q^2 >> m^2$, the running coupling is at $\mathcal{O}(\alpha^2)$:

$$\alpha_{eff}(q^2) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\log(-q^2 e^{-5/3}/m^2)} + \mathcal{O}(\alpha^3) \qquad (-q^2 >> m^2)$$

which increases with $-q^2$. At the Z-boson mass, $\alpha(m_Z^2) \simeq \frac{1}{128}$

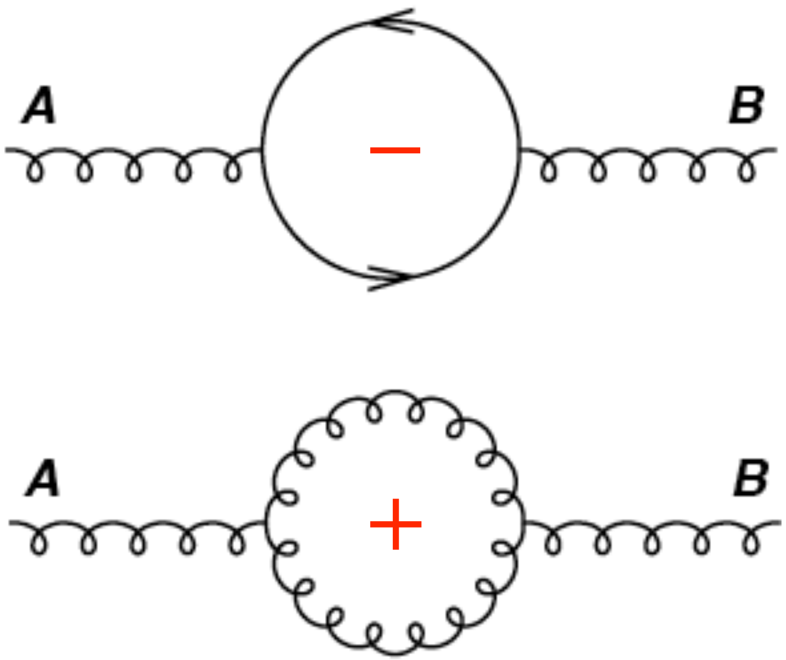
The effective coupling is infinite at

$$\sqrt{-q^2} = m \exp\left(\frac{3\pi}{2\alpha} + \frac{5}{6}\right) \approx 10^{119}\ \text{GeV} \qquad \text{“Landau pole”}$$

However, before the Landau pole is reached the perturbative expression breaks down. And the Planck scale is “only” 10^{19} GeV.

The running of α in QCD

In 1969, non-abelian gauge theory was a rather exotic topic. No one quite expected that these theories would have asymptotic freedom. After all, there was a physical argument for why the effective charge increases with Q^2 for abelian (QED) theory.



Nevertheless, two graduate students set out to do the calculation, and found that in QCD

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\log(Q^2/\Lambda^2_{QCD})}$$

$\rightarrow 0$ for $Q^2 \rightarrow \infty$!

IN QCD the gluon loop diagram contributes with opposite sign compared to the fermion loop.

$\Lambda_{QCD} \approx 200$ MeV ≈ 1 fm^{-1}

This discovery in 1972 made QCD a strong candidate theory for the strong interactions.

Quantum Chromodynamics (1972)

The QCD Lagrangian defines the interactions of quarks and gluons:

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f (i\partial\!\!\!/ - gA\!\!\!/ - m_f)\psi_f - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Looks like QED! But the gauge symmetry is SU(3) of color (Greenberg, 1964)

$$\psi \to U\psi \qquad U \subset SU(3) \qquad \psi = \begin{pmatrix} q \\ q \\ q \end{pmatrix}$$

All quarks u,d,s,c,b,t have the same strong coupling g

$$\alpha_s = \frac{g^2}{4\pi}$$

Note: In QED the abelian U(1) symmetry allows different electric charges *e* for different particles

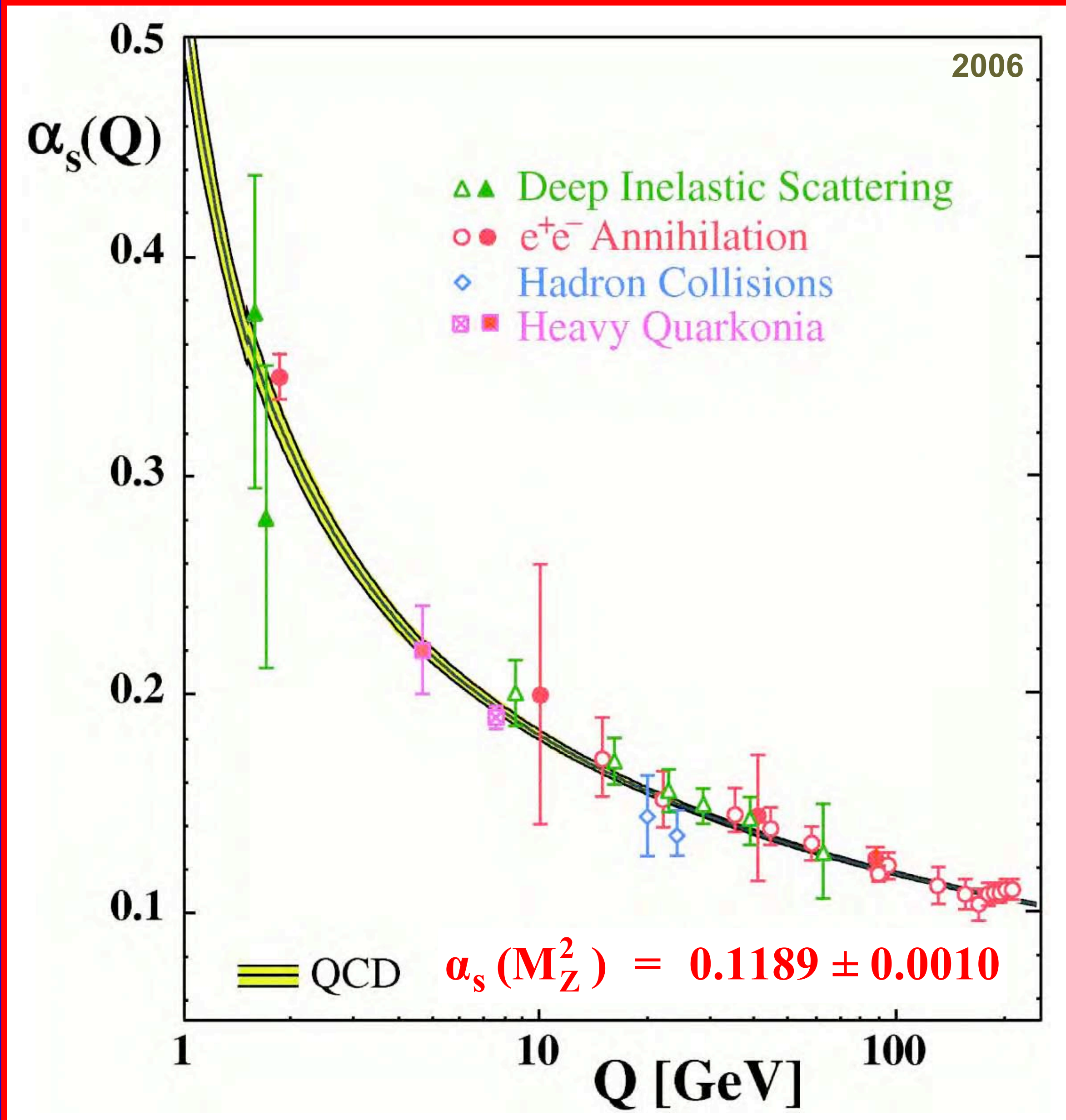
$$e_u = -\frac{2}{3}e_e$$

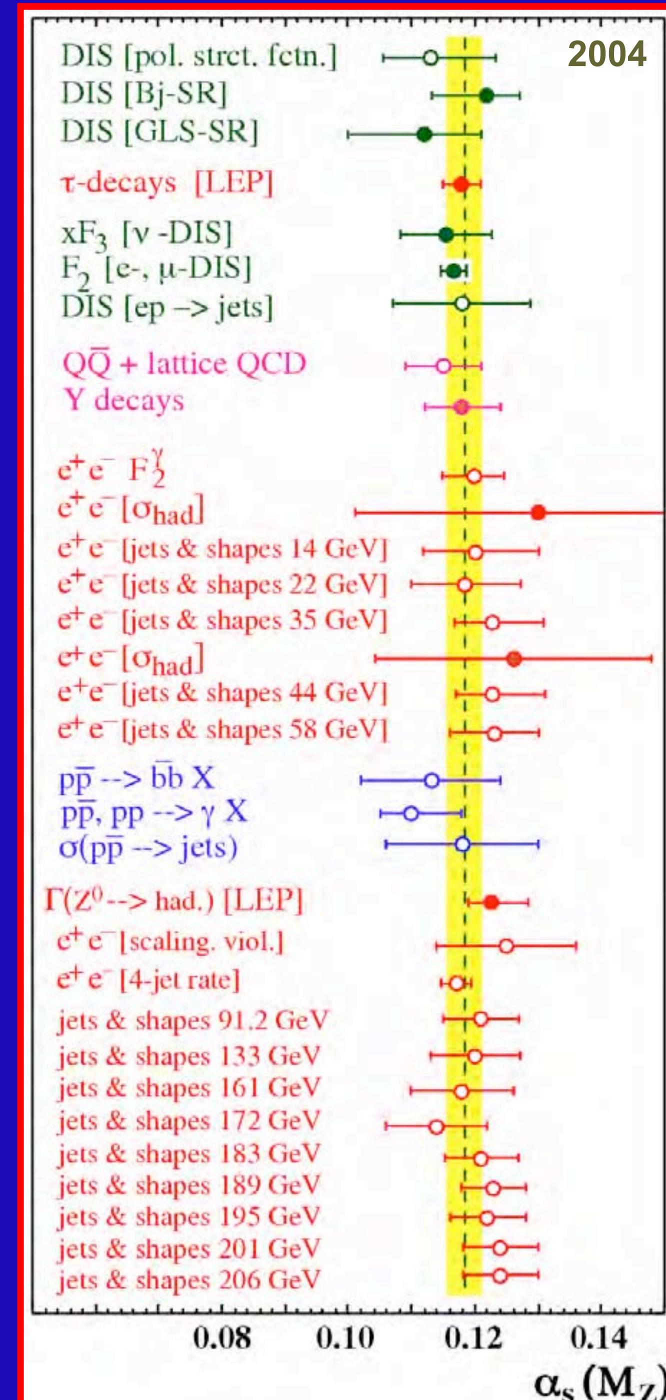
It remains a mystery why: $|e_p + e_e|/e < 1.0 \cdot 10^{-21}$

MEASUREMENTS OF α_s

$\alpha_s(M_Z^2) = 0.1182 \pm 0.0027$

S. Bethke





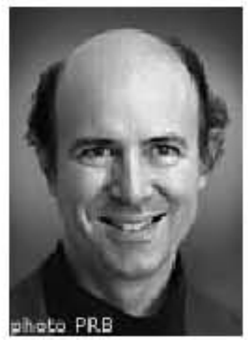
Asymptotic Freedom

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross
1/3 of the prize
USA
Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA
b. 1941

H. David Politzer
1/3 of the prize
USA
California Institute of Technology
Pasadena, CA, USA
b. 1949



Frank Wilczek
1/3 of the prize
USA
Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA
b. 1951

"What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to each other, the force is so weak that they behave almost as free particles. This phenomenon is called 'asymptotic freedom'. The converse is true when the quarks move apart: the force becomes stronger when the distance increases."

$\alpha_S(r)$

1/r

From P. Skands

Data creates Theory

Data		Phenomenological Models		(Solvable) Theory
Discriminating Observables	↔	Individual Essential Features	↔	Complete description
Fits		Quark Model, Eightfold Way, ...		Quantum Chromo-Dynamics

Paul Hoyer Mugla 2010

Puzzles of QCD

QCD must have Color Confinement: Only color singlet mesons and baryons can propagate over long distances.

Color confinement is verified numerically in numerical lattice simulations and modelled phenomenologically, but the mechanism is still poorly understood.

Baryons and mesons are bound states of quarks. What are the wave functions?

The quarks and gluons bound in hadrons are highly relativistic.
Data and models exist, but no calculations comparable to QED atoms.

How can we compare data on hadron final states with perturbative QCD?

Factorization theorems allow to express physical measurements in terms of hard, perturbative subprocesses and universal, measurable quark and gluon distributions.

Perspective: The divisibility of matter

Since ancient times we have wondered whether matter can be divided into smaller parts *ad infinitum*, or whether there is a smallest constituent.

Democritus, ~ 400 BC
Vaisheshika school

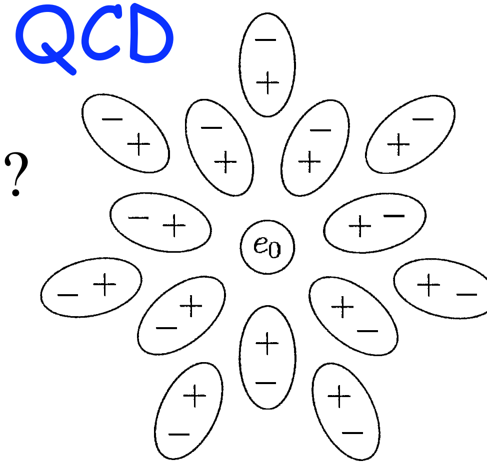
Common sense suggest that these are the two possible alternatives. However, physics requires us to refine our intuition.

Quantum mechanics shows that atoms (or molecules) are the identical smallest constituents of a given substance
– yet they can be taken apart into electrons, protons and neutrons.

Hadron physics gives a new twist to this age-old puzzle: Quarks can be removed from the proton, but cannot be isolated. Relativity – the creation of matter from energy – is the new feature which makes this possible.

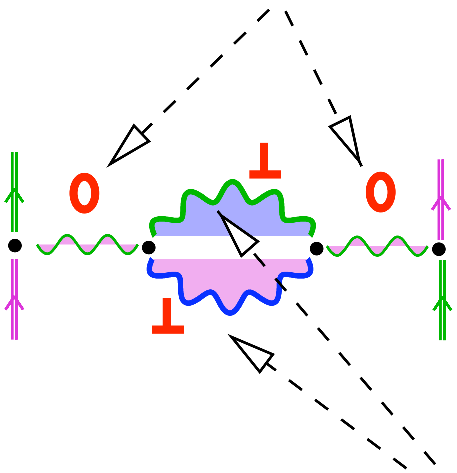
We are fortunate to be here to address – and hopefully develop an understanding of – this essentially novel phenomenon!

Understanding charge screening in QCD



What went wrong with the nice physical argument of QED?

Instantaneous Coulomb interaction

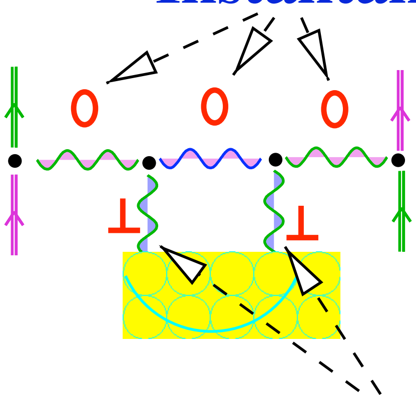


$$= -N_c * \frac{1}{3} \; - n_f * \frac{2}{3}$$

Physical, transverse gluons screen as QED

Transverse gluons (and quarks)

Instantaneous Coulomb interaction



$$= +N_c * 4$$

Coulomb gluons give the opposite sign!

Vacuum fluctuations of transverse fields

Yu. L. Dokshitzer hep-ph/0306287

Quark masses: Nature's gift

In QCD, mass terms can be directly introduced in the lagrangian.

In the SM all masses result from "Yukawa interactions" involving the Higgs field, but they are unconstrained by the theory.

The quark masses inferred from experiment are:

$m_u = 1.5 \ldots 3.3$ MeV

$m_d = 3.5 \ldots 6.0$ MeV

} $m \ll \Lambda_{QCD}$, mass effects small
Isospin invariance, chiral symmetry

$m_s = 104 \pm 30$ MeV ⇐ Λ_{QCD} 200 MeV = 1 fm^{-1} — Scale of strong interactions

$m_c = 1.27 \pm .10$ GeV

$m_b = 4.20 \pm .15$ GeV

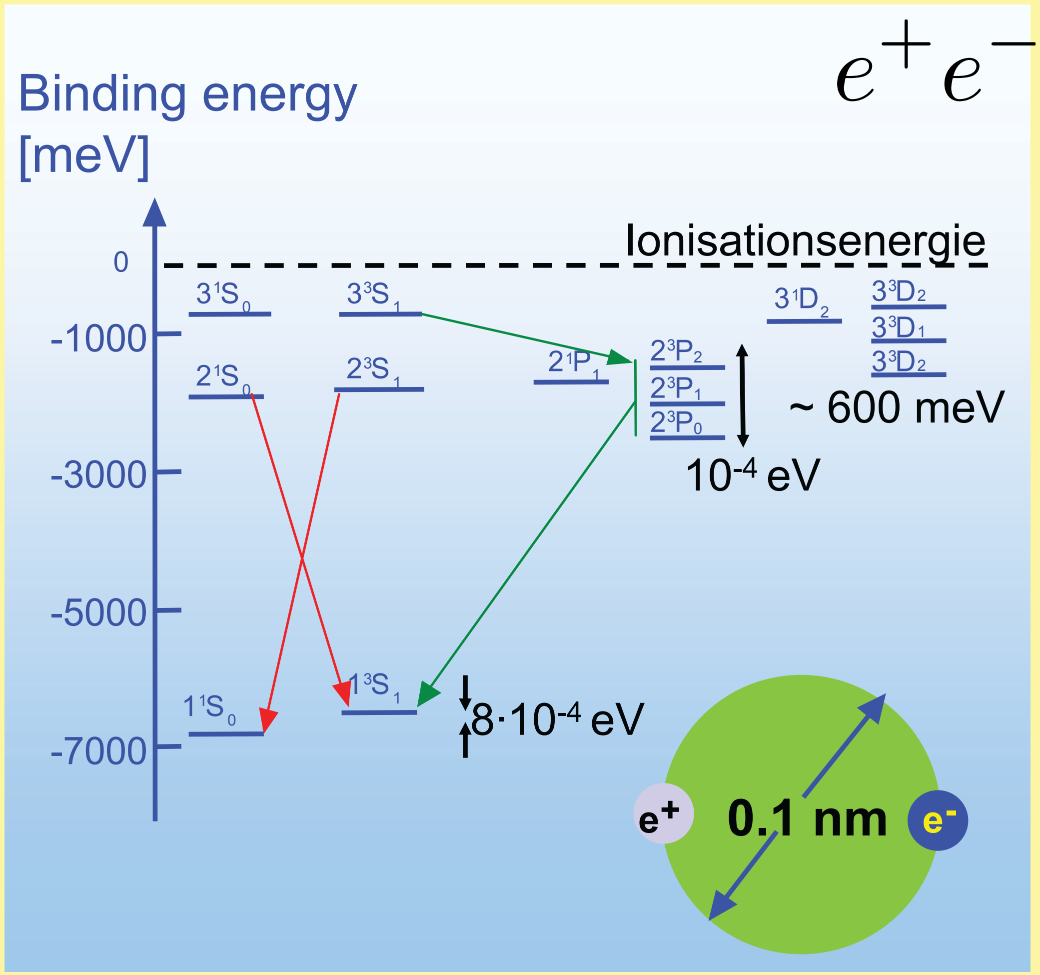
$m_t = 171.2 \pm 2.1$ GeV

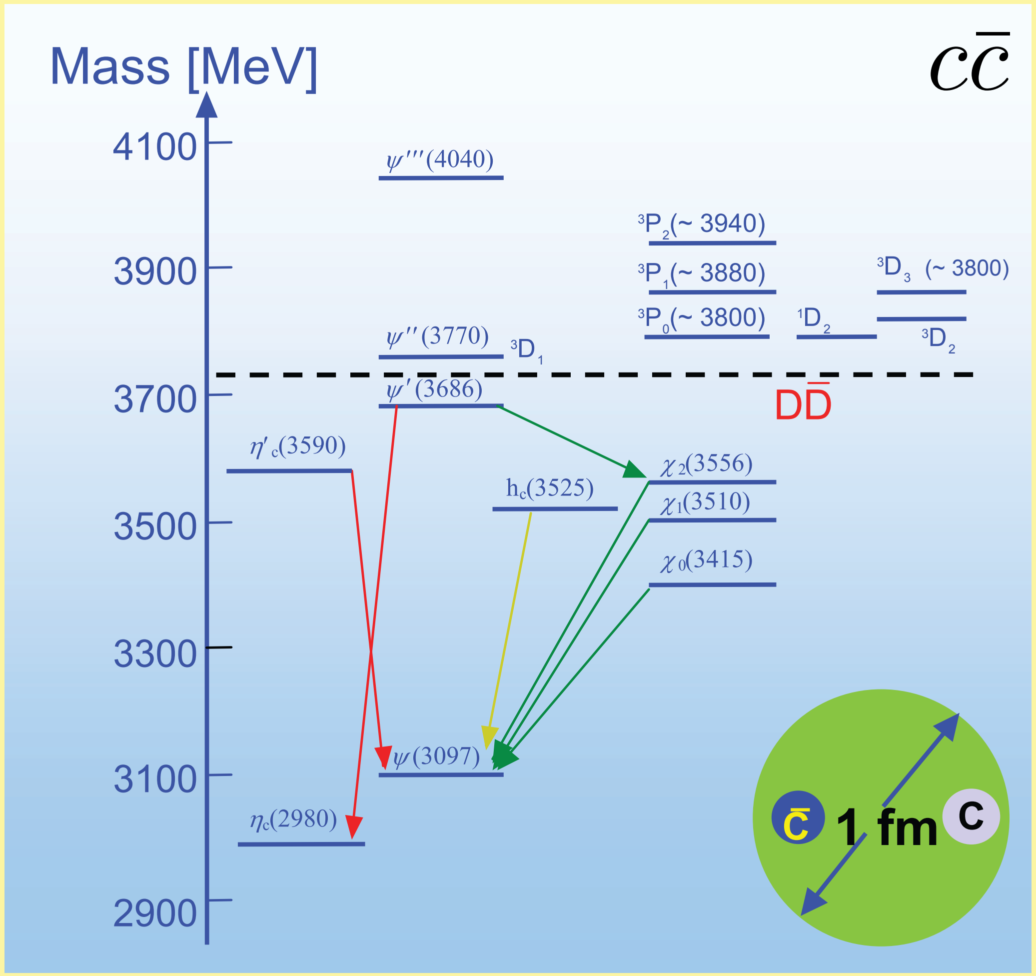
} $m \gg \Lambda_{QCD}$, mass effects large
Non-relativistic bound states

Charmonium – the Positronium of QCD

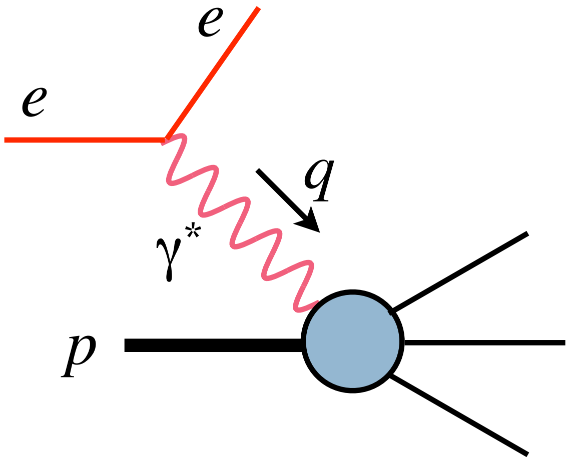
Since $m_c \gg \Lambda_{QCD}$, the $c\bar{c}$ mesons are nearly non-relativistic

- Positronium
- Charmonium



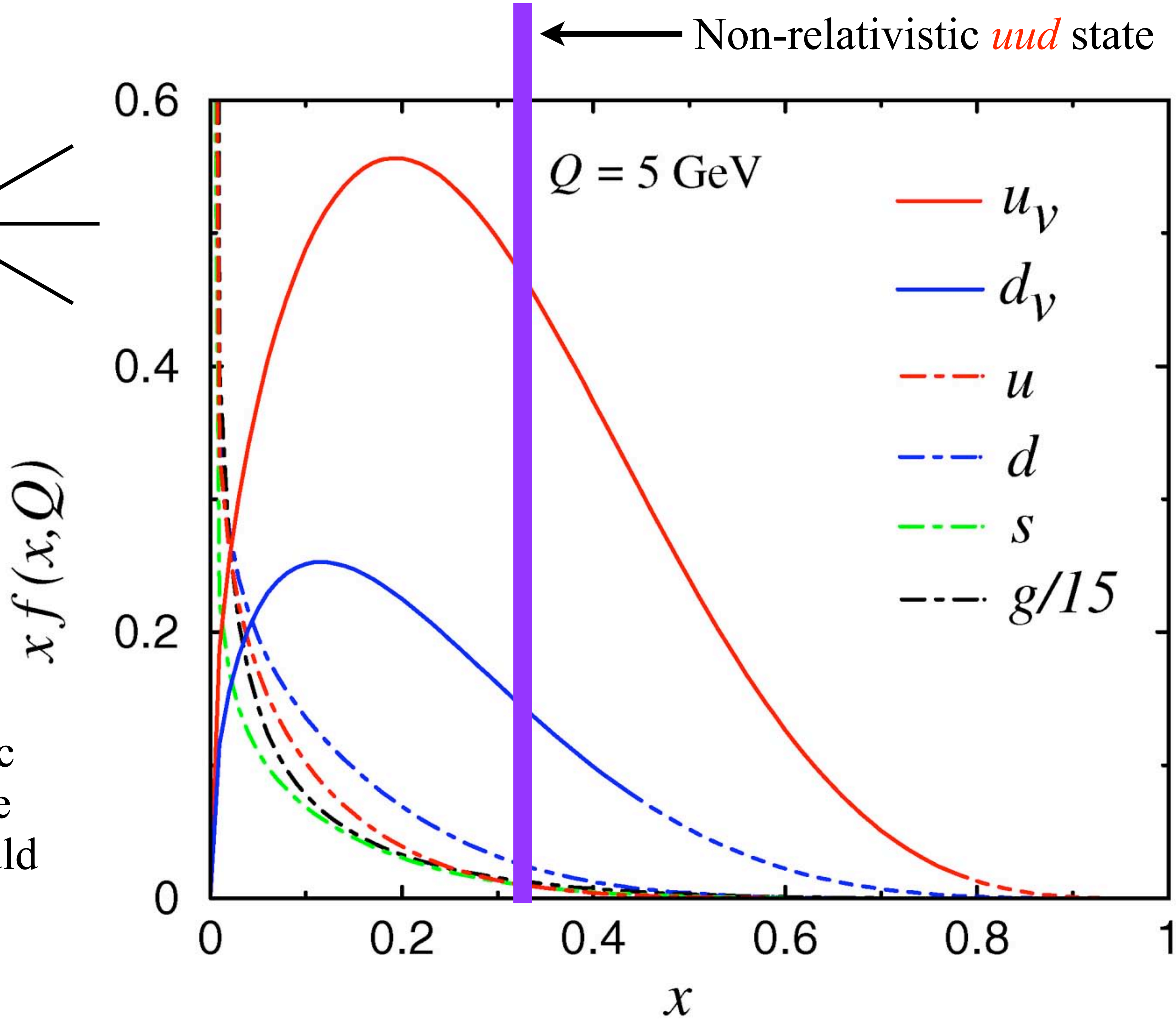


Quarks move relativistically inside the proton



DIS measures the fraction x of the proton energy which is carried by the quarks, anti-quarks and gluons.

For non-relativistic internal motion the x-distribution would be sharply peaked



Origin of the proton mass

The u, d quarks in the proton have small masses

$$\frac{2m_u + m_d}{m_p} \simeq \frac{10\ \text{MeV}}{938\ \text{MeV}} \simeq 1\%$$

99% of the proton mass is due to interactions!
1% is due to Higgs.
⇒ Ultra-relativistic state

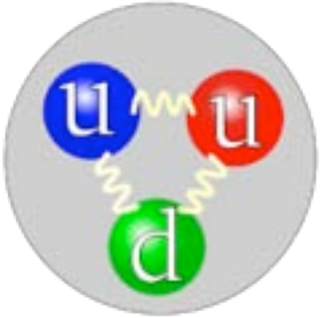
Compare this with positronium (e^+e^-), the lightest QED atom:

$$\frac{2m_e}{m_{pos}} \simeq 100.00067\%$$

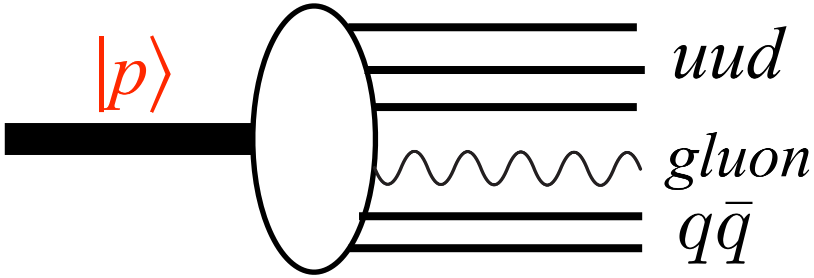
Binding energy is tiny *wrt* mc^2
⇒ Nonrelativistic state

The compatibility of the non-relativistic $|p\rangle = |uud\rangle$ quark model description of the proton with its ultra-relativistic parton model picture remains a mystery – but a mystery that we can address within QCD.

Both are supported by data:



$\overset{??}{=}$

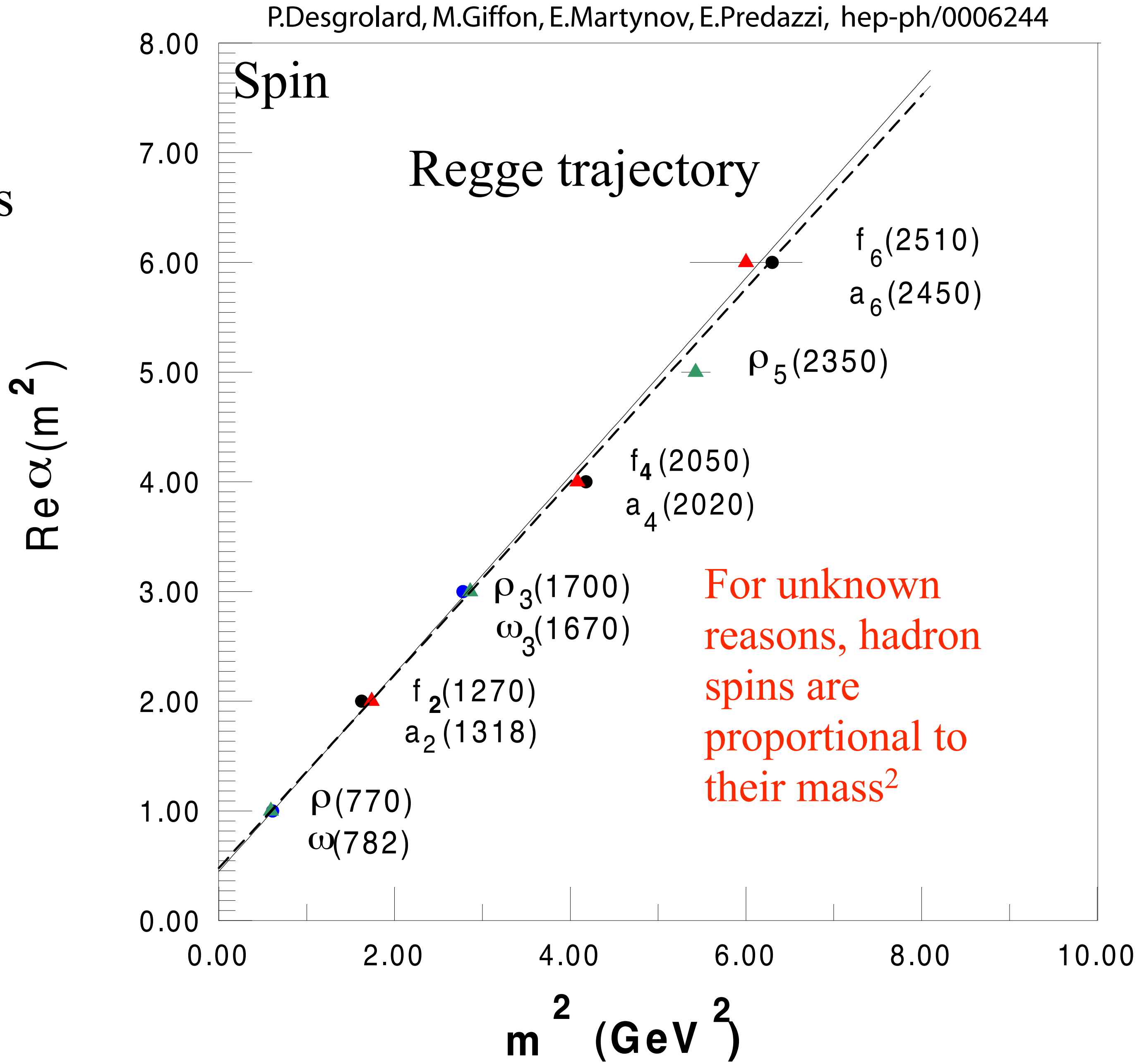


Hadron mass spectrum is relativistic

Unlike atoms, hadrons have no ionization threshold, where the quark constituents would be liberated.

Hadron masses are generated by the potential and kinetic energies of the constituents

Not yet explained by QCD



Total and Elastic Hadron Cross Sections

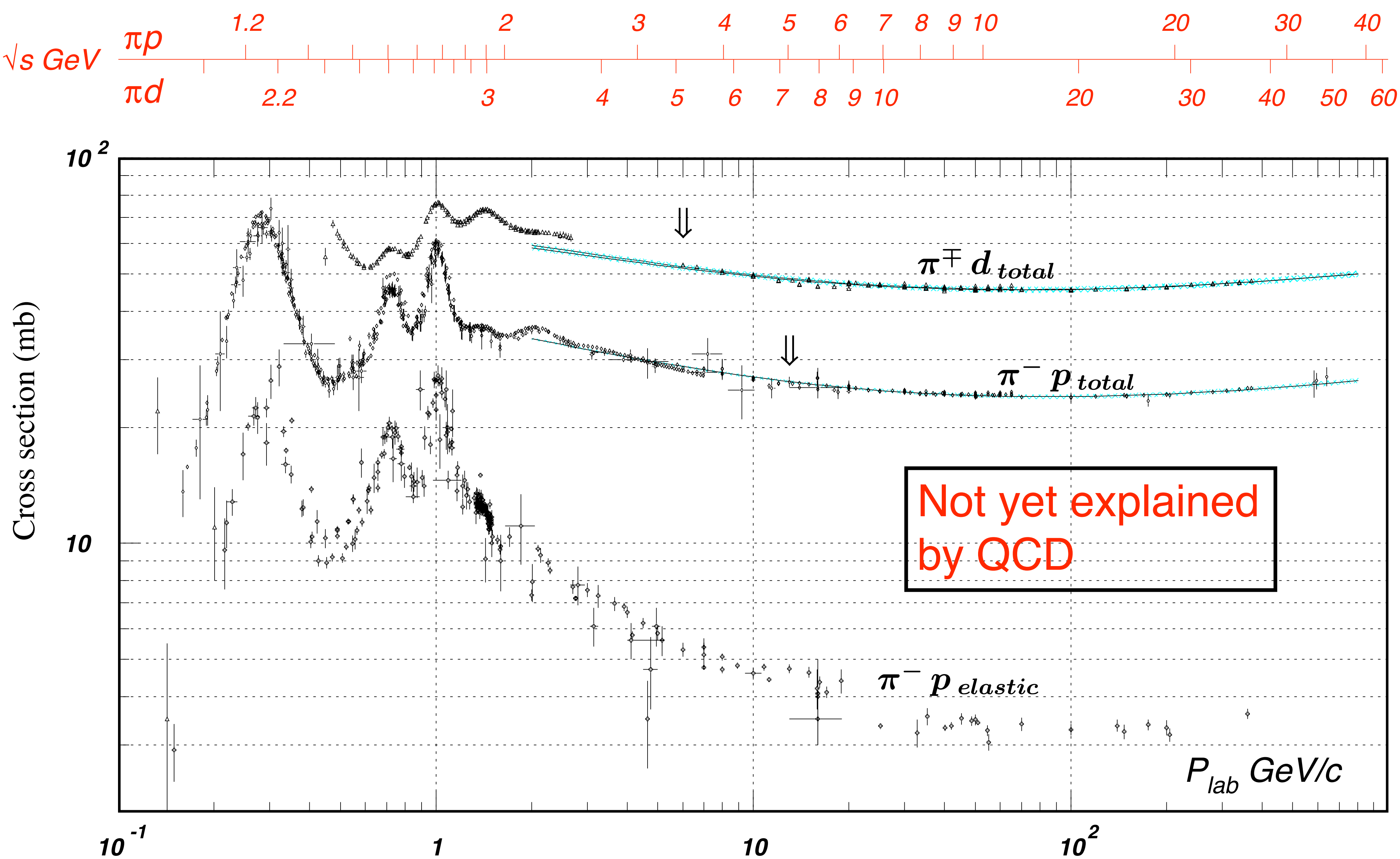
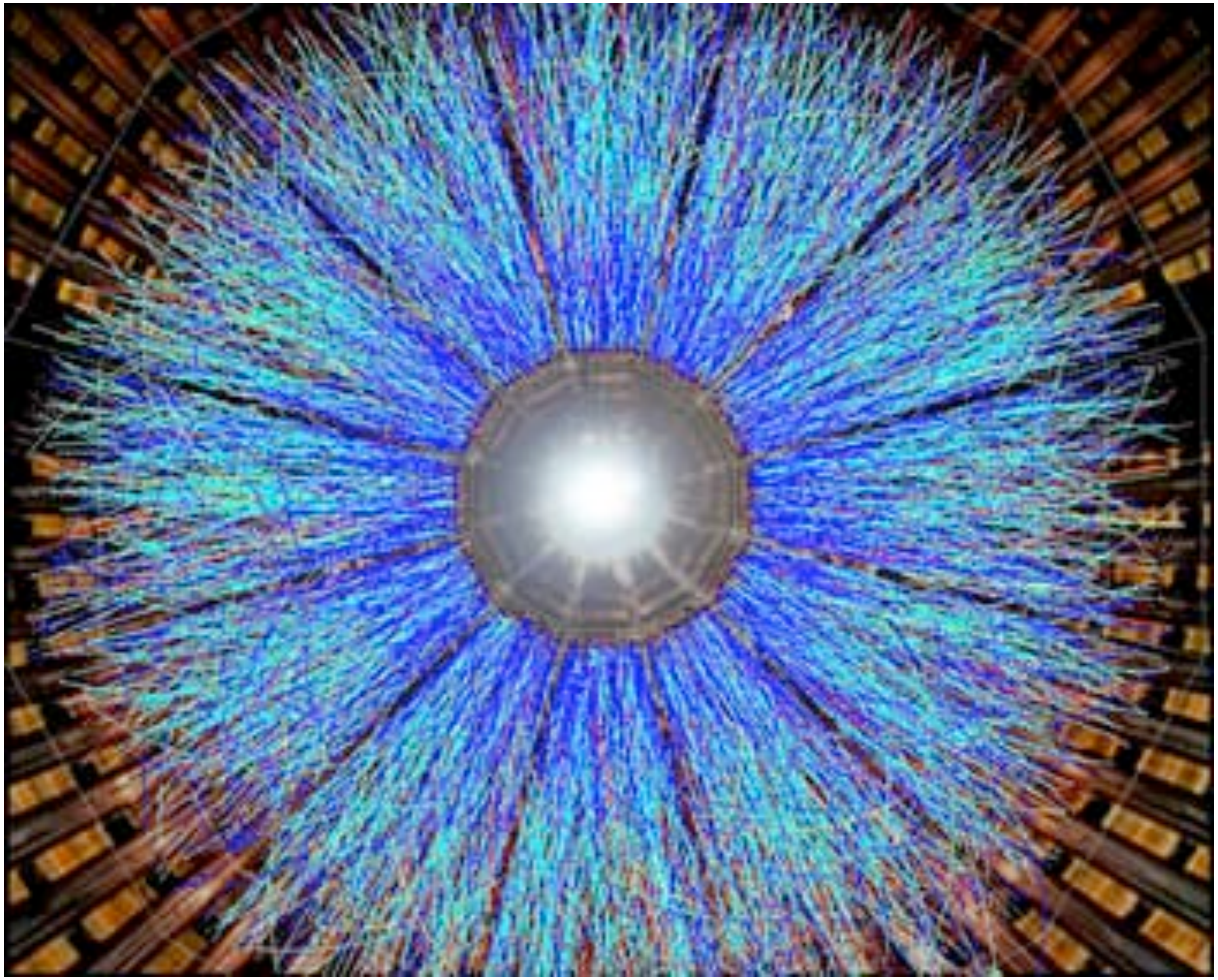


Figure 40.13: Total and elastic cross sections for $\pi^{\pm}p$ and $\pi^{\pm}d$ (total only) collisions as a function of laboratory beam momentum and total center-of-mass energy. Corresponding computer-readable data files may be found at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS Group, IHEP, Protvino, August 2005)

Heavy Ion Collisions

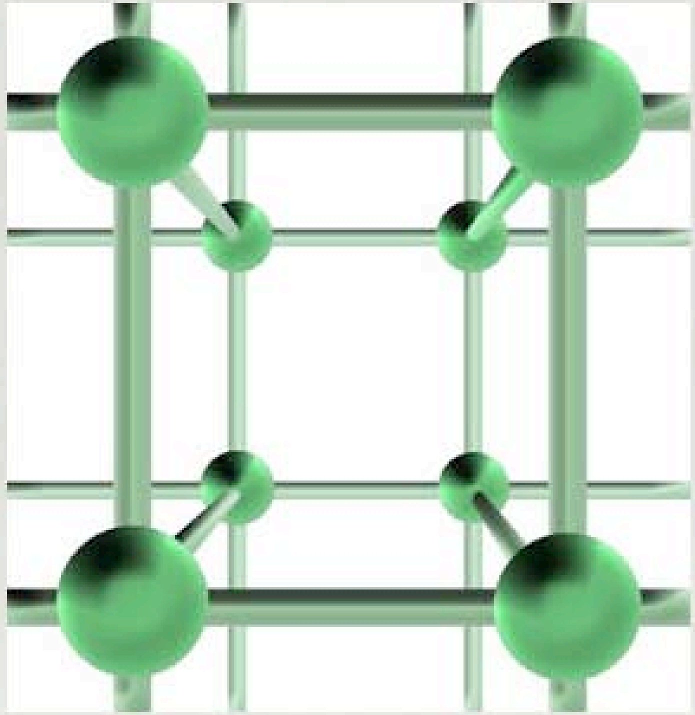
The quest for a new phase of matter at RHIC, LHC,...



Lattice QCD

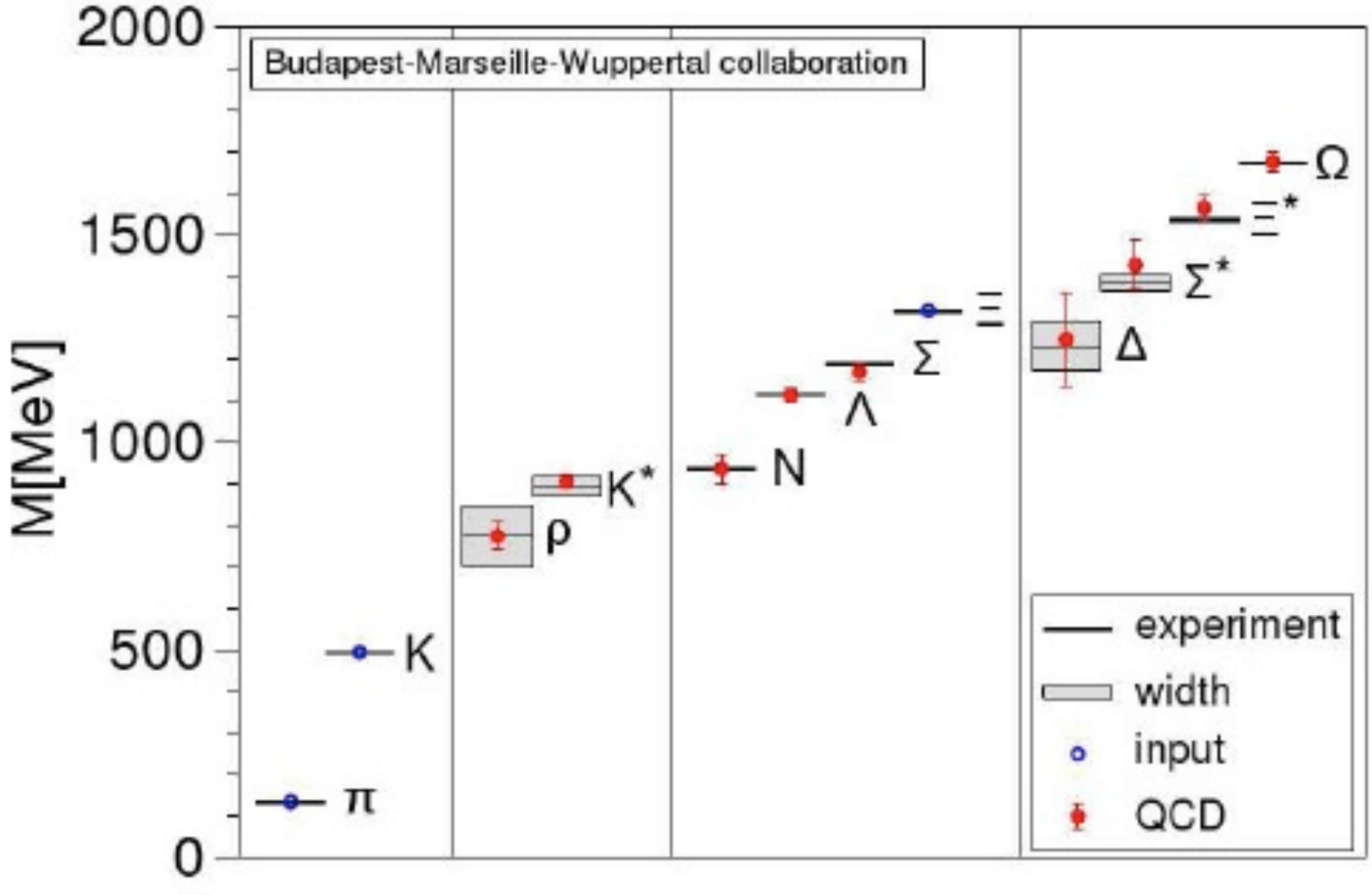
Spacetime

- Approximated by
- 4D (Euclidean) box of points
  - Similar to crystal lattice (with imaginary time) 3fm/c ≈ 10 yoctoseconds
- Symmetries
  - Full Lorentz → Hypercubic
  - But gauge invariance ok ✓
- "Discretization Errors" → 0
  - in limit of infinitely small lattice spacing, a



P. Skands

Hadron masses from Lattice calculations



Durr et al '08

The accuracy of QED perturbation theory

Many of our most accurate predictions come from QED atoms.
For example, the $2S_{1/2} - 8S_{1/2}$ splitting in Hydrogen:

$$\Delta(2S_{1/2} - 8S_{1/2})_H = 770\ 649\ 350\ 012.0(8.6)\ \text{kHz} \quad \text{EXP}$$
$$= 770\ 649\ 350\ 016.1(2.8)\ \text{kHz} \quad \text{QED}$$

U.D. Jentschura et al, PRL 95 (2005) 163003

The QED result is based on perturbation theory:
– an expansion in $\alpha = e^2/4\pi \approx 1/137.035\ 999\ 11(46)$

However, the series must diverge since for any $\alpha = e^2/4\pi < 0$ the electron charge e is imaginary: The Hamiltonian is not hermitian and probability not conserved.

F. Dyson

The perturbative expansion is believed to be divergent (asymptotic). The good agreement of data with QED is fortuituous, from a theoretical point of view.

For a discussion of the truncation effects in asymptotic expansions see Y. Meurice, hep-th/0608097

QCD perturbation theory has many features in common with that of QED.

Applications of perturbative QCD to data

The QCD perturbative expansion successfully describes short distance processes, which involve high virtualities Q^2 and small $\alpha_s(Q^2)$.

Long distance dynamics is "universal", *i.e.,* independent of the hard process. Measuring the soft quark distribution and fragmentation functions in one process one can then predict measurable hadron cross sections.

The applications of PQCD depends on Factorization Theorems, which hold to all orders in α_s at "Leading Twist", for sufficiently inclusive processes.

The "Higher Twist" corrections are power-suppressed in the hard scale, $\propto 1/Q^2$ and can usually not be predicted.

Factorization Theorem

Factorization: expresses the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b}\sum_{f}\int_{\hat{X}_f} f_a(x_a,Q_i^2)\, f_b(x_b,Q_i^2)\, \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a,x_b,f,Q_i^2,Q_f^2)}{\mathrm{d}\hat{X}_f}\, D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$

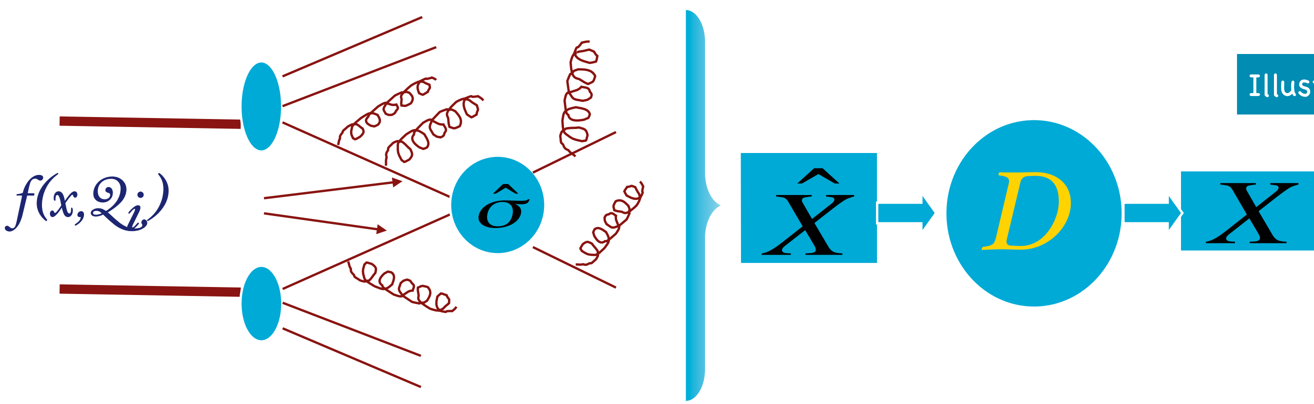


Illustration by M. Mangano

$\vec{p}_j = x\,\vec{P}_{proton}$

$f_a(x_a, Q_i^2)$ Parton distribution functions (PDF)

- sum over long-wavelength histories leading to a with x_a at the scale Q_i^2 (ISR)

$D(\hat{X}_f \to X, Q_i^2, Q_f^2)$ Fragmentation Function (FF)

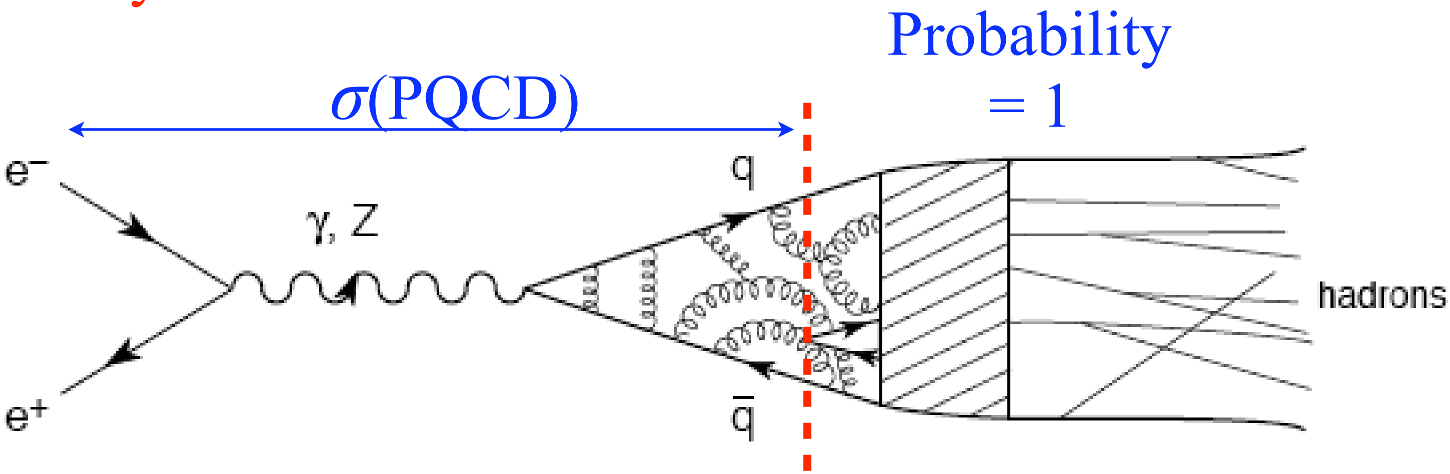
- Sum over long-wavelength histories from $\hat{X}_f$ at Q_f^2 to X (FSR and Hadronization)

+ (At H.O. each of these defined in a specific scheme, usually $\overline{\mathrm{MS}}$)

P. Skands

Simplest QCD prediction: $e^+e^- \to$ hadrons

In the total e^+e^- hadronic cross section we sum over the poorly understood processes by which quarks turn into hadrons, which occur with probability = 1

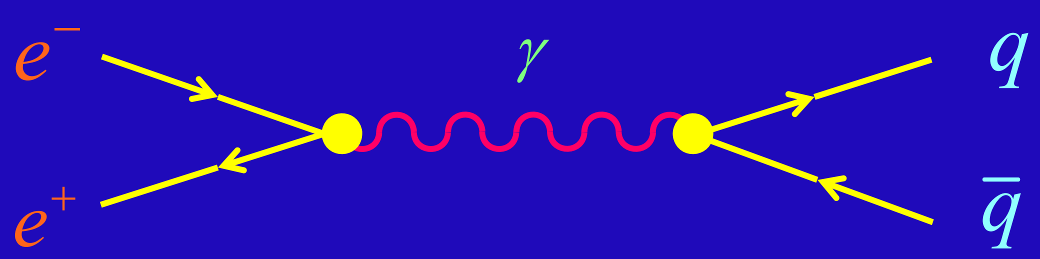


$e^+e^- \to$ hadrons.

There are no hadrons in the initial state, hence no quark distribution functions $f_q(x,Q^2)$.

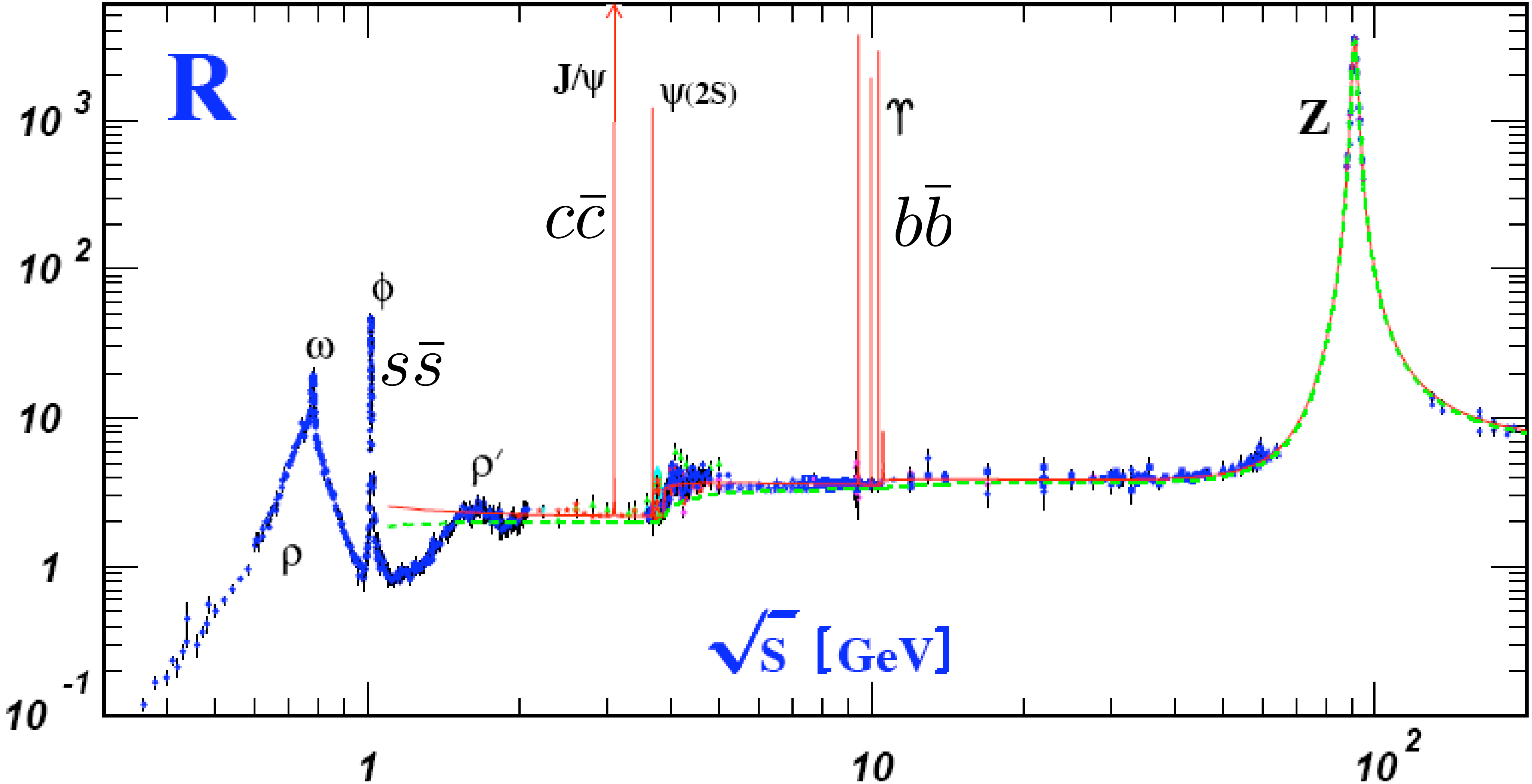
There are no fragmentation functions $D_q(z,Q^2)$, due to the sum over all hadronic final states.

$\Longrightarrow$ The PQCD prediction depends only on $\alpha_s(Q^2)$!



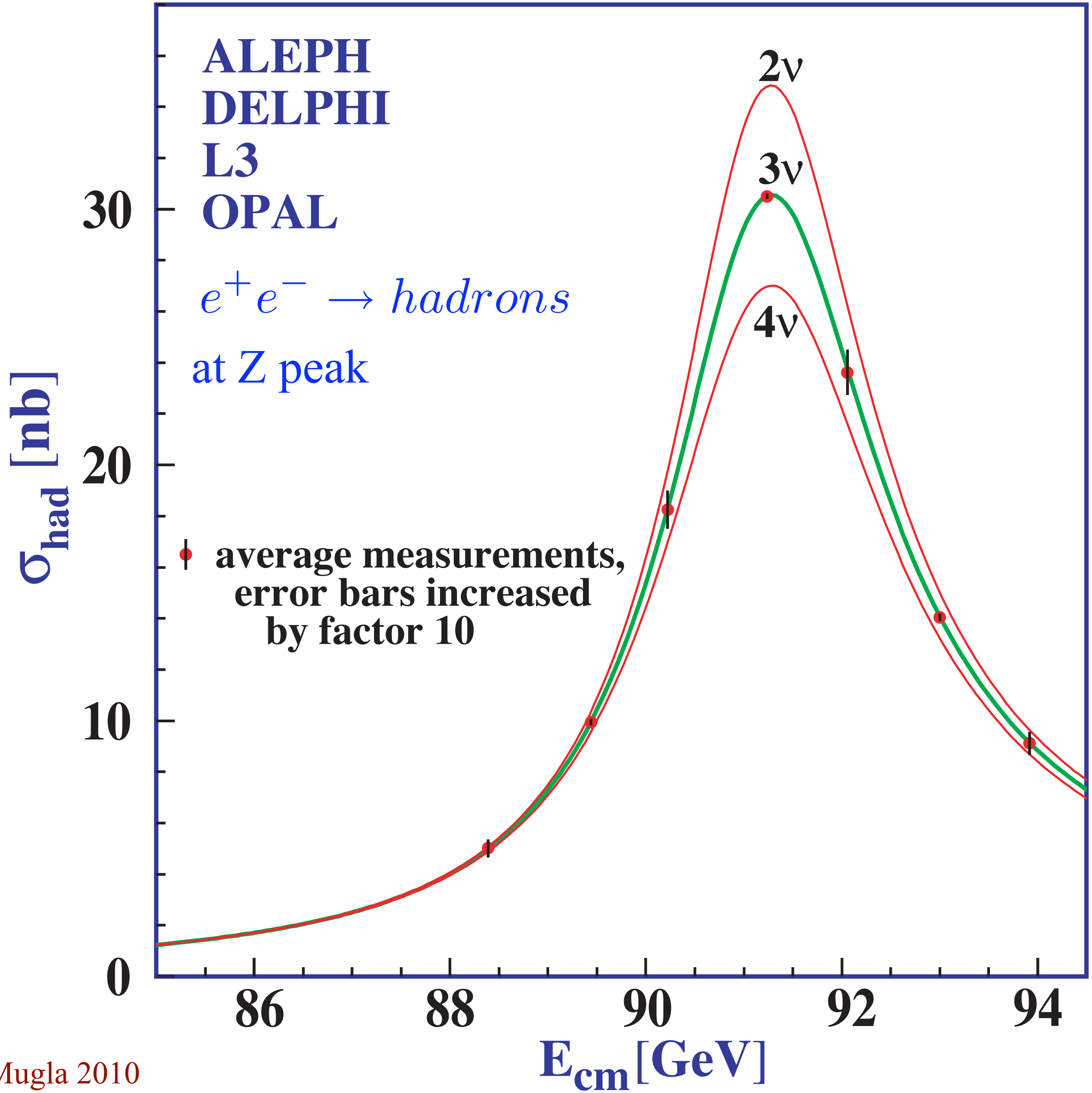
$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq \frac{\sum_q \sigma(e^+e^- \to q\,\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} \simeq N_C \sum_q Q_q^2$$

$$= \begin{cases} \frac{2}{3} N_C \quad , & (u, d, s) \\ \frac{10}{9} N_C \quad , & (u, d, s, c) \\ \frac{11}{9} N_C \quad , & (u, d, s, c, b) \end{cases}$$



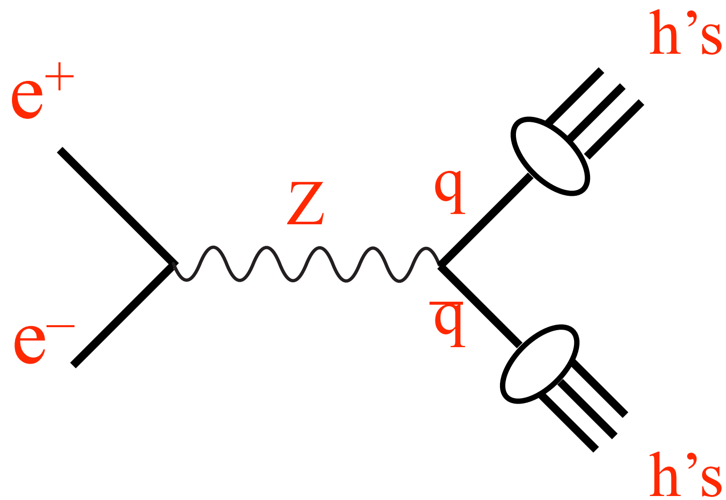
$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q e_q^2\left(1+\frac{\alpha_s}{\pi}\right)$$

LEP determination of neutrino number



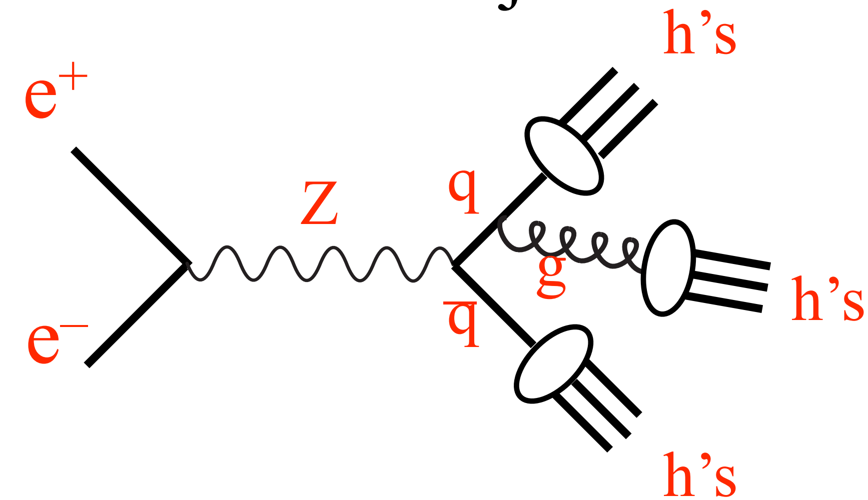
Traces of Quarks and Gluons in the final state?!

$e^+ e^- \rightarrow$ 2 jets

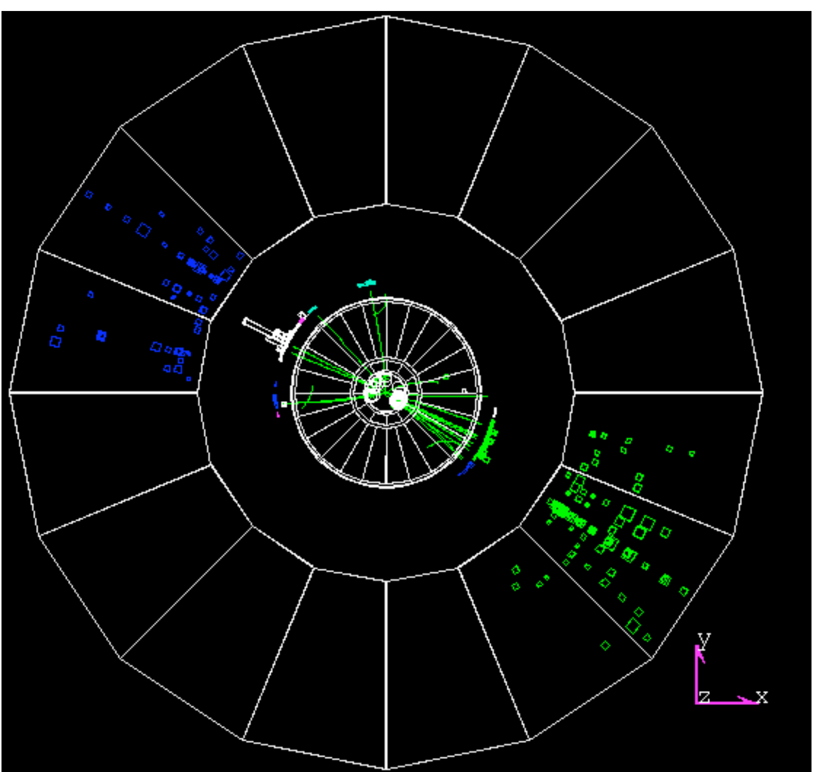


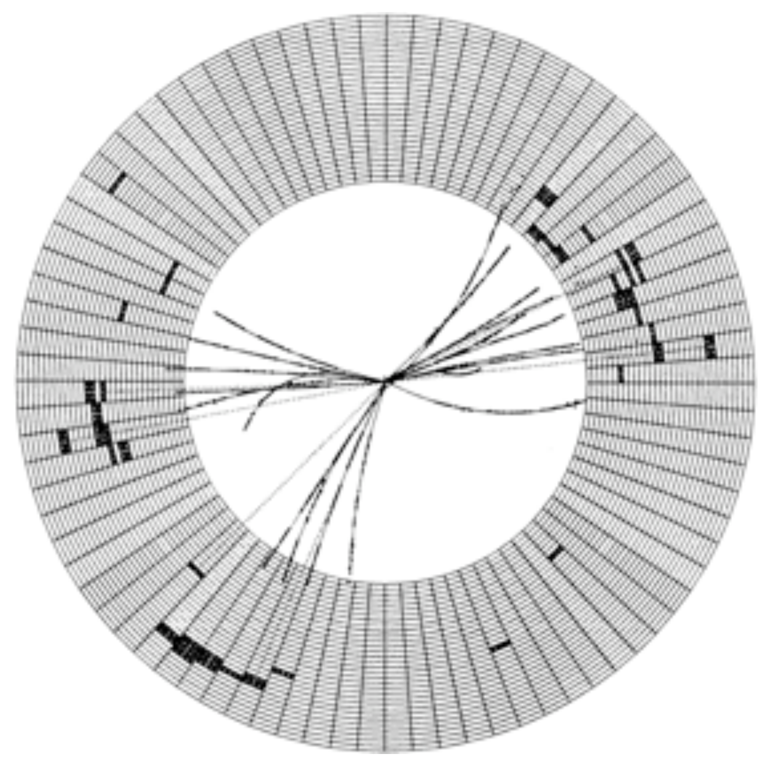
$$Q^2 = m_Z^2$$

$e^+ e^- \rightarrow$ 3 jets



5:02 PM





No exclusive amplitudes for charged particles (I)

The identification of the data on $e^+e^- \to 2\ jets$ with the QCD process $e^+e^- \to q\bar{q}$ requires care, since the latter does not exist!

This is a general feature of gauge theories, and as such is best illustrated by the QED process $e^+e^- \to \mu^+\mu^-$

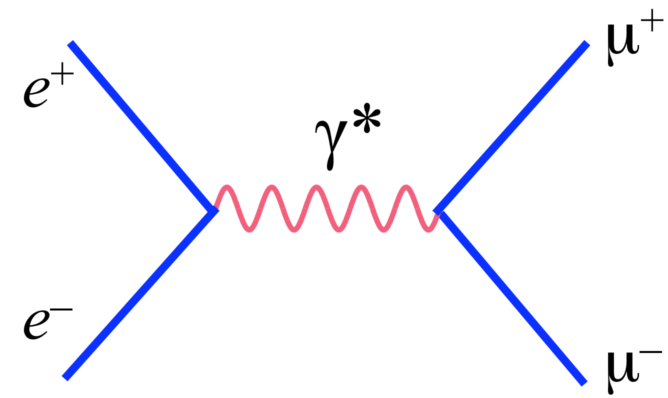
Gauge invariance dictates that amplitudes with external charged particles vanish: $A(e^+e^- \to \mu^+\mu^-) = 0$

This is because the amplitude must be invariant under local $U(1)$ gauge transformations. Multiplying one of the external fermions by

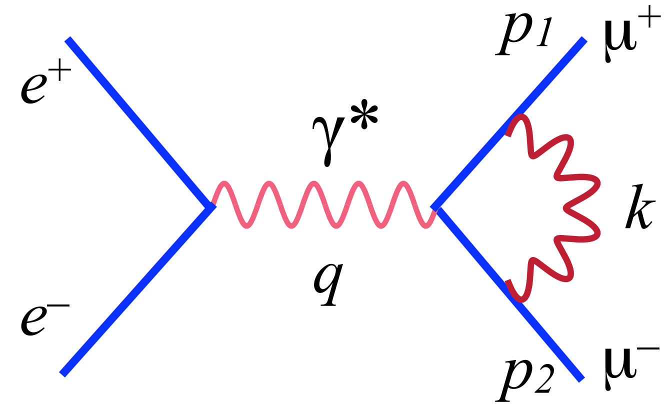
$U = e^{i\pi} = -1$ we get $A \to -A$.

No exclusive amplitudes for charged particles (II)

In the perturbation expansion the Born term is well-defined and $\neq 0$:



This problem shows up at order α^2 as an infrared singularity in the loop integral for $k \to 0$:



This may be seen without calculation:

The two fermion propagators $\propto k$, *e.g.:*

$$(p_1 - k)^2 - m_\mu^2 = -2p_1 \cdot k + k^2 \propto k$$

The photon propagator $\propto k^2$, giving a log singularity at $k = 0$

$$\int_0 \frac{d^4k}{k^4}$$

$\Longrightarrow$ The exclusive process $e^+e^- \to \mu^+\mu^-$ is ill defined.

No exclusive amplitudes for charged particles (III)

Two charged particles at different positions *x, y* must be connected by a photon string to be gauge invariant:

$$\bar{\psi}(y)\exp\left(ie\int_x^y dz_\nu A^\nu(z)\right)\psi(x)$$

The photon (gauge) field serves as a connection, which "informs" about the choice of gauge at each point in space.

In perturbation theory, the missing string causes an infrared singularity at the one photon correction level.

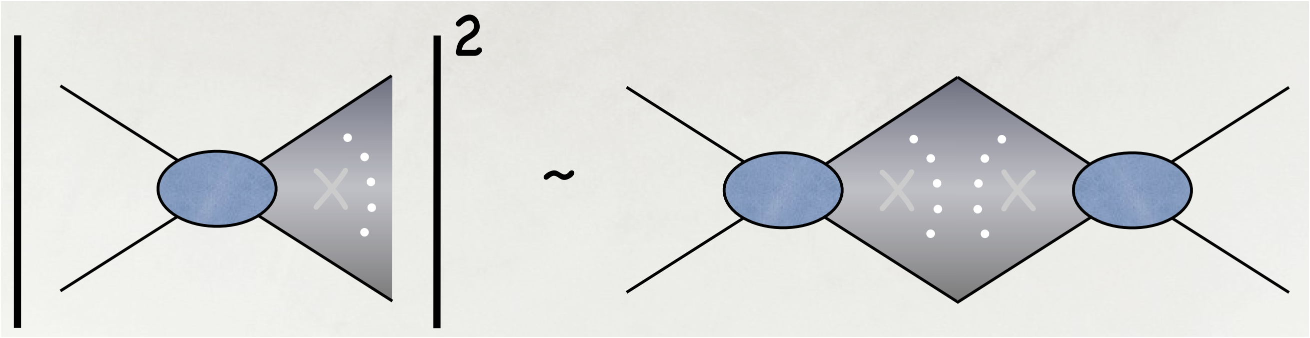
But we previously saw that there is no problem with the total e^+e^- cross section, which includes the $\mu^+\mu^-$ final state?!

To see how the IR singularity cancels in the total cross section we may use the optical theorem.

Optical Theorem

As a consequence of the unitarity of the scattering matrix: $S\,S^\dagger = 1$
the total cross section may be expressed in terms of the imaginary part of the forward elastic amplitude:

$$\sigma_{tot}(s) = \sum_X \int d\Phi_X\, |M_X|^2 = \frac{8\pi}{\sqrt{s}}\,\mathrm{Im}\,[M_{el}(\theta = 0)]$$

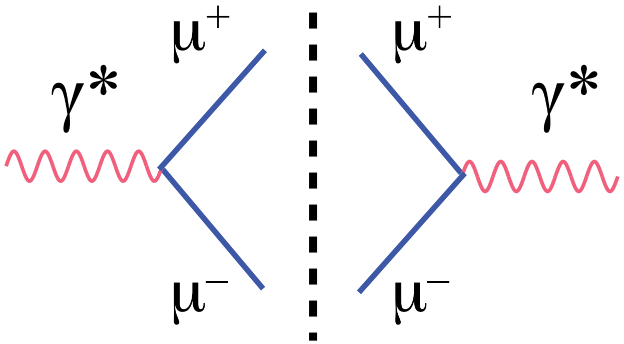


The sum over all states X becomes a completeness sum on the rhs.

QED satisfies unitarity at each order of α.

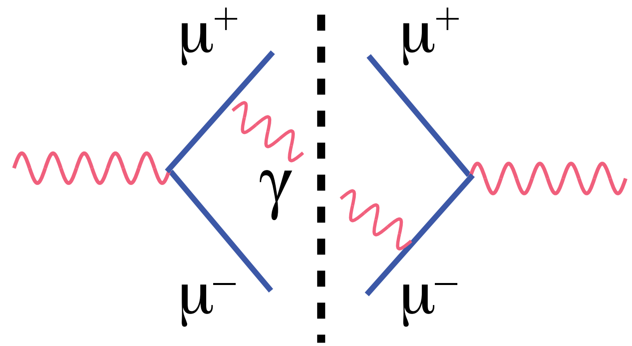
Optical Theorem for $\sigma_{tot}(e^+e^-)$ in QED (I)

$\mathcal{O}(\alpha)$

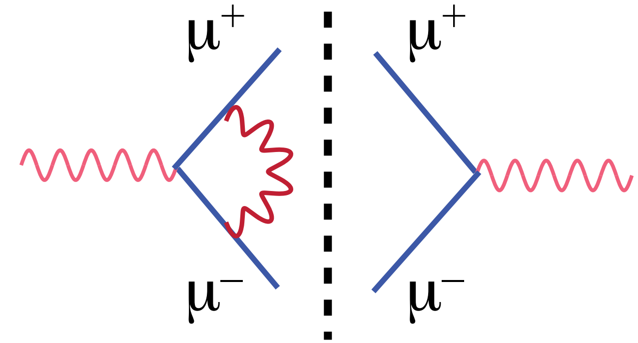


$\gamma^* \to \mu^+\mu^-$

$\mathcal{O}(\alpha^2)$



$\gamma^* \to \mu^+\mu^-\gamma$



$\gamma^* \to \mu^+\mu^-$

At $\mathcal{O}(\alpha^2)$ there are two contributions to the imaginary part (dashed line).
The IR singularity cancels between them, i.e., between different final states!

Since the $\gamma^* \to \gamma^*$ amplitude does not have external charges, it "has to be" regular.

It means that even in QED we must define cross sections such that they include (arbitrarily soft) photons. There are no free, "bare electrons".

Optical Theorem for $\sigma_{tot}(e^+e^-)$ in QED (II)

Since the IR singularity is only at $k = 0$, it suffices to include only photons with $k < k_0$, for arbitrarily small k_0.

The criterion is to sum over all states that are degenerate in energy, since these have an asymptotically long formation time $\Delta t \sim 1/\Delta E$

Kinoshita-Lee-Nauenberg (KLN) theorem

In QED, collinear IR singularities are regulated by the electron mass:

$$E_e = \sqrt{\boldsymbol{p}^2 + m_e^2} \qquad E_{e+\gamma} = \sqrt{(x\boldsymbol{p})^2 + m_e^2} + (1-x)|\boldsymbol{p}|$$

For $|\boldsymbol{p}| >> m_e$ the collinear $e + \gamma$ state has nearly the same energy as the single electron e.

⟹ In QED, collinear bremsstrahlung is enhanced by a factor $\log(|\boldsymbol{p}|/m_e)$.

Removing IR sensitivity in QCD

In QCD even soft gluons can change the color of the quark

$\Rightarrow$ we can never measure the color of a quark!

Also: Want to sum over all soft and collinear divergences up to a "Factorization scale" μ, large enough to apply PQCD for $Q > \mu$.

Define IR safe cross sections, which can be calculated in PQCD using the factorization theorem, and measured in experiments.

Infrared Safe observables

An observable is infrared safe if it is **insensitive** to

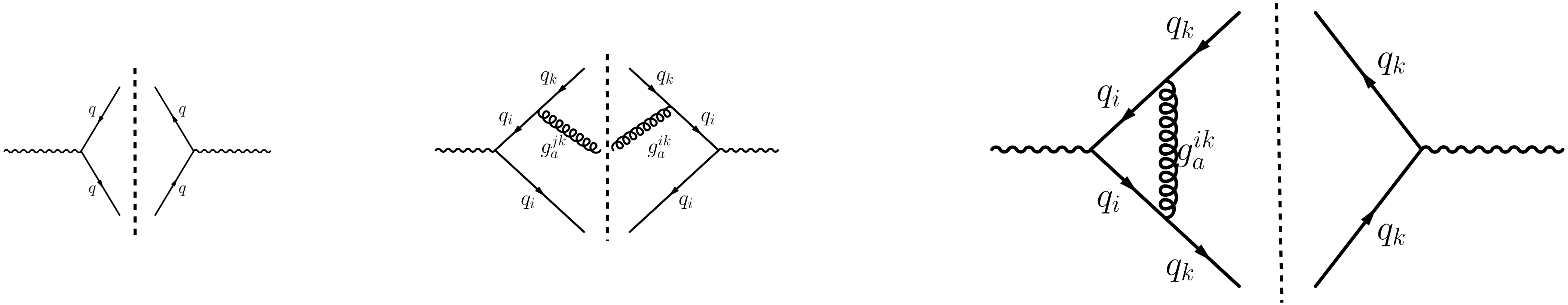
SOFT radiation:

Adding any number of infinitely soft particles should not change the value of the observable

COLLINEAR radiation:

Splitting an existing particle up into two comoving particles each with half the original momentum should not change the value of the observable

Removing IR sensitivity in QCD at NLO



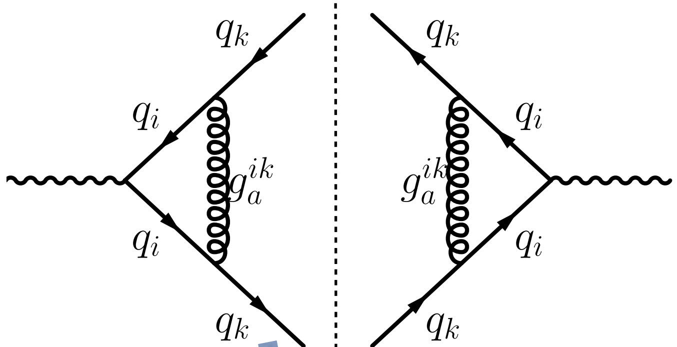
$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

after some hard work ...

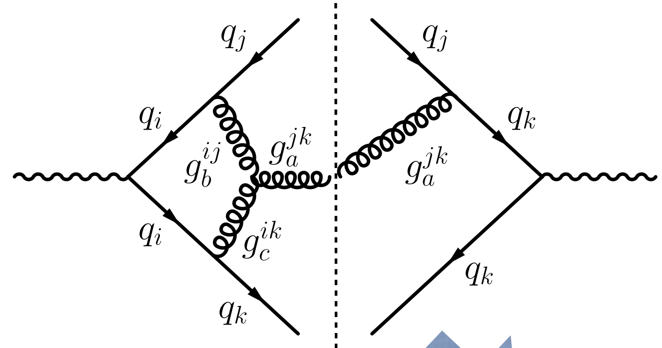
$$= \sigma_{\mathrm{Born}} + \mathrm{Finite}\left\{\int |M_{X+1}^{(0)}|^2\right\} + \mathrm{Finite}\left\{\int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]\right\}$$

Removing IR sensitivity in QCD at NNLO

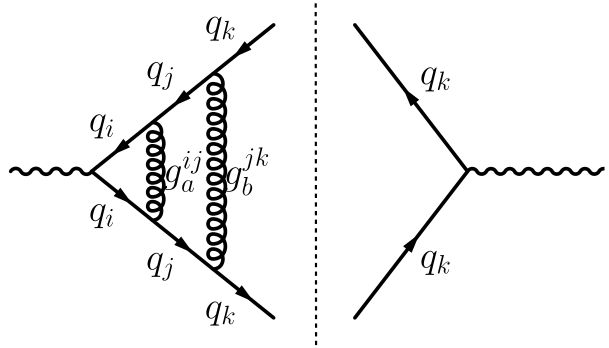
1-Loop × 1-Loop



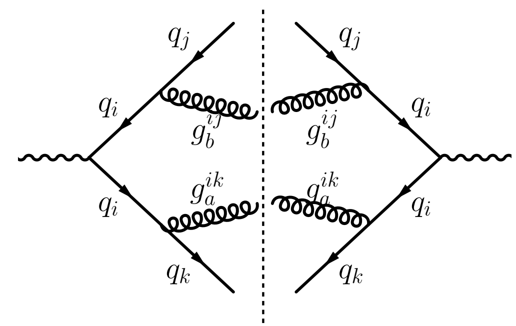
1-Loop × Real (X+1)



$$\sigma_{\rm X}^{\rm NNLO} = \sigma_X^{\rm NLO} + \int \left(|M_X^{(1)}|^2 + 2{\rm Re}[M_X^{(2)} M_X^{(0)*}] \right) + \int 2{\rm Re}[M_{X+1}^{(1)} M_{X+1}^{(0)*}] + \int |M_{X+2}^{(0)}|^2$$



Two-Loop × Born Interference



Real × Real (X+2)

and so on, at each fixed order in α_s

QCD cross sections at fixed order in α_s

Distribution of observable: O

In production of X + anything

Sum over identical amplitudes, then square

Momentum configuration

Fixed Order (all orders)

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{O}}\right|_{\mathrm{ME}} = \sum_{k=0} \int \mathrm{d}\Phi_{X+k} \left| \sum_{\ell=0} M^{(\ell)}_{X+k} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Phase Space

Cross Section differentially in O

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

In practice, matrix elements can be calculated only for the first few orders in α_s .

Event generators

At high energies many gluons are radiated. Then one needs to include high orders in α_s for which complete, IR regulated matrix elements are not available.

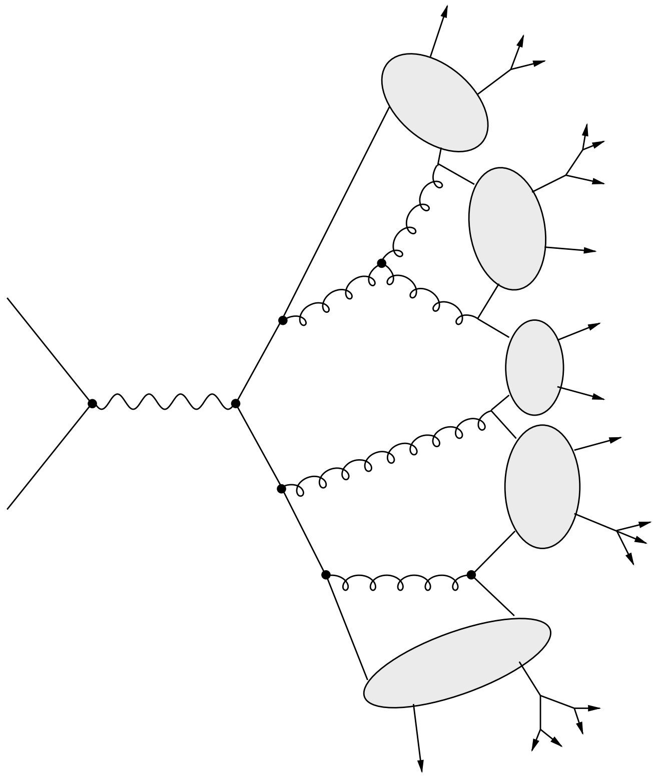
⇒ Shower Monte Carlo methods:

Include only the log enhanced terms of tree digrams (no loops)

Use phenomenological hadronization model for $Q < \mu$.

This gives approximate results for hadron distributions in the final state.

The reliability is tested by including next-to-leading logarithms and comparing several hadronization models.

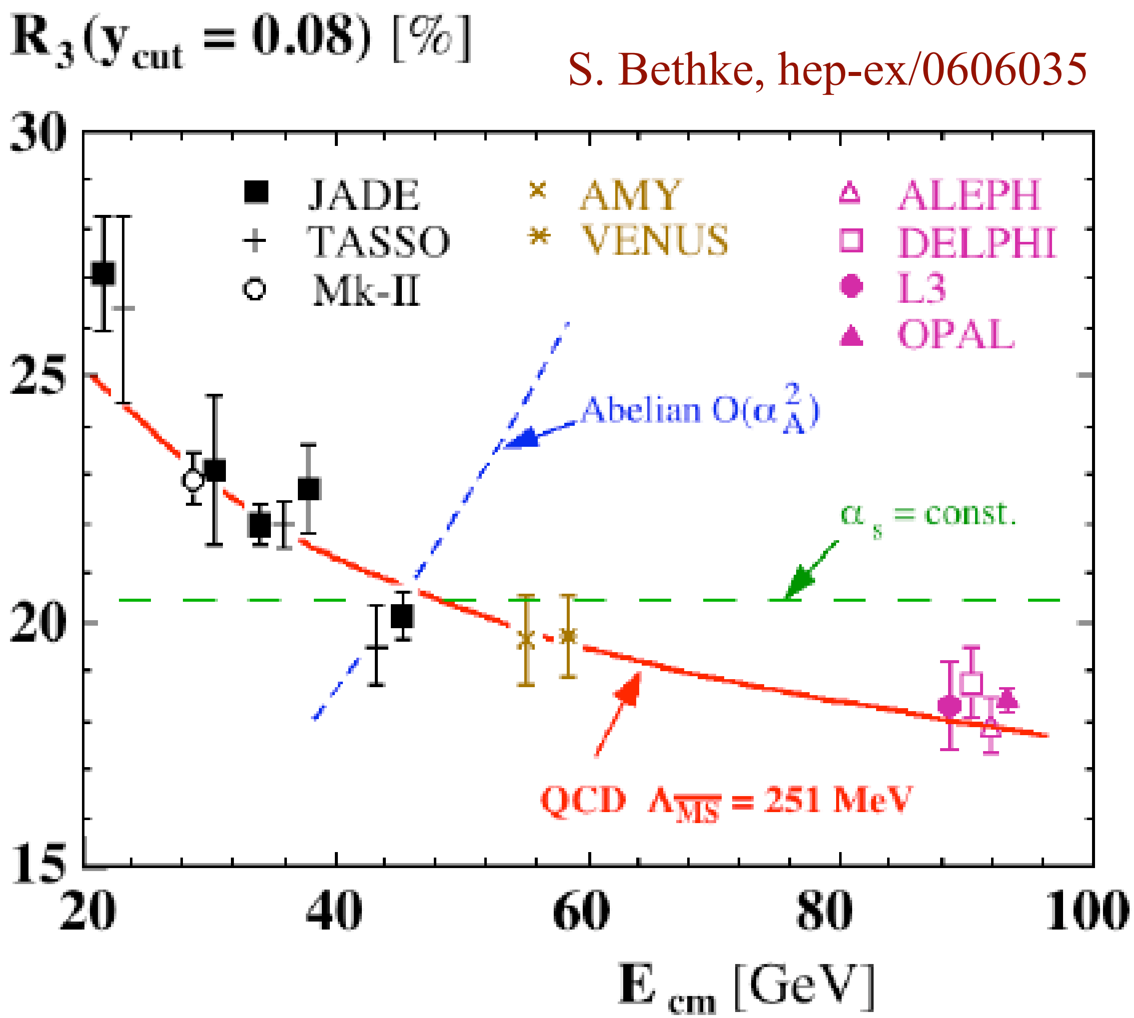


QCD is very successful in comparisons with data

Rate of 3-jet events in $e^+ e^-$ annihilations

$e^+ e^- \to q\, q\, g$

$\to$ 3 jets



Ex: Estimate the CM energy in e+ e– annihilations at which 2-jet structure emerges. In quark fragmentation, pions get an average fraction $\langle z \rangle \approx 0.1$ of the quark energy, and $\langle p_\perp \rangle \approx 350$ MeV.

Figure 8: Energy dependence of 3-jet event production rates, measured using the JADE jet finder at a scaled jet energy resolution $y_{cut} = 0.008$. The errors are experimental. The data are not corrected for hadronisation effects. They are compared to theoretical expectations of QCD, of an abelian vector gluon model, and to the hypothesis of a constant coupling strength.

S. Bethke, hep-ex/0606035

Angular distribution of 4-jet events in $e^+ e^-$ annihilations

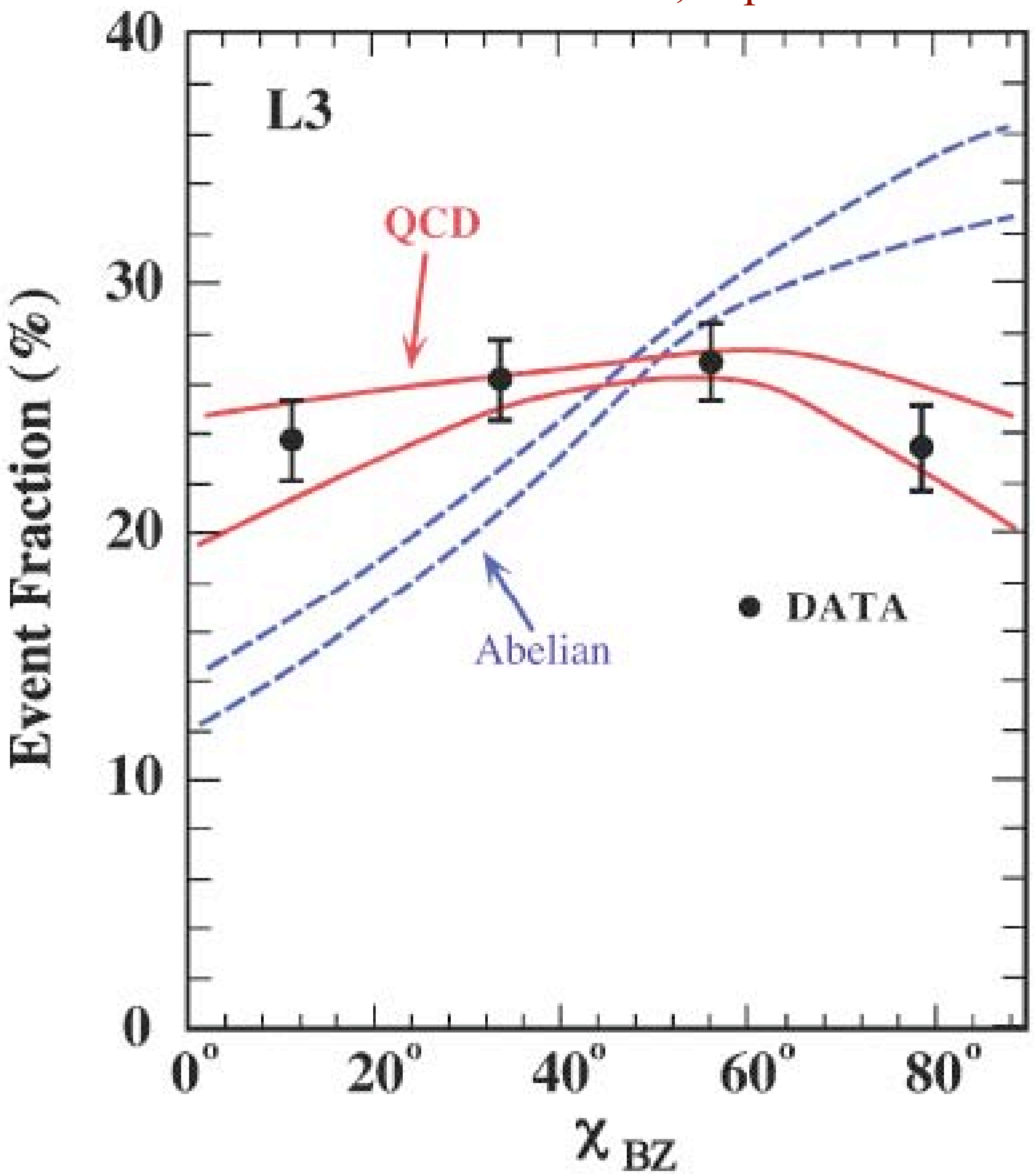
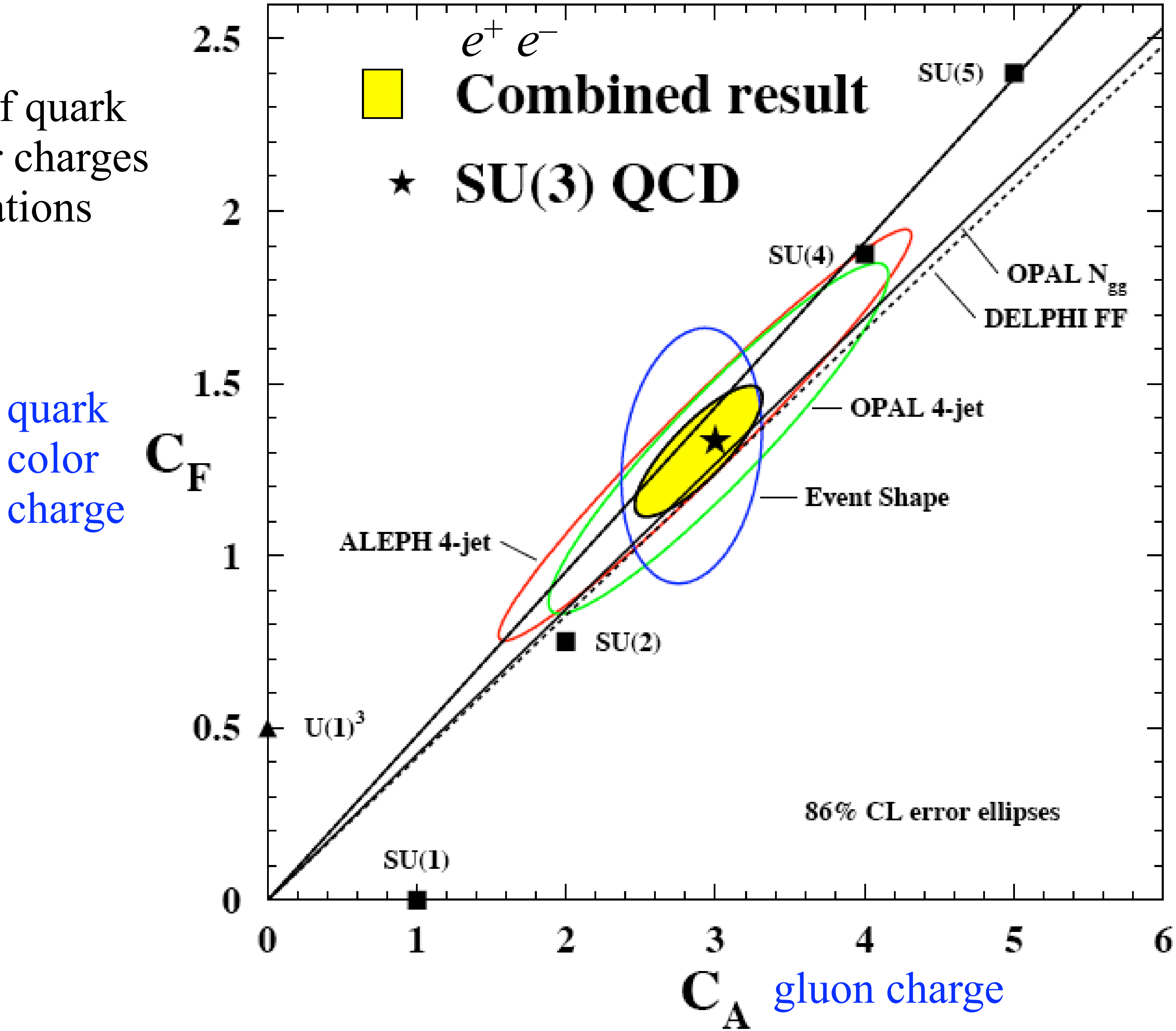


Figure 10: Distribution of the azimuthal angle between two planes spanned by the two high- and the two low-energy jets of hadronic 4-jet events measured at LEP [54], compared to the predictions of QCD and of an abelian vector gluon model where gluons carry no colour charge [27].

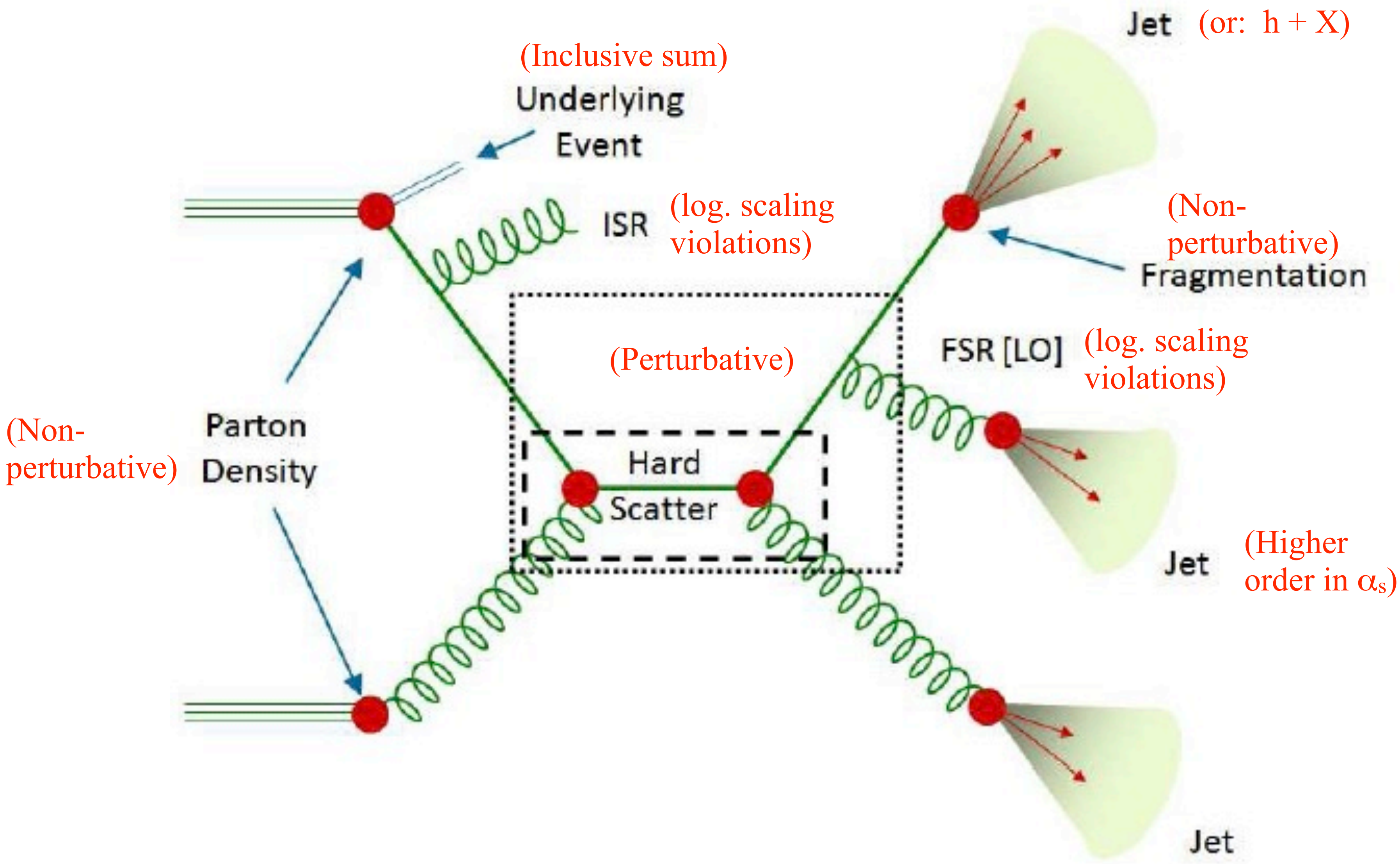
S. Bethke, hep-ex/0606035

Measurement of quark and gluon color charges in $e^+ e^-$ annihilations

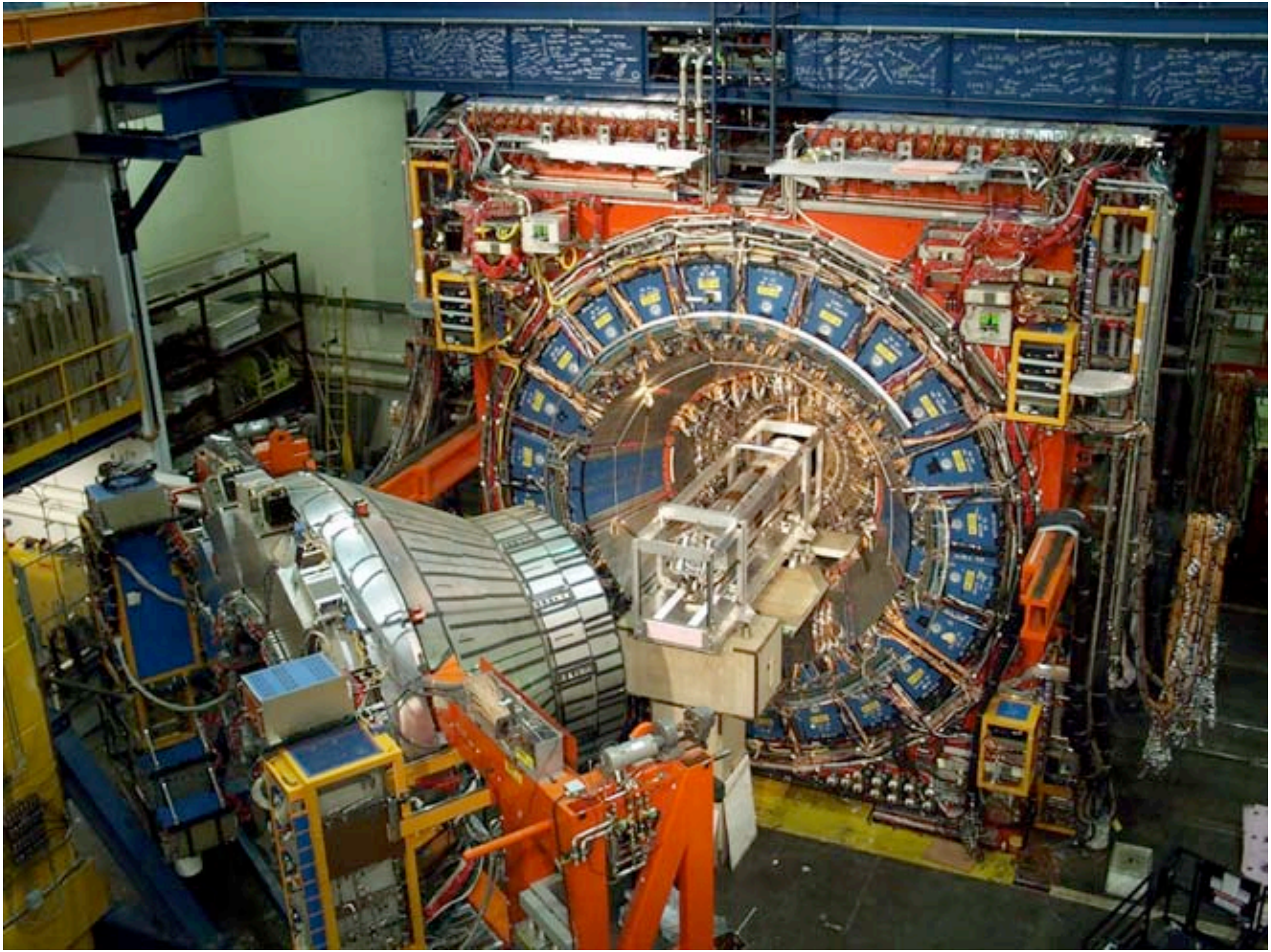


Jet production in hadron collisions

arXiv:1002.1708



The CDF Detector

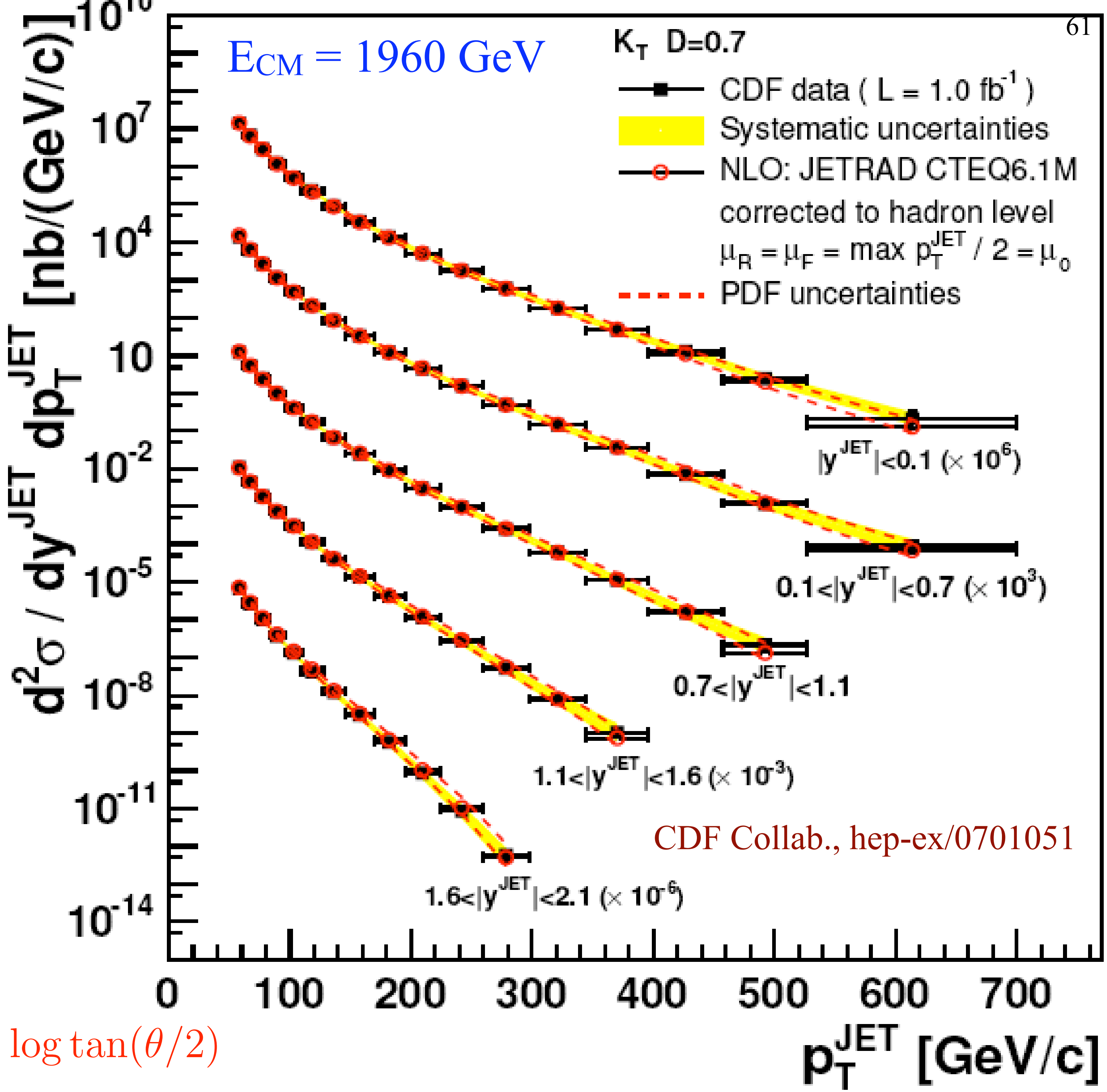


Fermilab:

$p\bar{p} \rightarrow \text{jet} + X$

$E_{CM} = 1.96$ TeV

Quarks and gluons are pointlike down to the best resolution that has been reached

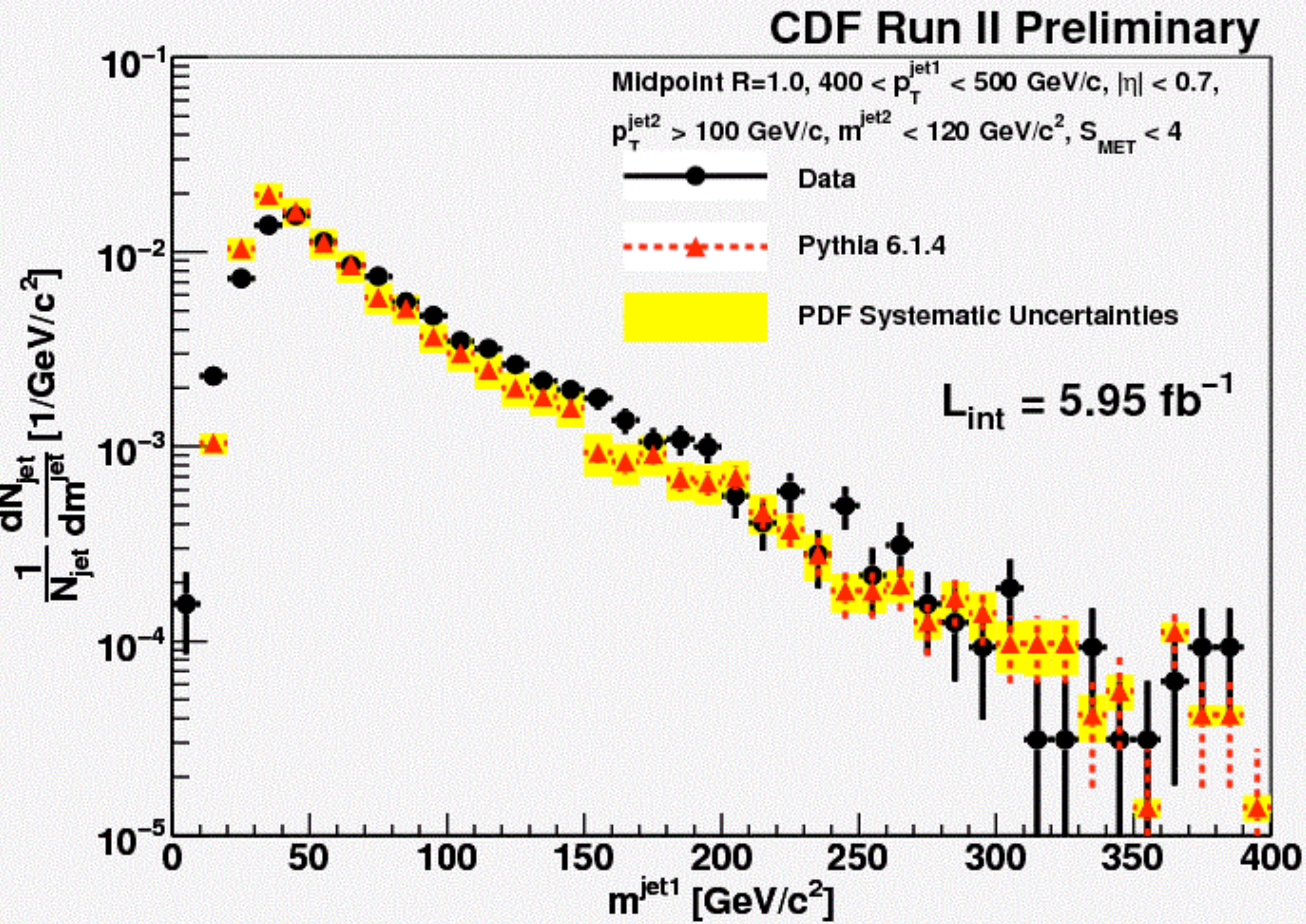


Ex: Estimate the maximum radius of quarks and gluons, given the agreement of QCD with the Fermilab jet data.

Rapidity:

$$y = \log \frac{E + p_\parallel}{\sqrt{m^2 + p_\perp^2}} \simeq -\log \tan(\theta/2)$$

$p\bar{p} \rightarrow$ jet + X: Jet mass distribution



Concluding remarks

Perturbative QCD has been successfully applied to hard collision data.

$$\alpha_s\,(M_Z^2) = 0.1189 \pm 0.0010$$

QCD effects constitutes the major background in the search for new physics at the Tevatron and the LHC. Hence much effort is expended on making the calculations as accurate as possible.

High intensity electron beams at Jefferson Lab, Mainz,... are mapping out hadron structure through Form factors and Generalized Parton Distributions.

Lattice QCD methods allow fast progress in the calculation of non-perturbative quantities, such as the hadron spectrum.

The simple Quark Model systematics of the hadron spectrum remains an encouraging mystery.