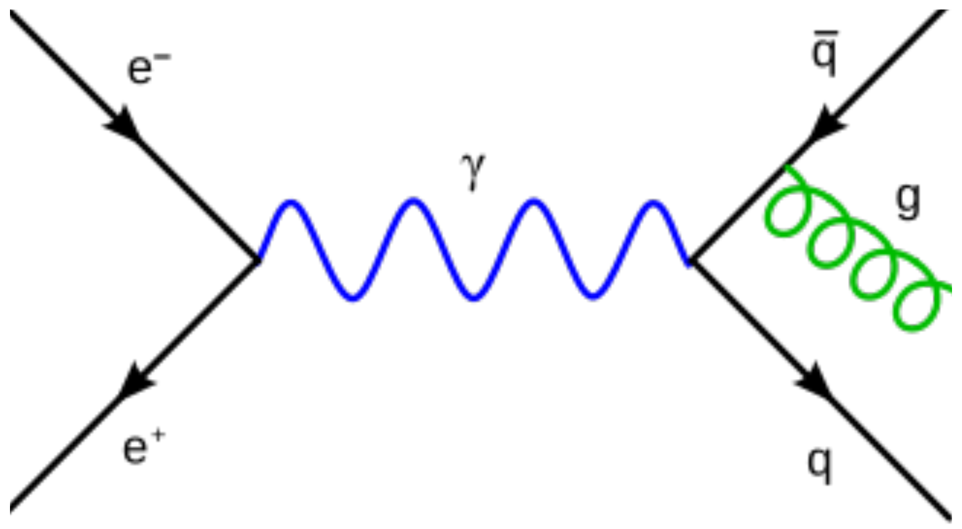
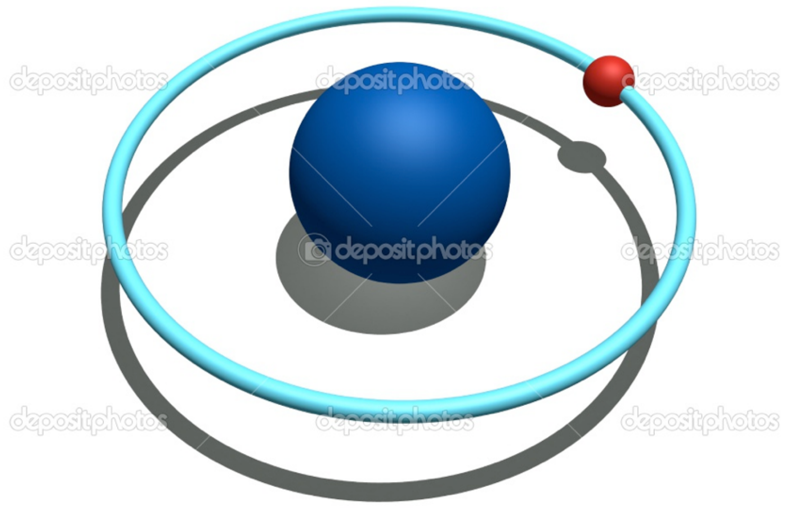
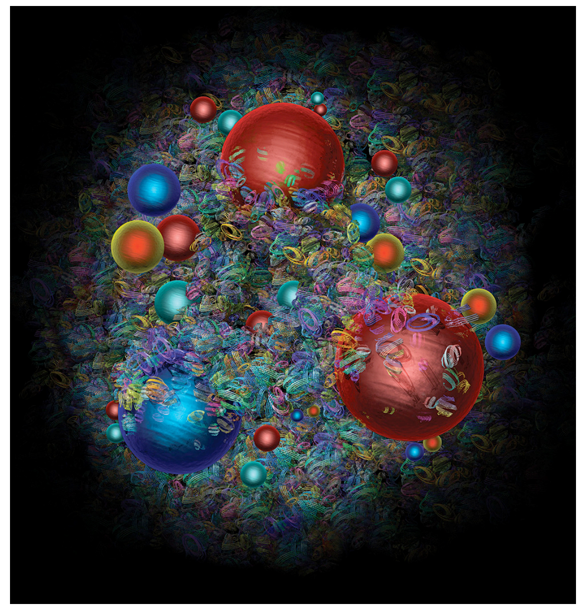


Gauge theory bound states

Paul Hoyer University of Helsinki

Higgs Center seminar on 6 May 2026





CERN Courier

Bound states vanish in QM → QFT transition

QFT textbooks omit atoms & hadrons

Lattice QCD has verified hadrons

An analytic framework is missing

Analogous to IP derivation of Feynman diagrams for scattering

Now: Discuss an analytic method for QED & QCD bound states

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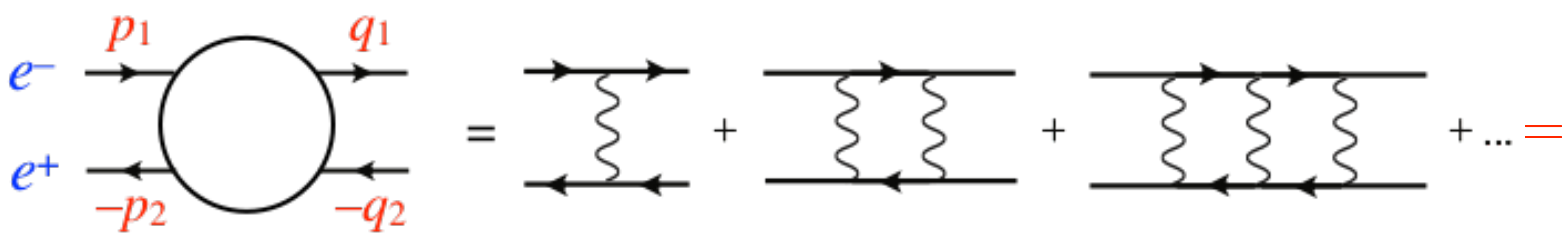
Atoms from Feynman diagrams

Schrödinger equation
from ladder diagrams



$$= \frac{R}{(p_1+p_2)^2 - M^2} + \ldots$$

Ladders dominate when $\alpha \ll 1$ and exchanged momenta $p = \mathcal{O}(\alpha m)$. Gives $V(r) = -\dfrac{\alpha}{r}$

NRQED expands in powers of $\dfrac{p}{m}$ around the Schrödinger atom

Positronium hyperfine splitting:

$$\frac{\Delta E}{m_e} = \frac{7}{12}\alpha^4 - \left(\frac{8}{9} + \frac{\ln 2}{2}\right)\frac{\alpha^5}{\pi} - \frac{5}{24}\alpha^6 \ln\alpha + \left[\frac{1367}{648} - \frac{5197}{3456}\pi^2 + \left(\frac{221}{144}\pi^2 + \frac{1}{2}\right)\ln 2 - \frac{53}{32}\zeta(3)\right]\frac{\alpha^6}{\pi^2} - \frac{7\alpha^7}{8\pi}\ln^2\alpha + \left(\frac{17}{3}\ln 2 - \frac{217}{90}\right)\frac{\alpha^7}{\pi}\ln\alpha + \mathcal{O}(\alpha^7)$$

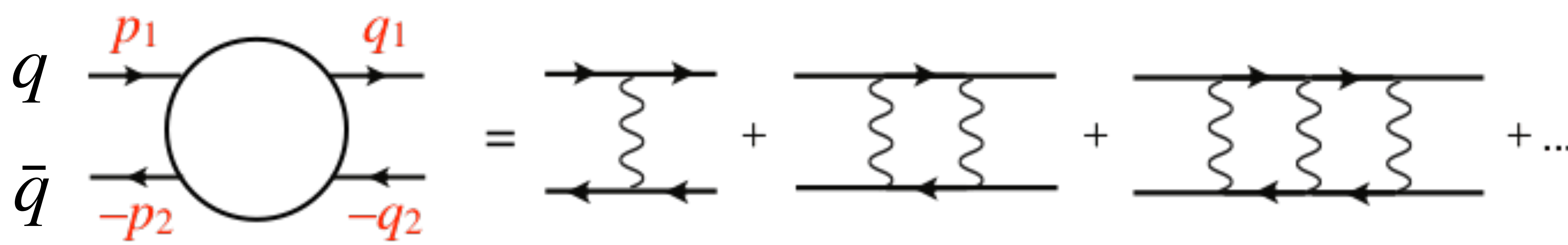
Agrees with experiment: $\Delta\nu_{QED} = 203.39169(41)$ GHz

$\Delta\nu_{EXP} = 203.394 \pm .002$ GHz

Bound state review:
Adkins, Cassidy, and Pérez-Ríos (2022)

No mesons from Feynman diagrams

Schrödinger equation from ladder diagrams

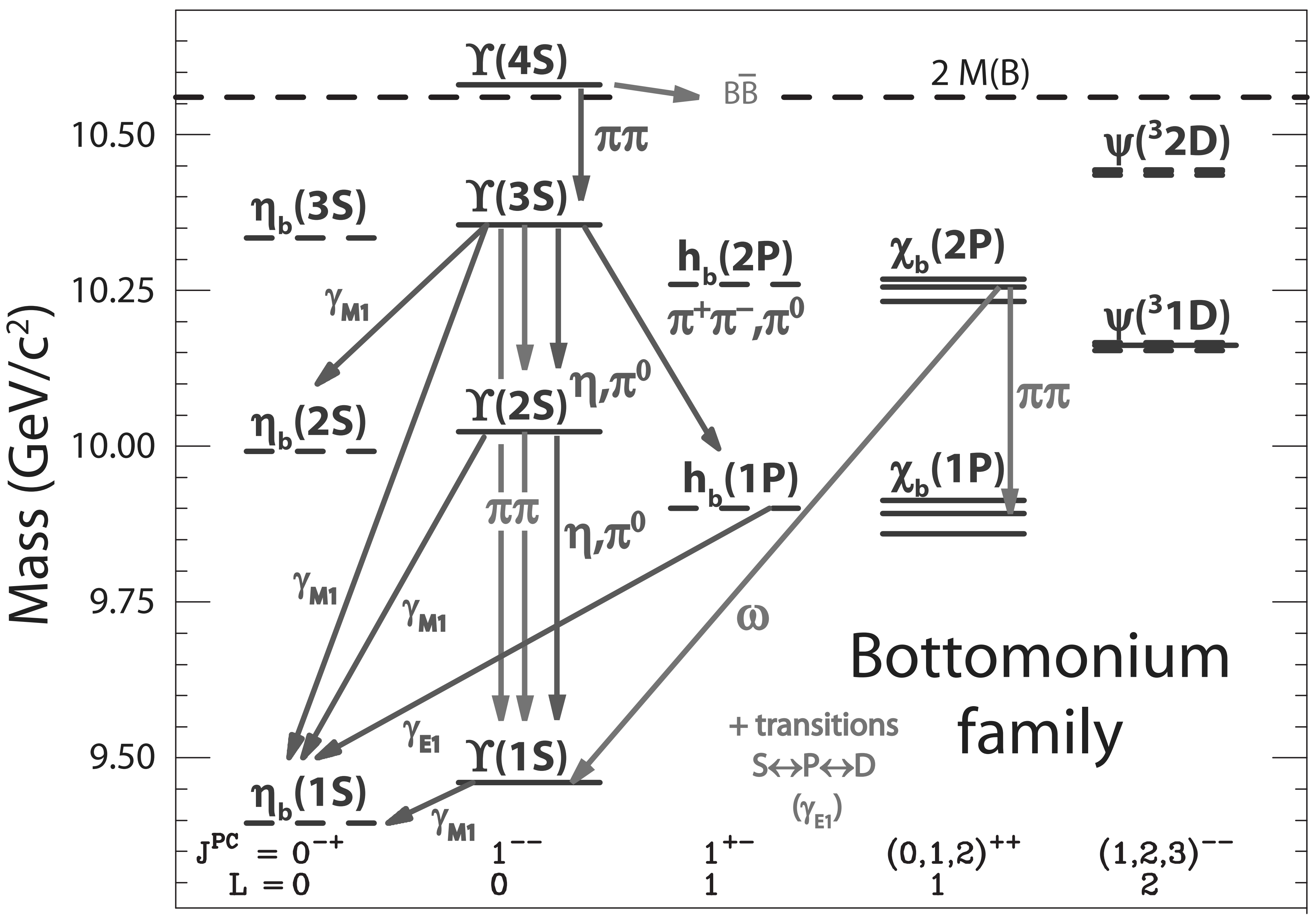


$$= \frac{R}{(p_1+p_2)^2 - M^2} + \ldots$$

QCD ladders dominate when $\alpha_s \ll 1$ and exchanged momenta $p = \mathcal{O}(\alpha_s m)$.

Gives: $V(r) = -C_F \dfrac{\alpha}{r}$ No confinement! Is $\alpha_s > 1$?

$b\bar{b}$: Atomic spectrum



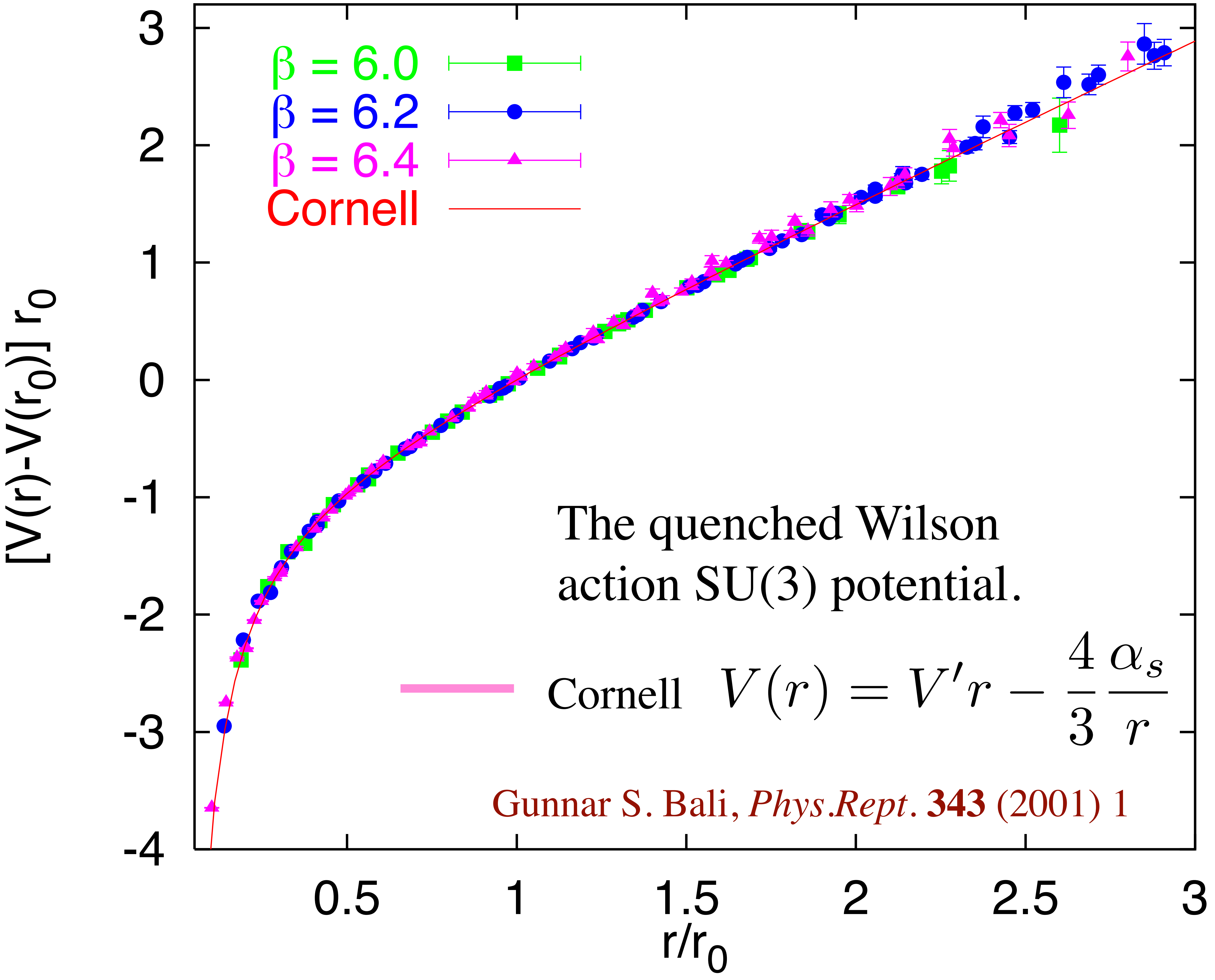
Quarkonium phenomenology:
Schrödinger equation with phenomenological

"Cornell" potential: $V(r) = V'r - \dfrac{4}{3}\dfrac{\alpha_s}{r}$

$V' \simeq 0.18\ \mathrm{GeV}^2,\ \ \alpha_s(m_c^2) \simeq 0.39$

Confinement scale V′ appears in the non-relativistic, Schrödinger (no-loop) framework

Lattice QCD confirmed the Cornell potential



How does the confinement scale V' arise in QCD?

From a boundary condition?

Need a bound state approach not based on free propagators (Feynman diagrams).

Start from scratch: Canonical quantization

Lose explicit Poincaré invariance

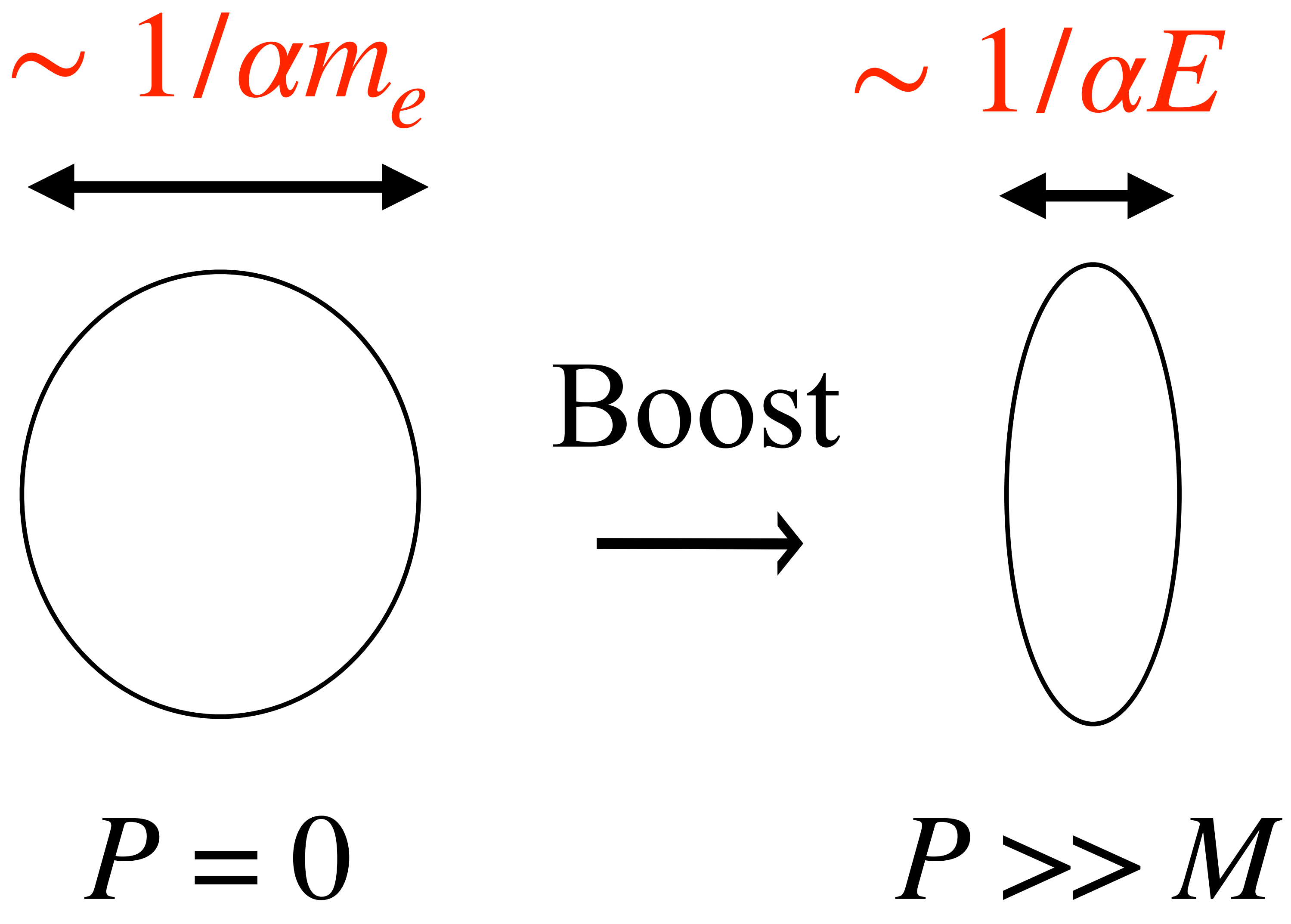
It is essential not to change the action (*ie.*, the theory)

Ensures the non-trivial Poincaré invariance of bound states

E.g.: Atoms in motion

Positronium in motion

Lorentz contraction of equal-time states:



The Coulomb potential $-\frac{\alpha}{r}$ grows with P, whereas excitation energies decrease with P:

$$\sqrt{P^2+(2m+E_B)^2}-\sqrt{P^2+4m^2}\simeq\frac{2m\,E_B}{P}$$

QED: $|e^-e^+\gamma\rangle$ Fock state contributes to E_B at leading $\mathcal{O}(m_e\alpha^2)$ for $P>0$.

It subtracts the large Coulomb energy, ensuring $E=\sqrt{M^2+P^2}$

M. Järvinen, Phys. Rev. D71 (2005) 085006 [hep-ph/0411208]

Temporal gauge: $A^0(t,\boldsymbol{x}) = 0$

$$-\frac{1}{4}\int dt\, d\boldsymbol{x} F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\int dt\, d\boldsymbol{x}\Big[(\partial_t \boldsymbol{A} + \nabla A^0)^2 - (\nabla \times \boldsymbol{A}_T)^2\Big] + \mathcal{O}(A^3, A^4)$$

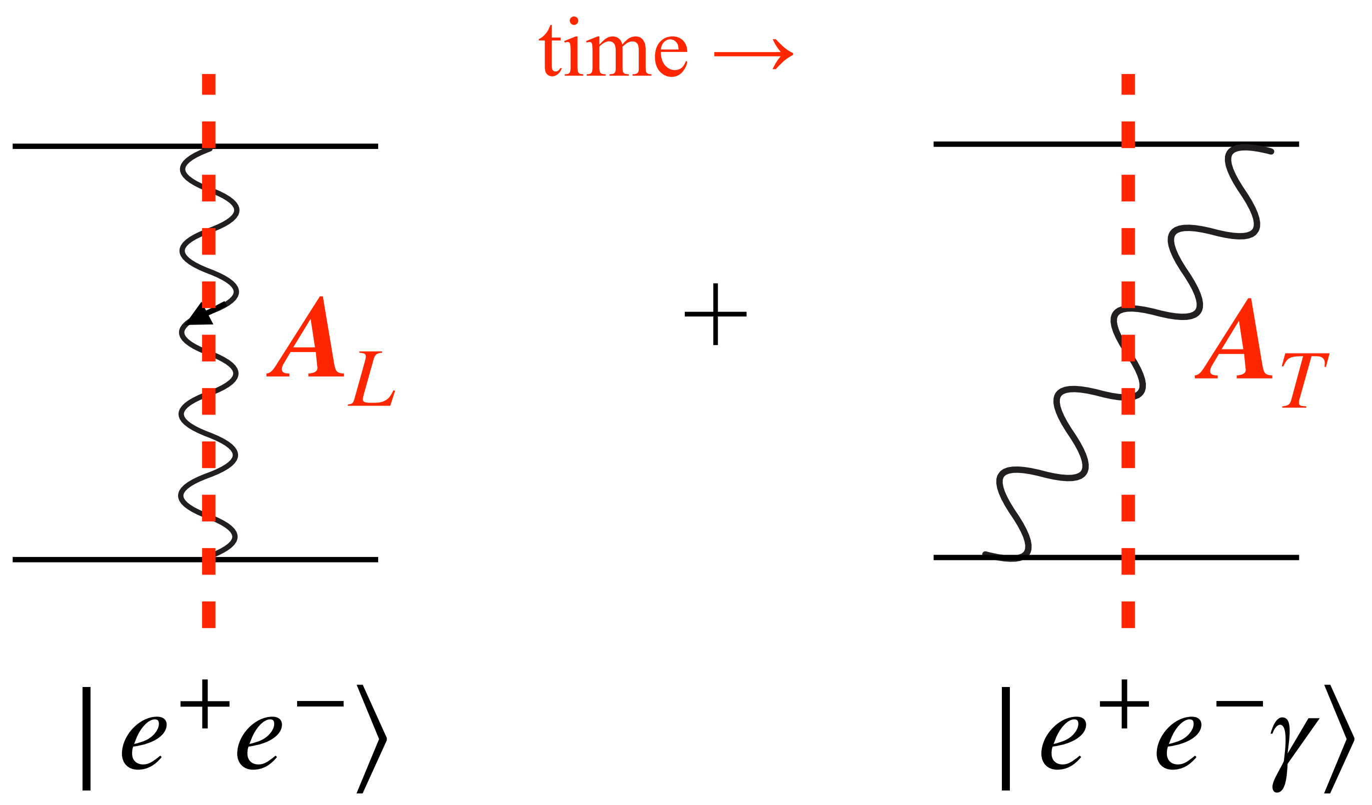
$\partial_t^2 A^0$, $\nabla^2 \boldsymbol{A}_L$ are missing

A^0 and $\boldsymbol{A}_L$ do not propagate

$A^0(t,\boldsymbol{x}) = 0$ gauge gives an instantaneous potential: Constraint at all $\boldsymbol{x}$ at an instant of time

Binding without propagating gauge bosons

Only gauge theory has truly instantaneous interactions



In Feynman gauge all four A^μ components propagate: $\mathcal{S}_{GF} = -\frac{1}{2}\int dt\, d\boldsymbol{x}\, (\partial_\mu A^\mu)^2 = -\frac{1}{2}\int dt\, d\boldsymbol{x}\, (\partial_t A^0 + \nabla \cdot \boldsymbol{A})^2$

Canonical quantization in $A^0(t, \boldsymbol{x}) = 0$ gauge

The electric field $\boldsymbol{E} = -\partial_t \boldsymbol{A}$ is conjugate to $-\boldsymbol{A}$.

A^0 and its conjugate field vanish

Canonical commutators:

$$\left[E^i(t, \boldsymbol{x}), A^j(t, \boldsymbol{y})\right] = i\,\delta^{ij}\delta(\boldsymbol{x} - \boldsymbol{y}) \qquad \{\psi_\alpha^\dagger(t, \boldsymbol{x}), \psi_\beta(t, \boldsymbol{y})\} = \delta_{\alpha\beta}\,\delta(\boldsymbol{x} - \boldsymbol{y})$$

Time-independent gauge transformations preserve $A^0 = 0$

$$G(t, \boldsymbol{x}) \equiv \frac{\delta \mathcal{S}_{QED}}{\delta A^0(t, \boldsymbol{x})} = \boldsymbol{\nabla} \cdot \boldsymbol{E}(t, \boldsymbol{x}) - e\psi^\dagger\psi(t, \boldsymbol{x})$$

Gauss' operator $G(t, \boldsymbol{x})$ generates time-independent gauge transformations

Physical states must be fully gauge fixed: $G(t, \boldsymbol{x})\,|\,phys\rangle = 0$

A free electron state $b^\dagger(\boldsymbol{p}, \lambda)\,|\,0\rangle$ is unphysical in temporal gauge.

Physical electrons have an A_L field

$$|e^-;\boldsymbol{p},\lambda,t\rangle = \int d\boldsymbol{x}\,\psi^\dagger(t,\boldsymbol{x})\,u(\boldsymbol{p},\lambda)\,e^{i\boldsymbol{p}\cdot\boldsymbol{x}}\,W(\boldsymbol{A}_L;t,\boldsymbol{x})\,|0\rangle$$

satisfies $G(t,\boldsymbol{x})\,|e^-;\boldsymbol{p},\lambda,t\rangle = 0$

$$G(t,\boldsymbol{x}) \equiv \frac{\delta\mathcal{S}_{QED}}{\delta A^0(t,\boldsymbol{x})} = \boldsymbol{\nabla}\cdot\boldsymbol{E}(t,\boldsymbol{x}) - e\psi^\dagger\psi(t,\boldsymbol{x})$$

$$W(\boldsymbol{A}_L;t,\boldsymbol{x}) = \exp\Big[i\int d\boldsymbol{y}\,\boldsymbol{A}_L(t,\boldsymbol{y})\cdot\nabla_y\Big(\frac{e}{4\pi|\boldsymbol{y}-\boldsymbol{x}|}\Big)\Big]$$

$|e^-;\boldsymbol{p},\lambda,t\rangle$ is spatially extended, includes a gauge field.

The Hamiltonian is a local functional of the electron and gauge fields:

$$H_{QED} = \int d\boldsymbol{x}\big[\tfrac{1}{2}(\boldsymbol{E}_L^2 + \boldsymbol{E}_T^2) + \tfrac{1}{4}F^{ij}F^{ij} + \psi^\dagger(-i\boldsymbol{\alpha}\cdot\nabla + m\gamma^0 - e\,\boldsymbol{\alpha}\cdot\boldsymbol{A})\psi\big]$$

Schrödinger equation for Positronium

Positronium at $t = 0$ in its rest frame:

$$\left|e^+e^-, t=0\right\rangle = \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar\psi_\alpha(\boldsymbol{x}_1)\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\psi_\beta(\boldsymbol{x}_2)\left|0\right\rangle$$

Physical e^- and e^+, distributed by a c-numbered wave function $\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$

$$W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2] \equiv \exp\Big[\frac{ie}{4\pi}\int d\boldsymbol{y}\,\boldsymbol{A}(\boldsymbol{y})\cdot\boldsymbol{\nabla}_y\Big(\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_2|}\Big)\Big]$$

The Coulomb potential is locally built from the gauge field energy density:

$$\tfrac{1}{2}\int d\boldsymbol{x}\,\big[\boldsymbol{E}_L^2(\boldsymbol{x}),W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\big] = -\frac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]$$

The interaction term $\left[\int d\boldsymbol{x}\,\psi^\dagger(-e\,\boldsymbol{\alpha}\cdot\boldsymbol{A})\psi\right]$ of H contributes at higher order in α

$$H_{QED}\left|e^+e^-\right\rangle = (2m+E_B)\left|e^+e^-\right\rangle$$

gives the NR Schrödinger equation for $\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)$ and structure of Positronium in motion.

$q\bar{q}$ bound states in QCD

$$|q\bar{q};t=0\rangle \equiv \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_A(\boldsymbol{x}_1)\Phi(\boldsymbol{x}_1-\boldsymbol{x}_2)W_{AB}[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\psi_B(\boldsymbol{x}_2)\,|0\rangle$$

$$W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2] \equiv \exp\Big[\frac{ig}{4\pi}\int_{\boldsymbol{x}_2}^{\boldsymbol{x}_1} d\boldsymbol{y}\, T^a \boldsymbol{A}_L^a(\boldsymbol{y})\cdot\boldsymbol{\nabla}_y\Big(\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_2|}\Big)\Big]$$

Gauss' constraint $G_a(t,\boldsymbol{x})\,|\,q\bar{q}\rangle = 0$:
$$\boldsymbol{\nabla}\cdot\boldsymbol{E}_L^a(\boldsymbol{x})\,|q\bar{q}\rangle = \big[-g\,f_{abc}\boldsymbol{A}_T^b\cdot\boldsymbol{E}_T^c(\boldsymbol{x}) + g\,\psi^\dagger T^a\psi(\boldsymbol{x})\big]\,|q\bar{q}\rangle$$
is satisfied.

The potential is not confining:
$$\tfrac{1}{2}\int d\boldsymbol{x}\,\big[\boldsymbol{E}_{L,a}^2(\boldsymbol{x}),W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\big] = -C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]$$

Boundary condition in Gauss constraint

We needed: $$\nabla_x^2\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}\right)=-4\pi\left[\delta(\boldsymbol{x}-\boldsymbol{x}_1)-\delta(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

Did we miss something?

Add a homogeneous term in $W[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]$: $$\nabla_x^2\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_2|}+\kappa\,\boldsymbol{x}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)\right)=-4\pi\left[\delta(\boldsymbol{x}-\boldsymbol{x}_1)-\delta(\boldsymbol{x}-\boldsymbol{x}_2)\right]$$

$$W_\kappa[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\equiv\exp\left\{i\int_{\boldsymbol{x}_2}^{\boldsymbol{x}_1}d\boldsymbol{y}\,T^a\boldsymbol{A}_L^a(\boldsymbol{y})\cdot\left[\frac{g}{4\pi}\nabla_y\left(\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_1|}-\frac{1}{|\boldsymbol{y}-\boldsymbol{x}_2|}\right)+\kappa\,(\boldsymbol{x}_1-\boldsymbol{x}_2)\right]\right\}$$

- $\kappa\,\boldsymbol{x}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)$ is invariant under rotations
- Linear in $\boldsymbol{x}$, hence electric field $\propto\nabla_x\boldsymbol{x}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)=\boldsymbol{x}_1-\boldsymbol{x}_2$ is isotropic (independent of $\boldsymbol{x}$)

Gives potential: $V(\boldsymbol{x}_1-\boldsymbol{x}_2)=\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2|-C_F\dfrac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$

Confinement scale $\Lambda^2\propto\kappa$ from a boundary condition.

Globally color singlet $q\bar{q}$:

$q(x)\to U\,q(x)\qquad U\neq U(x)$

$T^a\boldsymbol{A}_L^a(y)\to U\,T^a\boldsymbol{A}_L^a(y)\,U^\dagger$

Confining potential from $\kappa\,\boldsymbol{x}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)$

Contribution of $\boldsymbol{A}_L$ field to Hamiltonian:

$$\frac{1}{2}\int d\boldsymbol{x}\big[\boldsymbol{E}_{L.a}^2(\boldsymbol{x}),W_\kappa[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\big] = W_\kappa[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\tfrac{1}{2}C_F\int d\boldsymbol{x}\Big[\frac{g}{4\pi}\nabla_x\Big(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1|}-\frac{1}{\boldsymbol{x}-\boldsymbol{x}_2}\Big)+\kappa(\boldsymbol{x}_1-\boldsymbol{x}_2)\Big]^2$$

$$= -C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+C_F g\kappa(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+\frac{1}{2}C_F\kappa^2(\boldsymbol{x}_1-\boldsymbol{x}_2)^2\int d\boldsymbol{x}$$

In the last term $\int d\boldsymbol{x} \propto$ Volume of space: Must be universal to be irrelevant.

Define $\kappa \equiv \dfrac{\Lambda^2}{C_F g}\dfrac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$ gives the energy density of the ground state: $E_\Lambda = \dfrac{\Lambda^4}{2C_F g^2}$

and potential energy $V(\boldsymbol{x}_1-\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\dfrac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$

Λ is the only free parameter,
All states have the same E_Λ .

Instantaneous potentials for other singlet states

$q(\boldsymbol{x}_1)q(\boldsymbol{x}_2)q(\boldsymbol{x}_3)$:

$$V_{qqq} = \frac{\Lambda^2}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+(\boldsymbol{x}_2-\boldsymbol{x}_3)^2+(\boldsymbol{x}_3-\boldsymbol{x}_1)^2} - \frac{2}{3}\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|}+\frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

$g(\boldsymbol{x}_1)g(\boldsymbol{x}_2)$:

$$V_{gg} = \sqrt{\frac{N_c}{C_F}}\,\Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - N_c\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$

$q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)g(\boldsymbol{x}_g)$:

$$V_{q\bar{q}g} = \frac{\Lambda^2}{\sqrt{C_F}}\sqrt{\tfrac{1}{4}\big(N_c-\tfrac{2}{N_c}\big)(\boldsymbol{x}_1-\boldsymbol{x}_2)^2+N_c\big(\boldsymbol{x}_g-\tfrac{1}{2}\boldsymbol{x}_1-\tfrac{1}{2}\boldsymbol{x}_2\big)^2} + \frac{\alpha_s}{2}\Big[\frac{1}{N_c}\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} - N_c\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_g|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_g|}\Big)\Big]$$

Consistency conditions:

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2)$$

$$V_{q\bar{q}g}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2)$$

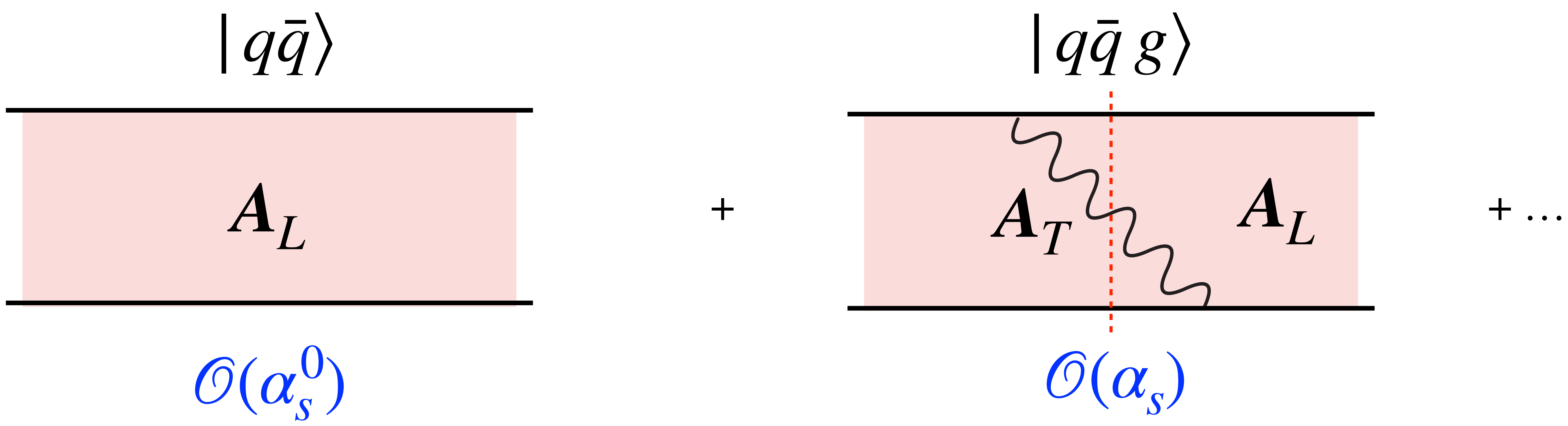
$$V_{q\bar{q}g}(\boldsymbol{x}_1,\boldsymbol{x}_1,\boldsymbol{x}_g) = V_{gg}(\boldsymbol{x}_1,\boldsymbol{x}_g)$$

Bound state equation for mesons

$$|q\bar{q};M,\boldsymbol{P}\rangle \equiv \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\,\bar{\psi}(\boldsymbol{x}_1)e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\,\Phi^{(P)}(\boldsymbol{x}_1-\boldsymbol{x}_2)W_\kappa[\boldsymbol{A}_L;\boldsymbol{x}_1,\boldsymbol{x}_2]\psi(\boldsymbol{x}_2)\,|0\rangle$$

BSE: $H_{QCD}|q\bar{q};M,\boldsymbol{P}\rangle = E_P|q\bar{q};M,\boldsymbol{P}\rangle$

Perturbative expansion: BSE at $\mathcal{O}(\alpha_s^n)$



$\mathcal{O}(\alpha_s^0)$: $V(\boldsymbol{x}) = \Lambda^2|\boldsymbol{x}|$, with $g=0$ in H_{QCD} Lowest order in perturbative expansion

$$i\boldsymbol{\nabla}\cdot\{\boldsymbol{\alpha},\Phi^{(P)}(\boldsymbol{x})\} - \tfrac{1}{2}\boldsymbol{P}\cdot\big[\boldsymbol{\alpha},\Phi^{(P)}(\boldsymbol{x})\big] + m\big[\gamma^0,\Phi^{(P)}(\boldsymbol{x})\big] = \big[E_P - V(\boldsymbol{x})\big]\Phi^{(P)}(\boldsymbol{x})$$

Poincaré covariant

$\boldsymbol{P}=0$: Radial & angular variables separate, J^{PC} of states determined

No quark model exotic states

Chiral symmetry for $m=0$: Parity doublets

Example: $-\eta_P = \eta_C = (-1)^j$ states at $\mathcal{O}(\alpha_s^0)$, P = 0

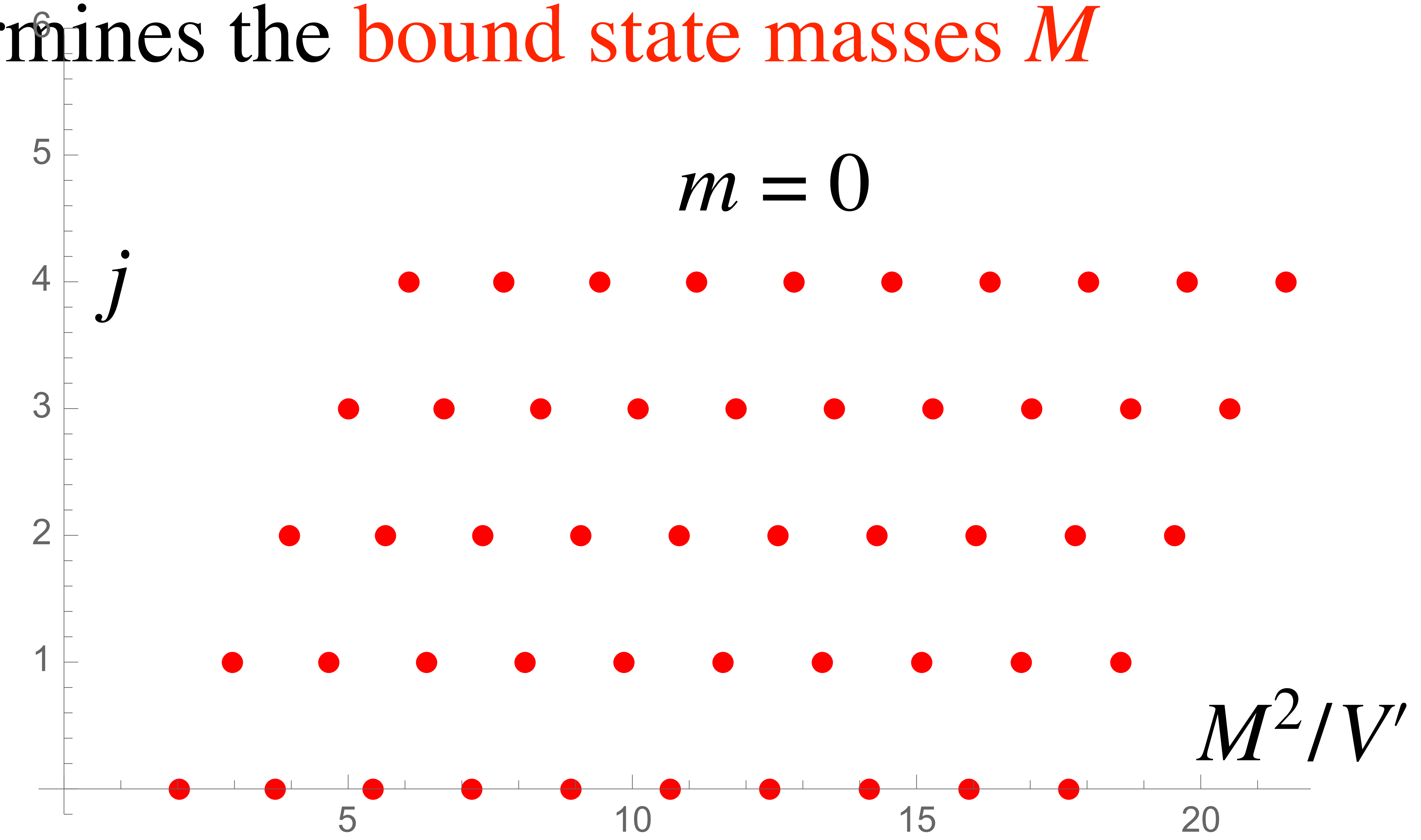
$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overset{\rightarrow}{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

$$F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\tfrac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$$ Radial equation

Regularity of the wave function determines the bound state masses M

Linear Regge trajectories with daughters:

Spectrum similar to dual models



Breaking of chiral symmetry at m=0

Massless ($M = 0$) $J^{PC} = 0^{++}$ state σ for $m = \boldsymbol{P} = 0$: Poincaré invariant, can mix with vacuum

$$\hat{\sigma}\,|0\rangle = \int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{q}(\boldsymbol{x}_1)\Phi_\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2)W_\kappa[\boldsymbol{A}_L; \boldsymbol{x}_1, \boldsymbol{x}_2]q(\boldsymbol{x}_2)\,|0\rangle \qquad q = \begin{pmatrix} u \\ d \end{pmatrix} \text{ massless isospin doublet}$$

BSE: $i\nabla \cdot \{\boldsymbol{\alpha}, \Phi_\sigma(\boldsymbol{x})\} + V'r\,\Phi_\sigma(\boldsymbol{x}) = 0$ $\qquad \Phi_\sigma(\boldsymbol{x}) = N_\sigma\Big[J_0(\tfrac{1}{4}V'r^2) + \dfrac{i\boldsymbol{\alpha}\cdot\boldsymbol{x}}{r}J_1(\tfrac{1}{4}V'r^2)\Big]$

Generator of chiral transformations: $Q_{5j} = \int d\boldsymbol{x}\, q^\dagger(\boldsymbol{x})\gamma_5 \tfrac{1}{2}\tau^j\, q(\boldsymbol{x})$

Massless $J^{PC} = 0^{-+}$ pion: $i\big[Q_{5j}, \hat{\sigma}\big] = \hat{\pi}_j$ with $\Phi_{\pi j}(\boldsymbol{x}) = -\,i\tau^j\gamma_5\,\Phi_\sigma(\boldsymbol{x})$

Define σ condensate vacuum: $|\sigma\rangle = \exp(\hat{\sigma})\,|0\rangle$ Ground state is not chiral invariant.

Build states with $|0\rangle \to |\sigma\rangle$: Preserves J^{PC} , but breaks chiral invariance: no parity doublets.

Preliminary suggestion for χSB. Requires further study.

THE STATE IS NOT ABOLISHED, IT WITHERS AWAY: HOW QUANTUM FIELD THEORY BECAME A THEORY OF SCATTERING

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12th November 2020

2011.0598

Learning quantum field theory (QFT) for the first time, after first learning quantum mechanics (QM), one is (or maybe, rather, I was) struck by the change of emphasis: The notion of the quantum state, which plays such an essential role in QM, from the stationary states of the Bohr atom, over the Schrödinger equation to the interpretation debates over measurement and collapse, seems to fade from view when doing QFT.

Summary

We need a framework for QED & QCD bound states, based on the action. Poincaré symmetry!

Feynman diagrams are questionable for bound states: Expand around free propagation.

Temporal gauge fixing gives an instantaneous interaction.

Physical states must satisfy Gauss' law. Physical electron has an $\boldsymbol{A}_L$ field.

Universal confinement scale Λ_{QCD} from a boundary condition on Gauss' constraint .

Confining potentials for (globally) color singlet states $(q\bar{q},\ qqq,\ gg,\ q\bar{q}g\ldots)$

Poincaré covariant eigenstates of H_{QCD} . No J^{PC} states that would be exotic in the quark model.

Chiral symmetry breaking at $m = 0$ by building states on a "σ condensate" vacuum.