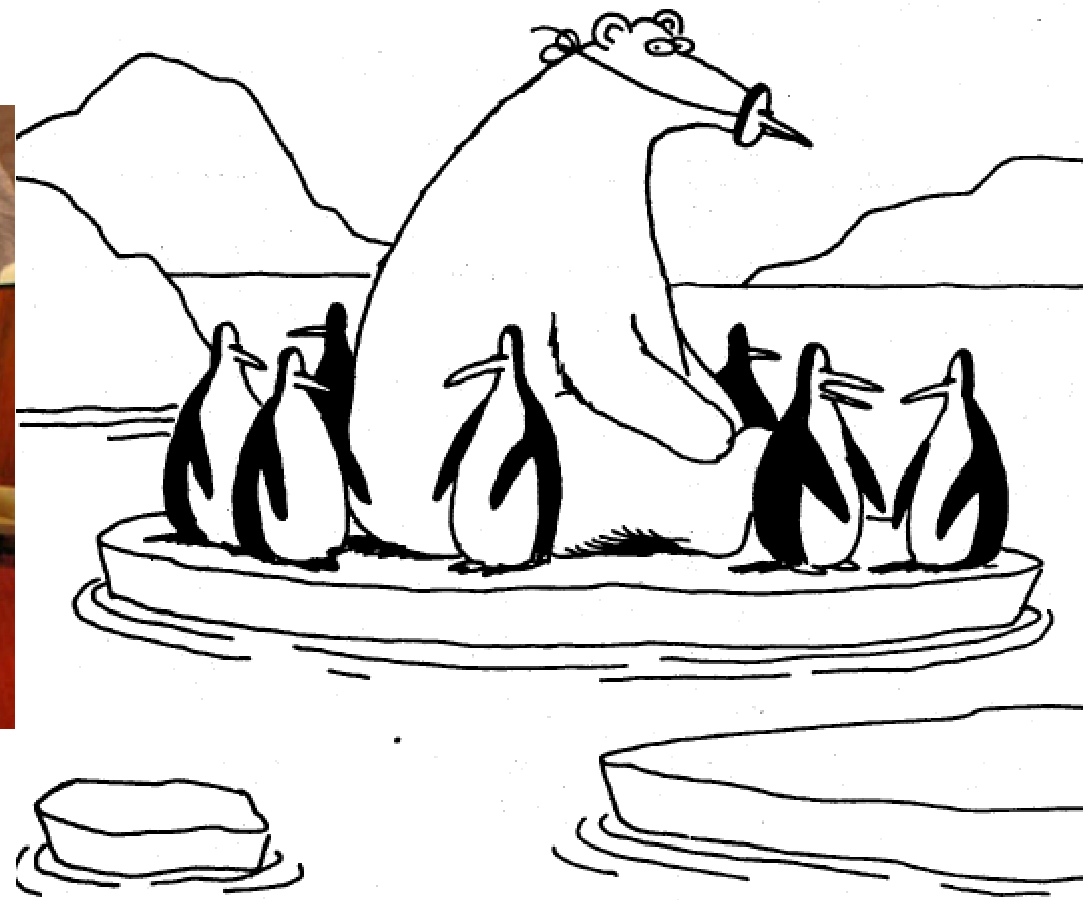
An elephant in the room: QFT Bound States

Paul Hoyer

Helsinki CosmoCoffee 25 January 2024

Gary Larson: The Far Side



"And now Edgar's gone. ...
Something's going on around here."

THE STATE IS NOT ABOLISHED, IT WITHERS AWAY: HOW QUANTUM FIELD THEORY BECAME A THEORY OF SCATTERING

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Learning quantum field theory (QFT) for the first time, after first learning quantum mechanics (QM), one is (or maybe, rather, I was) struck by the change of emphasis: The notion of the quantum state, which plays such an essential role in QM, from the stationary states of the Bohr atom, over the Schrödinger equation to the interpretation debates over measurement and collapse, seems to fade from view when doing QFT.

2011.0598

QED atoms are omitted from QFT textbooks

There is no systematic discussion of bound states in QFT, analogous to that of the S-matrix in the Interaction Picture.

See: C. Itzykson and J.-B. Zuber: Quantum Field Theory (1980)

10-3 HYPERFINE SPLITTING IN POSITRONIUM

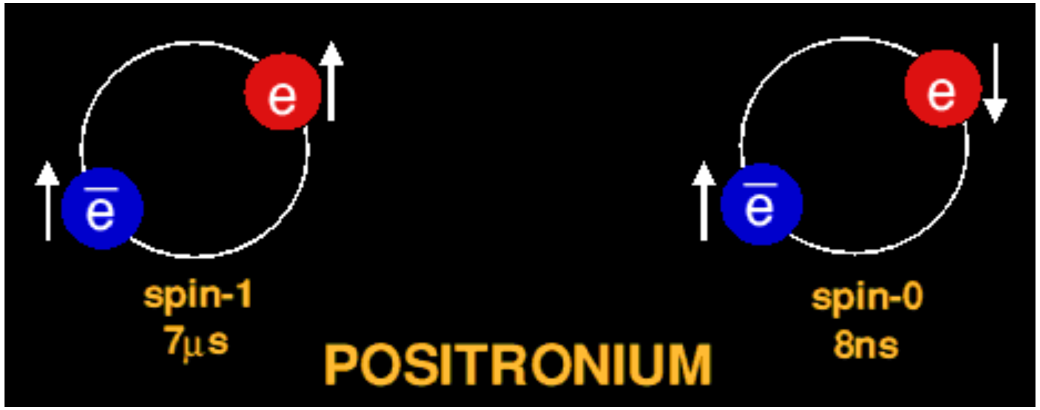
It should not be concluded that relativistic weak binding corrections cannot be obtained for two-body systems that agree with experiment. On the contrary, the positronium states give an example of a successful agreement. This will serve to illustrate the theory. To be completely fair, we should admit that accurate predictions require some artistic gifts from the practitioner. As yet no systematic method has been devised to obtain the corrections in a completely satisfactory way.

I & Z do not discuss how the Schrödinger equation follows from the QED action.

State of the art: Hyperfine splitting in Positronium

Atomic calculations start from the NR Schrödinger atom at rest, with its $\mathcal{O}(\alpha^\infty)$ wave function

$$\Psi(\boldsymbol{x}) \sim \exp(-\alpha m r/2)$$



Example: Hyperfine splitting in Positronium

G. S. Adkins, Hyperfine Interact. **233** (2015) 59

$$\begin{aligned}\Delta\nu_{QED} &= m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right.\\ &+\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right]\\ &\left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}\end{aligned}$$

$\Delta\nu_{EXP}$ = 203.394± .002 GHz

Hadrons are Paradoxical

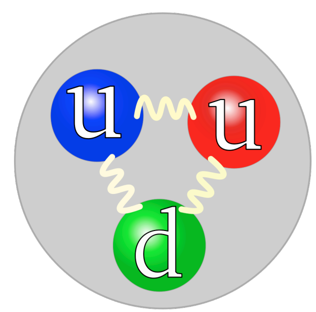
Relativistic mass spectrum but $q\bar{q}$ quantum numbers

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)^\ddagger$		$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)^\dagger$		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)^\ddagger$	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

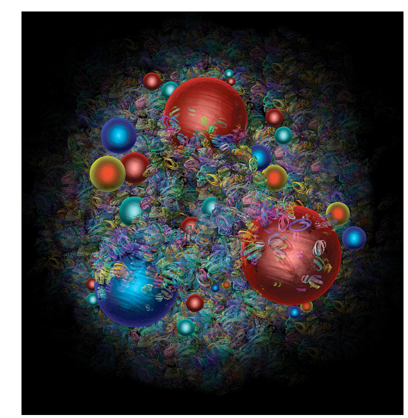
Particle Data Group

Strong color fields should create many constituents

Why do hadron quantum numbers agree with the non-relativistic quark model?

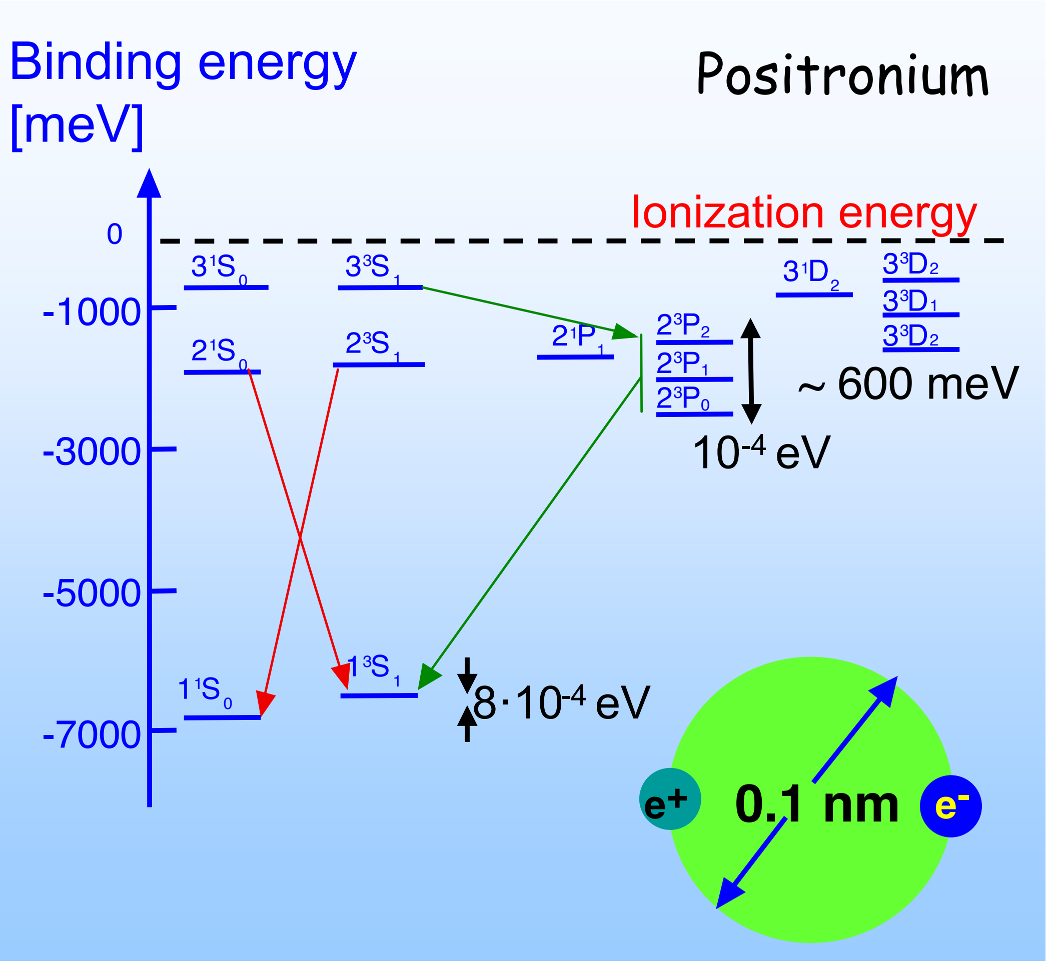


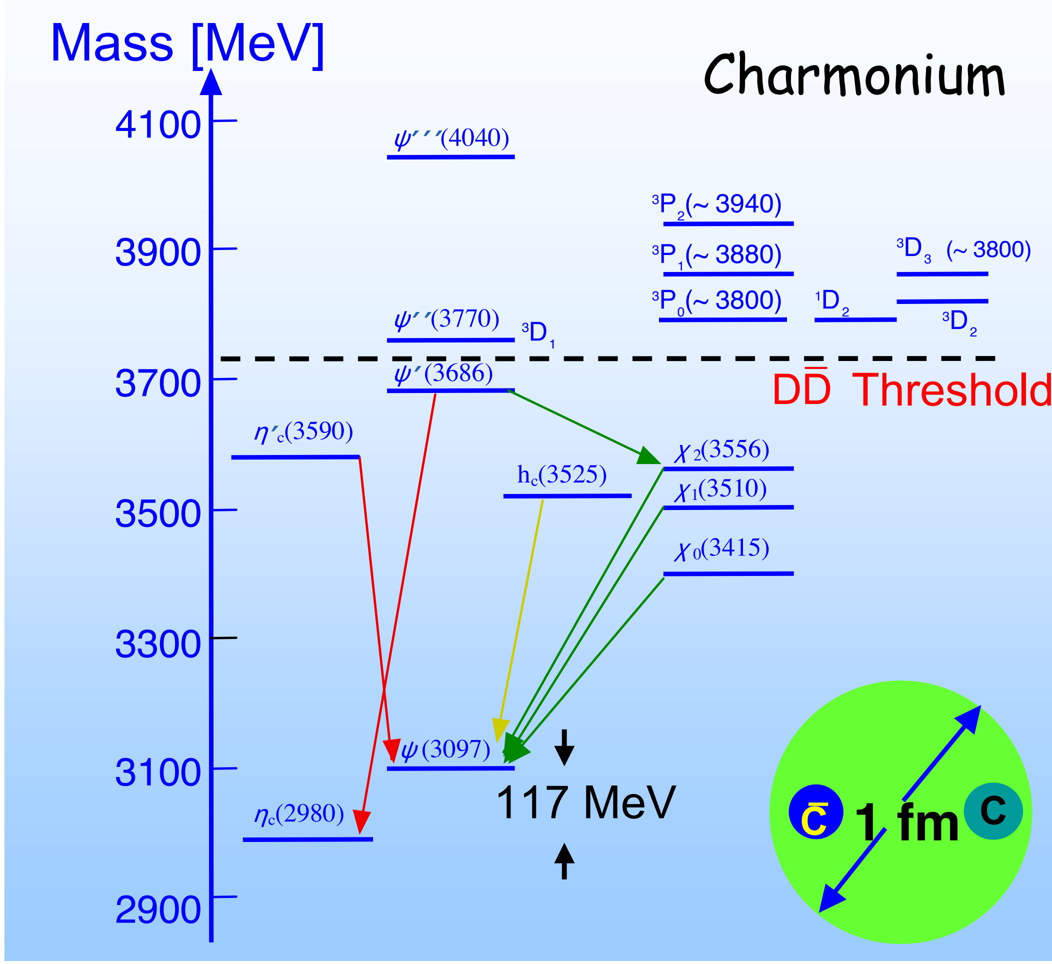
$\stackrel{?}{=}$



We know the QCD action:
We can find out!

Quarkonia are like atoms with confinement





$$V(r) = -\frac{\alpha}{r}$$

$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r} \quad (1980)$$

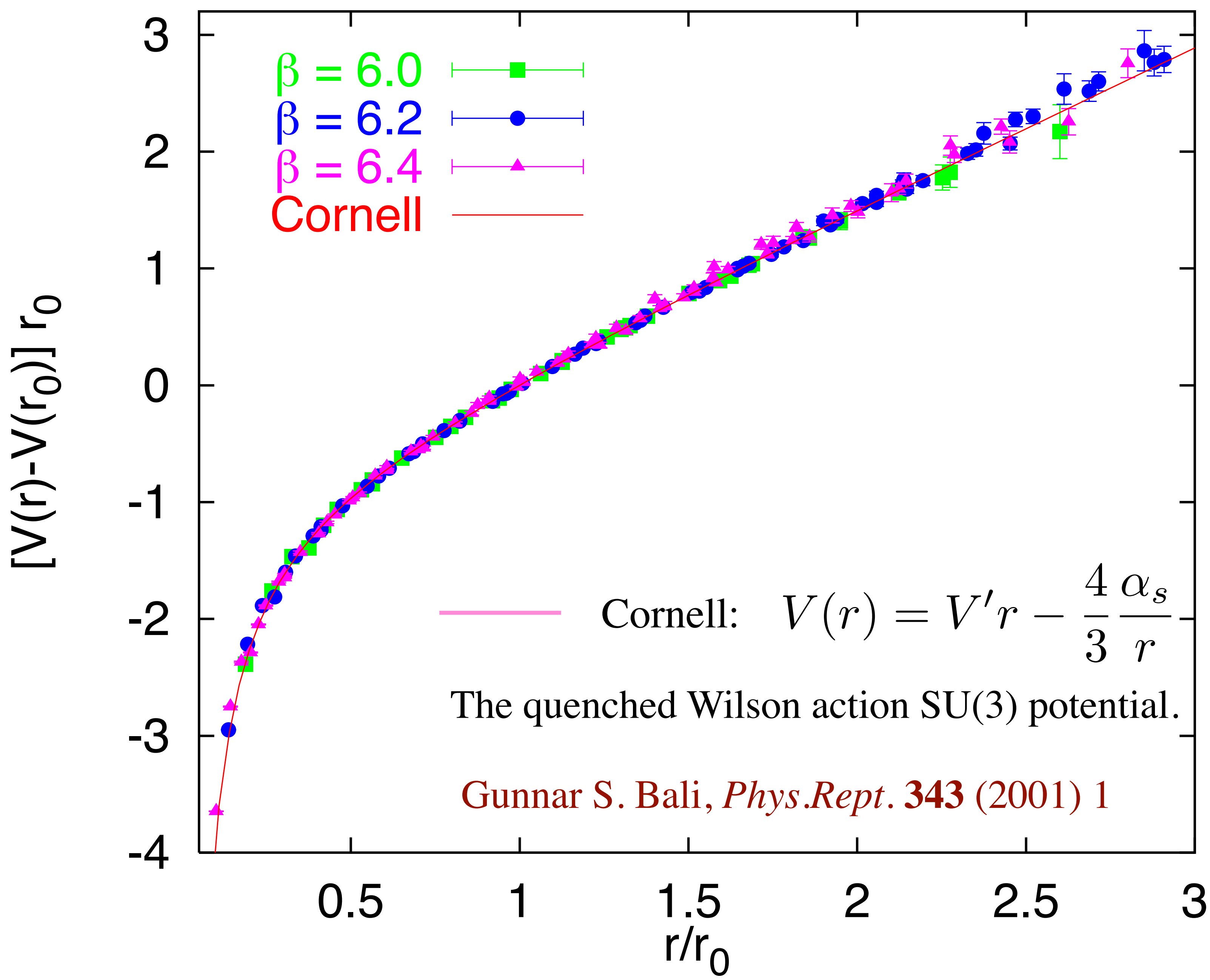
"The J/ψ is the Hydrogen atom of QCD"

Since 1974

$$V' \simeq 0.18\ \mathrm{GeV}^2, \quad \alpha_s \simeq 0.39$$

E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys. **80** (2008) 1161

Lattice QCD agrees with the Cornell potential



Generalized laws must reproduce earlier facts (1)

Relativistic → non-relativistic dynamics:

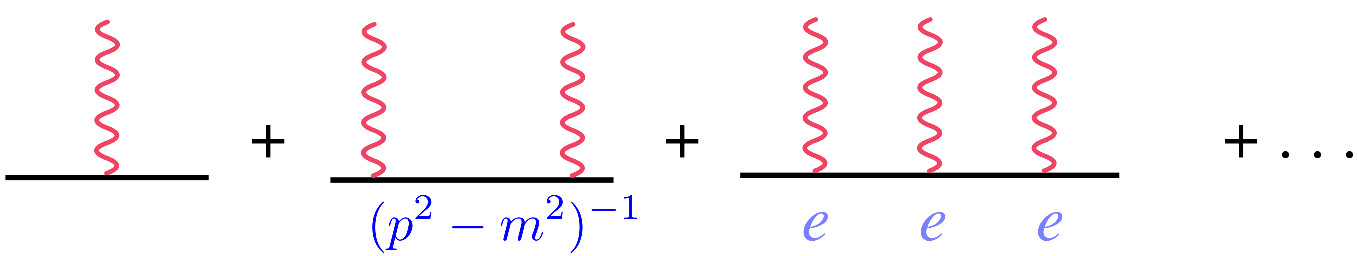
$$\sqrt{m^2+p^2} = m + \frac{p^2}{2m} + \mathcal{O}\left(p^4\right)$$

$$R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu} = \kappa T_{\mu\nu} \quad \to \quad F = G_N\,\frac{m_1\,m_2}{r}$$

New physical laws must reproduce earlier facts (2)

	Quantum	$\rightarrow$	**Classical**
Free propagation:	$\dfrac{i}{p^2 - m^2 + i\epsilon}$	$\rightarrow$	$\boldsymbol{x} = \dfrac{\boldsymbol{p}}{E}\, t$
	QFT		Newton's 1st. law
With EM interactions:	$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} +$ $\bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi$	$\rightarrow$	$\partial_\mu F^{\mu\nu} = j^\nu$ $\partial_\mu \tilde{F}^{\mu\nu} = 0$ $\mathbf{F} = e\big(\boldsymbol{E} + \mathbf{v} \times \boldsymbol{B}\big)$
	QED		Lorentz force postulated

Note: Motion in a field is a "non-perturbative":



Series diverges when
$p^2 - m^2 << e^2 p^2$

Yet $\mathbf{F} \propto e$

New physical laws must reproduce earlier facts (3)

Quantum Field Theory $\longrightarrow$ **Quantum Mechanics**

$? \nearrow$

$$\Big(-\frac{\boldsymbol{\nabla}^2}{2m}+V\Big)\Phi(\boldsymbol{x}) = E\,\Phi(\boldsymbol{x})$$

Schrödinger equation

$$-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+$$

$$\bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi$$

QED

Electron moves in a classical potential
Atoms are "non-perturbative"

$? \searrow$

$$(i\boldsymbol{\nabla}\cdot\boldsymbol{\gamma} - eA\!\!\!/ - m)\psi(\boldsymbol{x}) = E\gamma^0\,\psi(\boldsymbol{x})$$

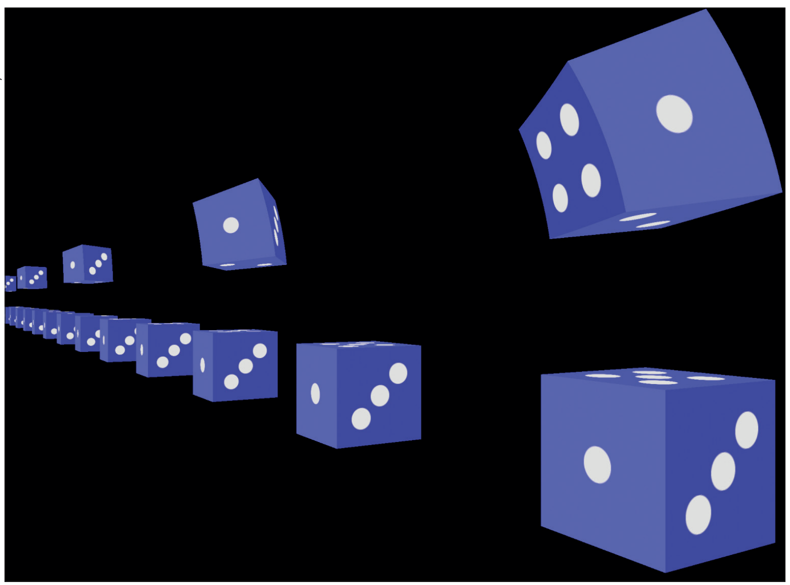
Dirac equation

What state does the Dirac wave function $\psi(\boldsymbol{x})$ describe?

Classical Lorentz Contraction

THE
VISUAL
APPEARANCE
OF *RAPIDLY MOVING OBJECTS*

V. F. Weisskopf
Physics Today **13** (1960) 24



The invisibility of length contraction

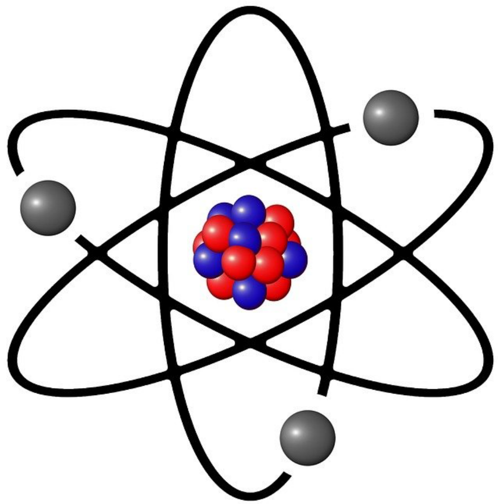
D. Appell
Physics World **32** (2019) 41

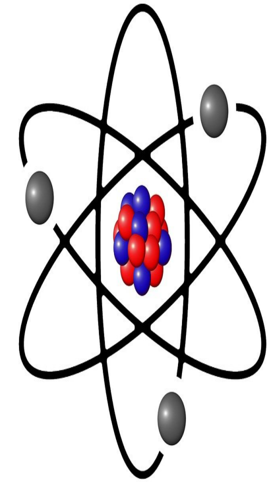
Curiouser and curiouser A row of stationary dice (bottom), with other dice moving from left to right (top) at 90% of the speed of light. All cubes, whether moving or at rest, have the same orientation. However, we cannot see the Lorentz contraction of the upper cubes, which instead are rotated. Indeed, due to the fact the speed of light is finite, we can actually see the “rear” sides of the upper cubes.

Do Atoms Lorentz Contract?

Quantum physics

$P = 0$





$P > m$

$$r = \frac{1}{\alpha m}$$

$$r = \frac{1}{\alpha E}$$

$$V = -\frac{\alpha}{r} \simeq -\alpha^2 m$$

$$\simeq E_{bind} = -\tfrac{1}{2}\alpha^2 m$$

$$V \simeq -\alpha^2 E$$

Relativistic dynamics?

$$P \to \infty$$

$$\sqrt{P^2 + (2m + E_{bind})^2} - \sqrt{P^2 + 4m^2} = \frac{2mE_{bind}}{P}$$

Relativistic kinematics

QED reconciles relativistic kinematics and dynamics

Preconceptions of hadrons

Hadrons are non-perturbative: Only numerical lattice approaches are feasible

The strong coupling $\alpha_s \geq 1$