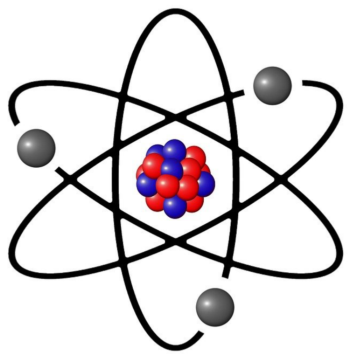
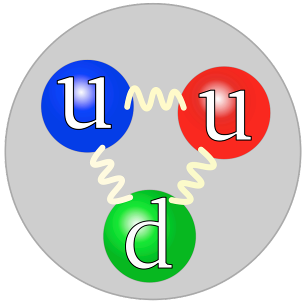
Bound States in Perturbative Quantum Field Theory

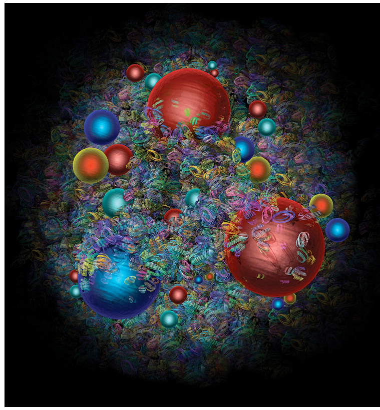
Helsinki Seminar 8 March 2022



Paul Hoyer

University of Helsinki





A less well known part of the Standard Model

QED bound states are not part of the standard QFT curriculum

There is a consensus that the QED methods are not applicable to hadrons

Yet there are similarities between hadrons and atoms

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Bodwin, Yennie and Gregorio, Rev. Mod. Phys. **57** (1985) 723

Introduction:

"Bound state theory is non-perturbative, but it is possible to develop expressions in increasing orders of α …
There is an art in developing theoretical expressions in this manner."

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Introduction:

"Bound state theory is non-perturbative, but it is possible to develop expressions in increasing orders of α …
There is an art in developing theoretical expressions in this manner."

Itzykson and Zuber, Quantum Field theory (1980)

Hyperfine splitting in Positronium (sect. 10.3):

"To be completely fair, we should admit that accurate predictions require some artistic gifts from the practitioner."

Contrast with the perturbative S-matrix

Contrast with the perturbative S-matrix

Interaction Picture: $H = H_0 + H_{int}$

The time dependence of the IP fields is given by H_0 ,

$$\psi_I(t,\boldsymbol{x}) = \int \frac{d\boldsymbol{k}}{(2\pi)^3 2E_k} \sum_\lambda \left[u_\alpha(\boldsymbol{k},\lambda) e^{-ik\cdot x} b_{\boldsymbol{k},\lambda} + v_\alpha(\boldsymbol{k},\lambda) e^{ik\cdot x} d^\dagger_{\boldsymbol{k},\lambda} \right]$$

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The perturbative S-matrix is derived to be

$$S_{fi} = {}_{out}\langle f,\, t \to \infty | \left\{ \mathrm{T} \exp \left[-i \int_{-\infty}^{\infty} dt\, H_I(t) \right] \right\} | i,\, t \to -\infty \rangle_{in}$$

where H_I is H_{int} (ψ_I), and the *in* and *out* states are free.

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There is little discussion of the principal differences between perturbation theory for bound states and scattering.

The state is not abolished, it withers away: how quantum field theory became a theory of scattering

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Max Planck Institute for the History of Science, Boltzmannstraße 22, 14195 Berlin, Germany

12th November 2020

2011.0598

Learning quantum field theory (QFT) for the first time, after first learning quantum mechanics (QM), one is (or maybe, rather, I was) struck by the change of emphasis: The notion of the quantum state, which plays such an essential role in QM, from the stationary states of the Bohr atom, over the Schrödinger equation to the interpretation debates over measurement and collapse, seems to fade from view when doing QFT.

Perturbative expansion: Scattering vs. bound states

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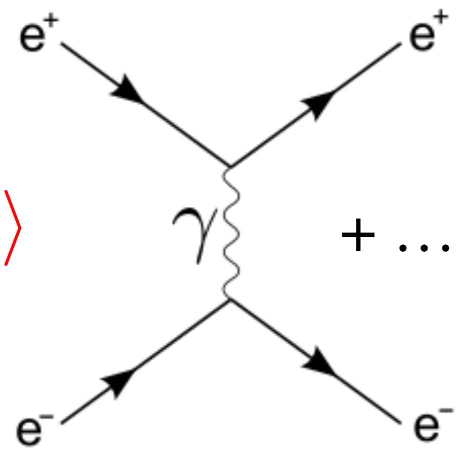
$$e^+e^- \rightarrow e^+e^-$$

$|in\rangle$



$+ \ldots \; |out\rangle$

Scattering amplitudes are expanded around free states

Perturbative expansion: Scattering vs. bound states

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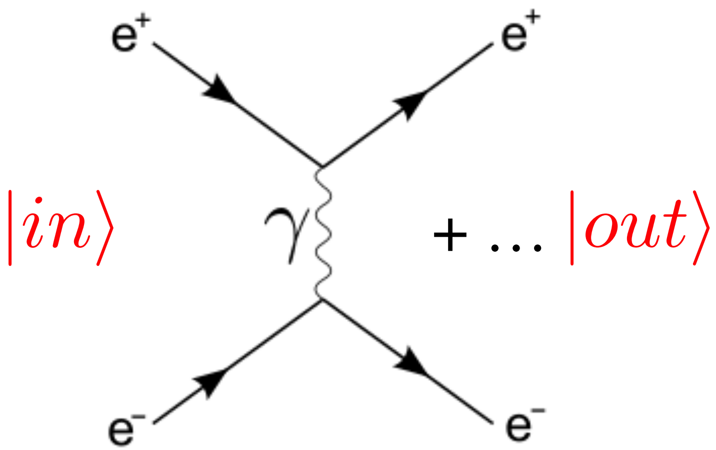
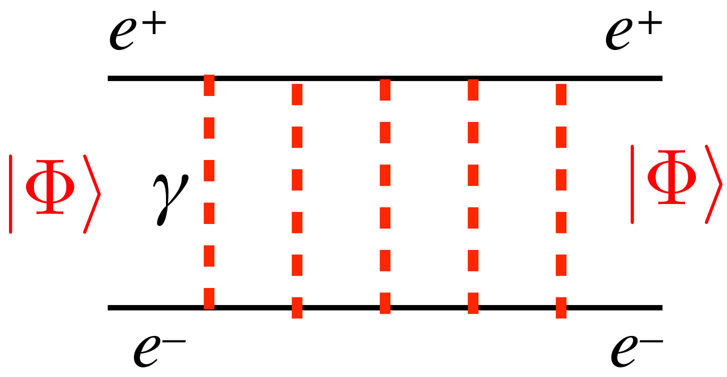


Scattering amplitudes are expanded around free states

Positronium



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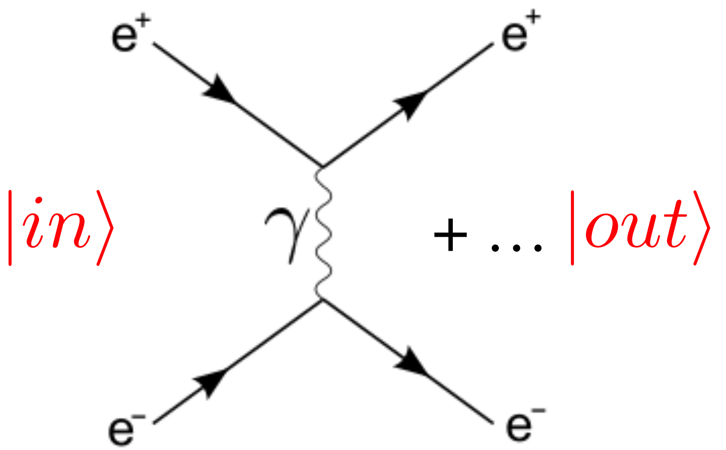

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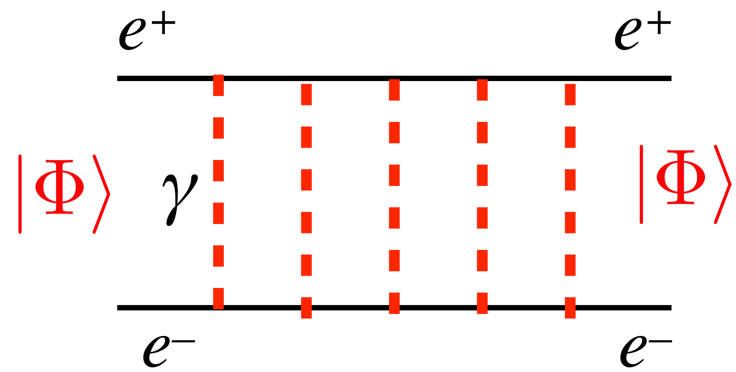

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$\langle\Phi|in\rangle = 0 \implies$ No Feynman diagram has a bound state pole

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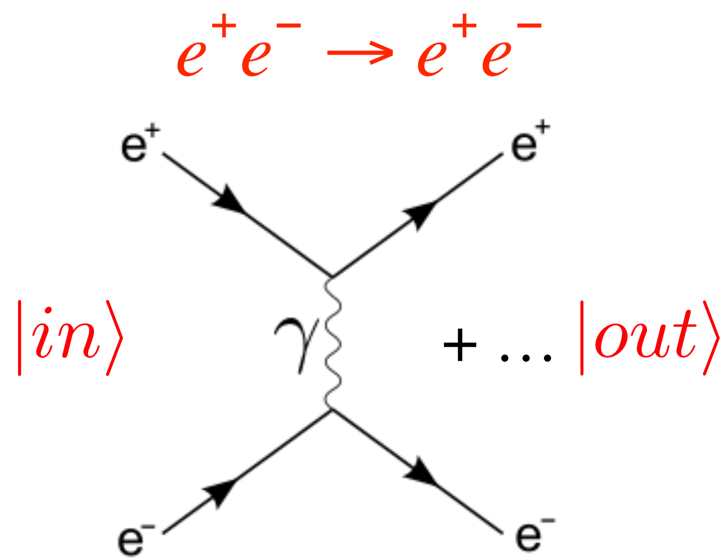

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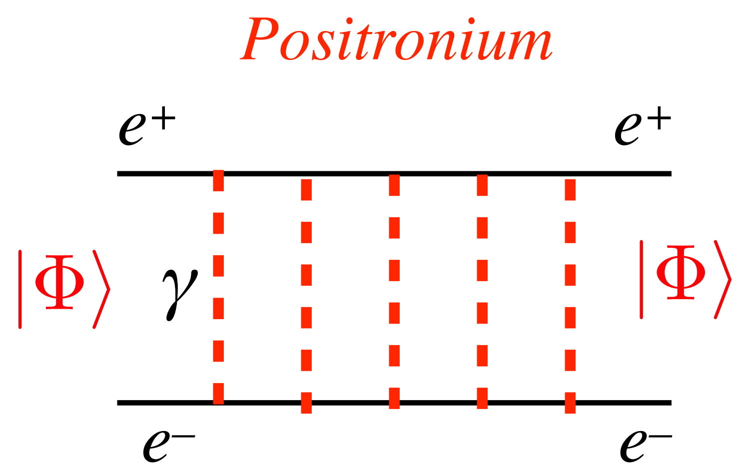

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Atomic wave functions $\Phi(\alpha)$ are non-polynomial (exponential) in α

Their higher order corrections $\Phi(\alpha)(1 + c_1\alpha + c_2\alpha^2 \ldots)$ depend on $\Phi(\alpha)$.

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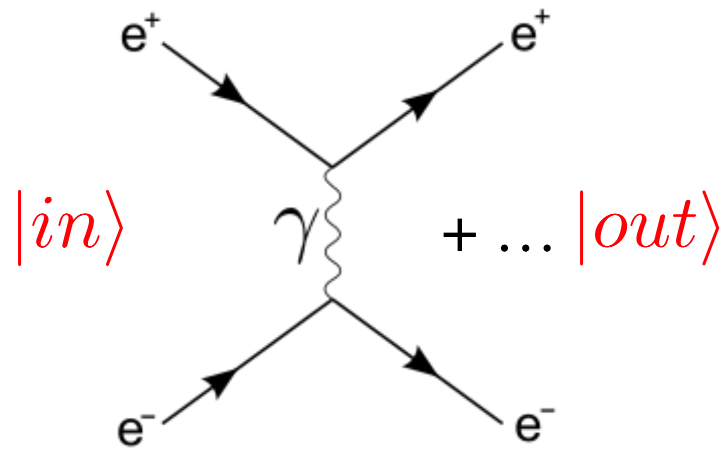

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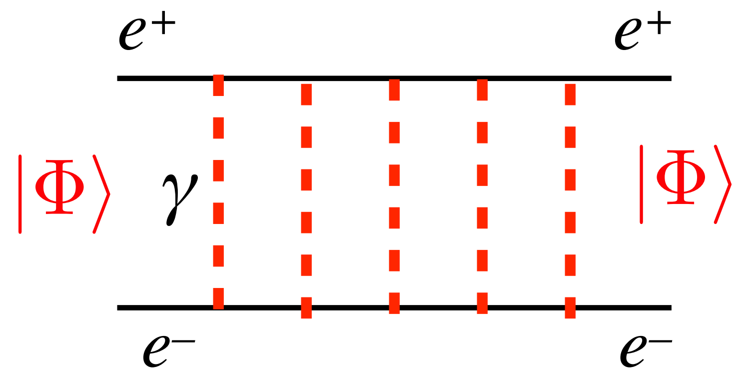

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The perturbative expansion for wave functions is not unique, it depends on the choice of initial state.

Caswell & Lepage (1975)

Atoms provide precision tests of QED

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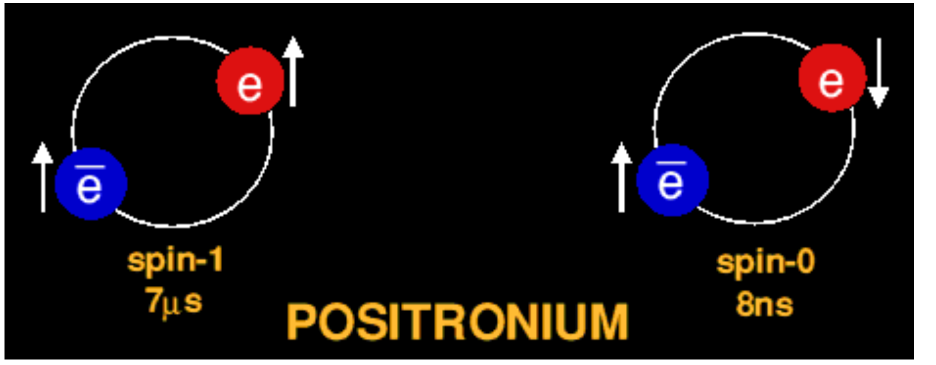
G. S. Adkins,
Hyperfine Interact. **233** (2015) 59



Hyperfine splitting in Positronium

$$\Delta\nu_{QED} = m_e\alpha^4\left\{\frac{7}{12} - \frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right) + \frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha + \frac{1367}{648} - \frac{5197}{3456}\pi^2 + \left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2 - \frac{53}{32}\zeta(3)\right] - \frac{7\alpha^3}{8\pi}\ln^2\alpha + \frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2 - \frac{217}{90}\right) + \mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}$$

$\Delta\nu_{EXP} = 203.394 \pm .002$ GHz

depends on $\ln\alpha$

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depends on lnα

Bound state expansions are not unique: Bethe-Salpeter (1950) NRQED (1986) and others

and they agree for measurable quantities, such as binding energies.

The Schrödinger equation from Feynman diagrams

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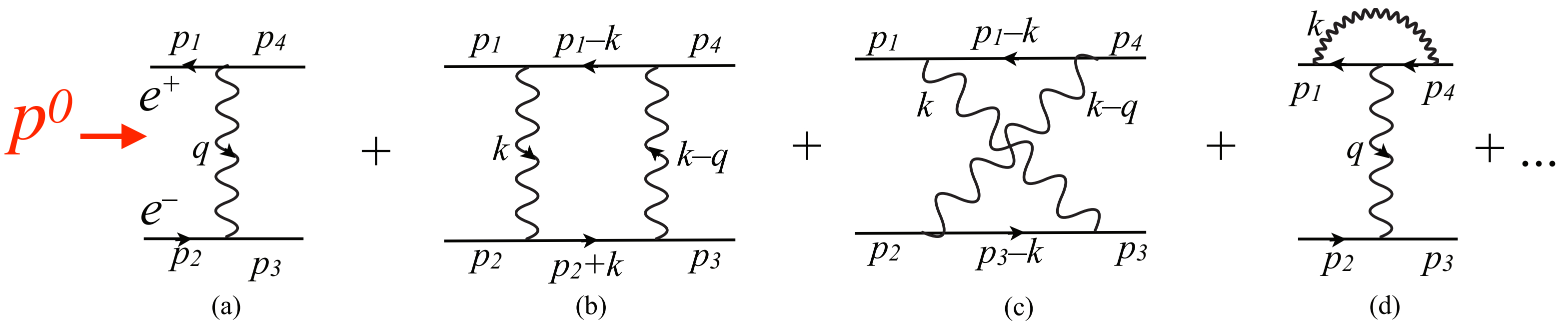

$$= \frac{|\varphi_{e^+e^-}|^2}{p^0 - E + i\varepsilon} + \ldots$$

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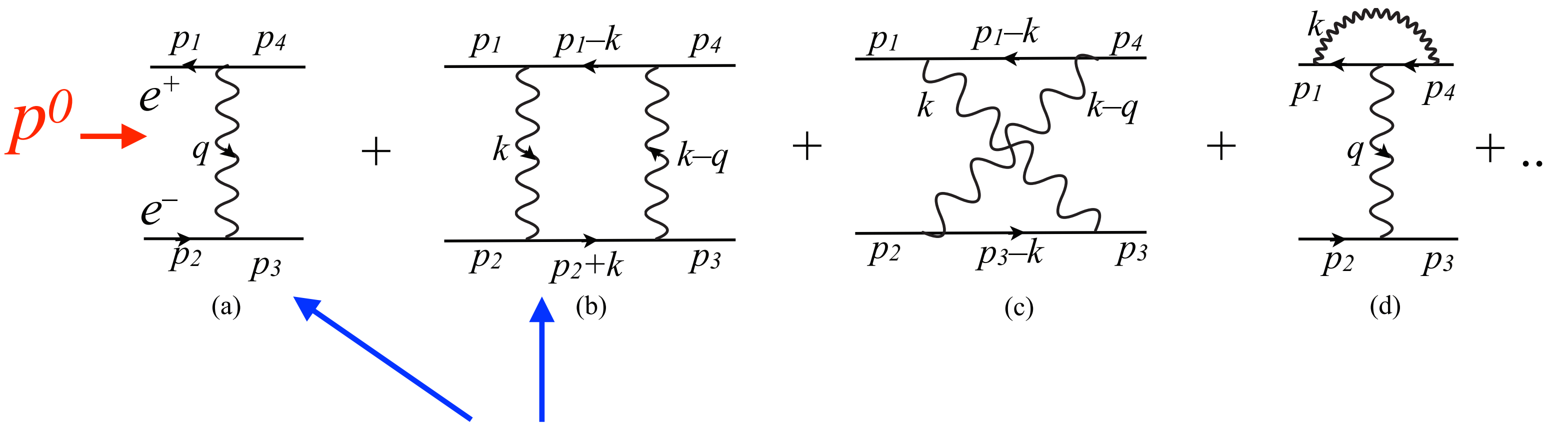

$$p^0 \rightarrow \quad \cdots = \frac{|\varphi_{e^+e^-}|^2}{p^0 - E + i\varepsilon} + \ldots$$

The sum of "ladder diagrams" (re)generates the classical potential:

$$\frac{i}{i\partial\!\!\!/ - m - eA\!\!\!/} = \frac{i}{i\partial\!\!\!/ - m} - \frac{i}{i\partial\!\!\!/ - m} ieA\!\!\!/ \frac{i}{i\partial\!\!\!/ - m} + \ldots \implies \quad V(r) = -\frac{\alpha}{r}$$

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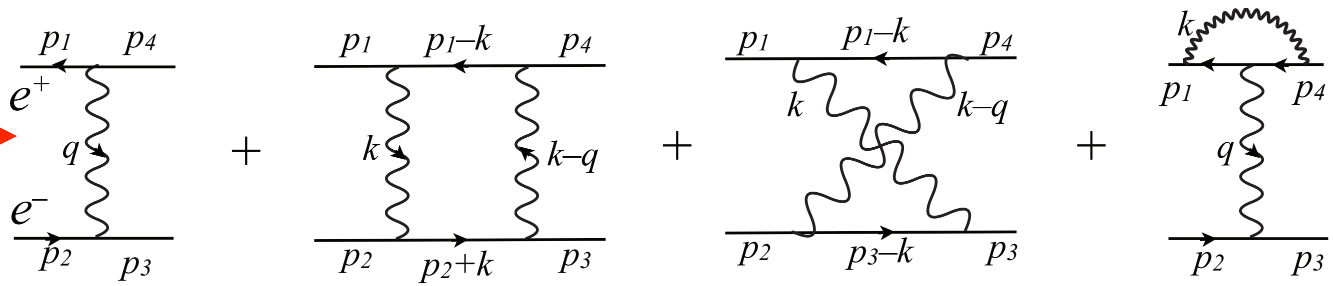

(a) (b) (c) (d)

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Ladder diagrams are unsuppressed at the Bohr scale $|\boldsymbol{q}| \sim \alpha m$: $1/\mathbf{q}^2 \propto 1/\alpha^2$

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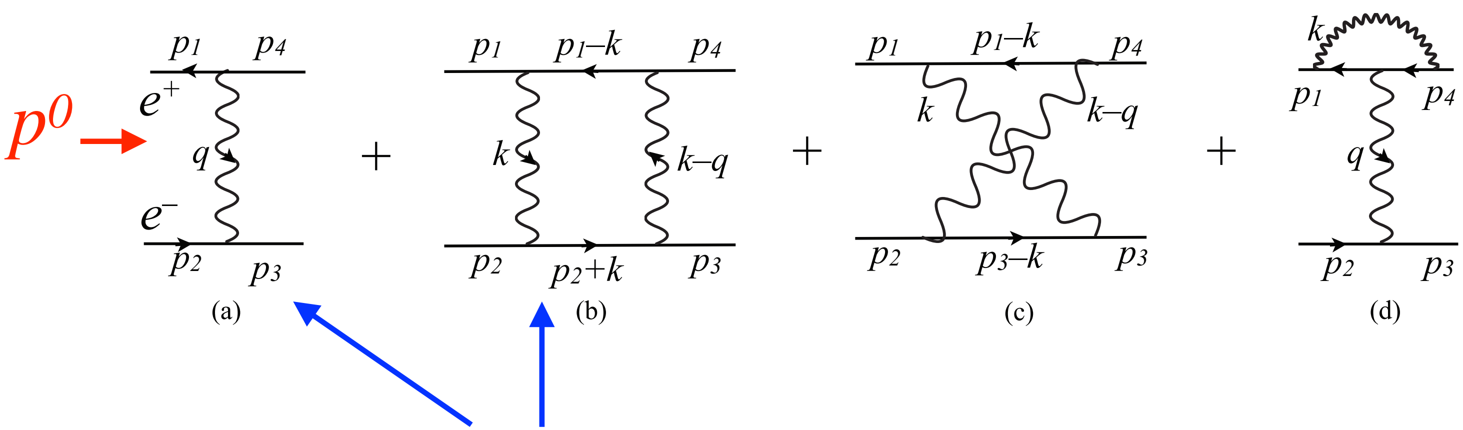

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Ladder diagrams are unsuppressed at the Bohr scale $|\boldsymbol{q}| \sim \alpha m$: $1/\mathbf{q}^2 \propto 1/\alpha^2$

For $|\boldsymbol{q}| << \alpha m$: classical physics dominates:

Atoms are at the borderline to classical physics

Basic issues in need of attention

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The Schrödinger equation is postulated in Introductory Quantum Mechanics.

In QFT it should be derived from S_{QED}.

C.f. Relativity: $\sqrt{M^2+P^2} \simeq M + P^2/2M$

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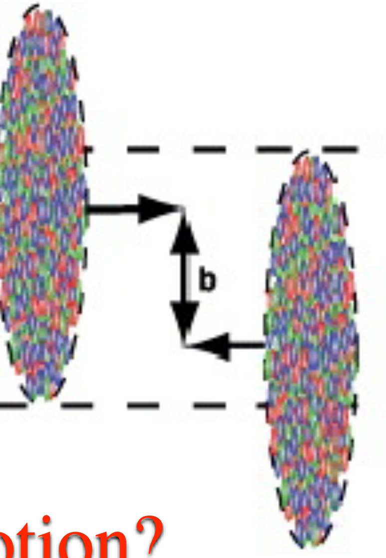
Moving bound states are often depicted as ellipses due to Lorentz contraction



(How) is the classical relativistic concept of contraction realised in QFT:

What is the wave function of Positronium in motion?

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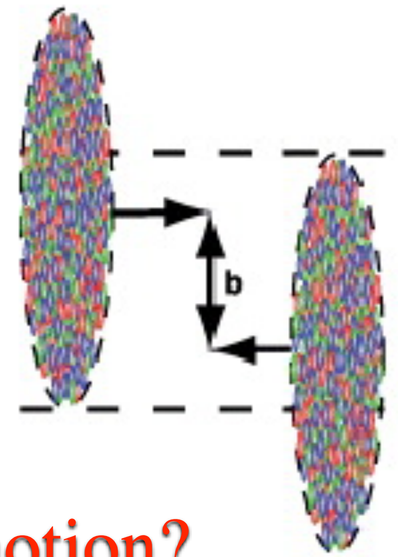

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Poincaré symmetry for extended states is interesting and non-trivial.

Recap: Bound state features

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Atomic constituents are bound by an instantaneous, classical potential V(r)

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QCD: Expanding around free quarks and gluons need not give confinement.

Hadron data shows similarities to atoms

Non-relativistic bound states

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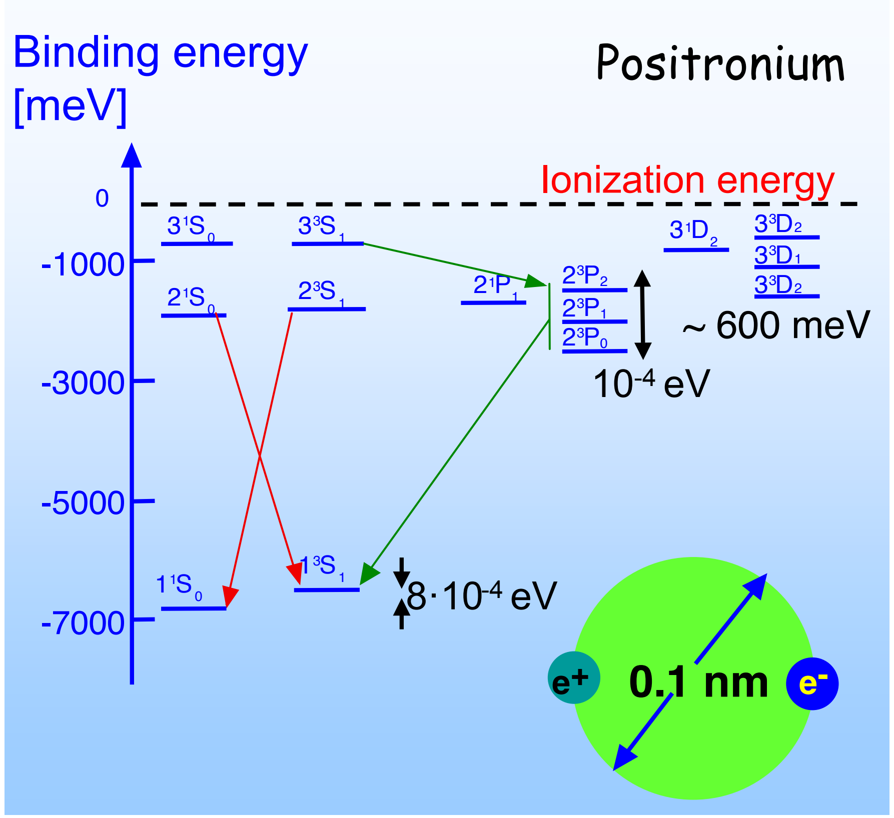
QED: e^+e^- atoms



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Non-relativistic bound states

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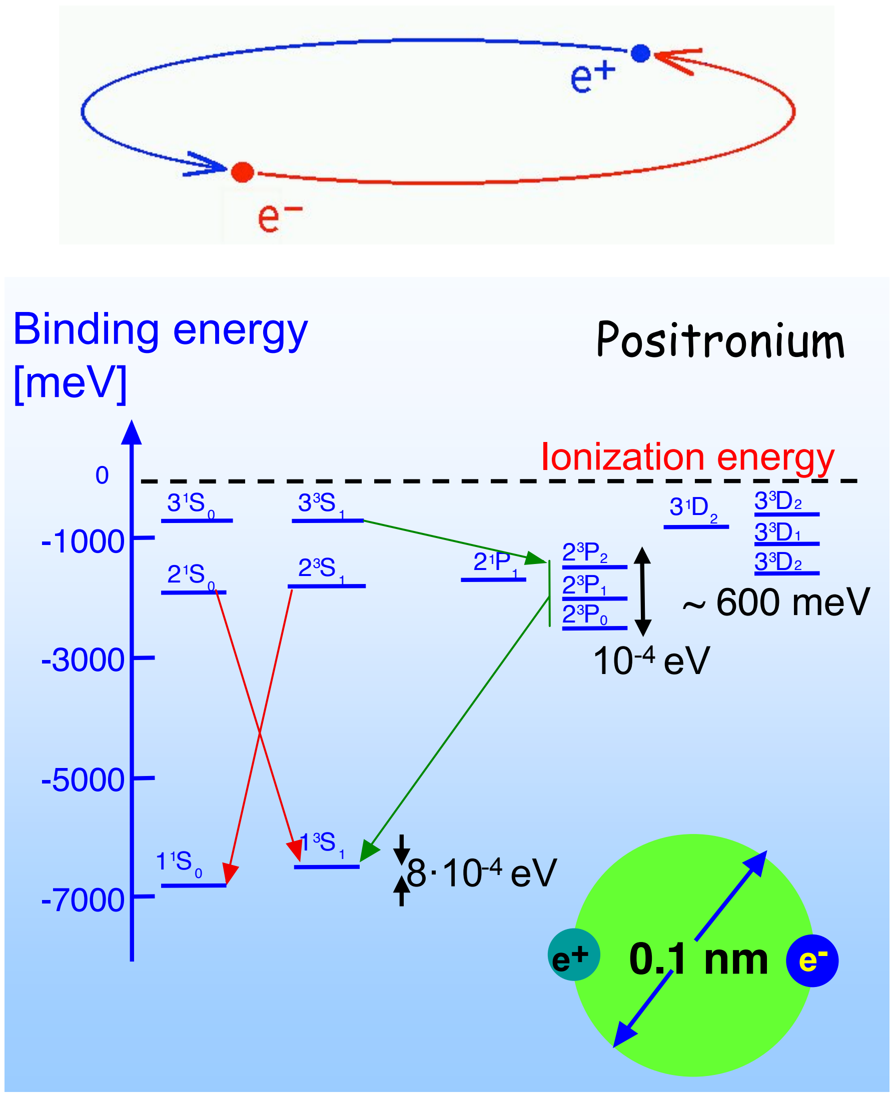

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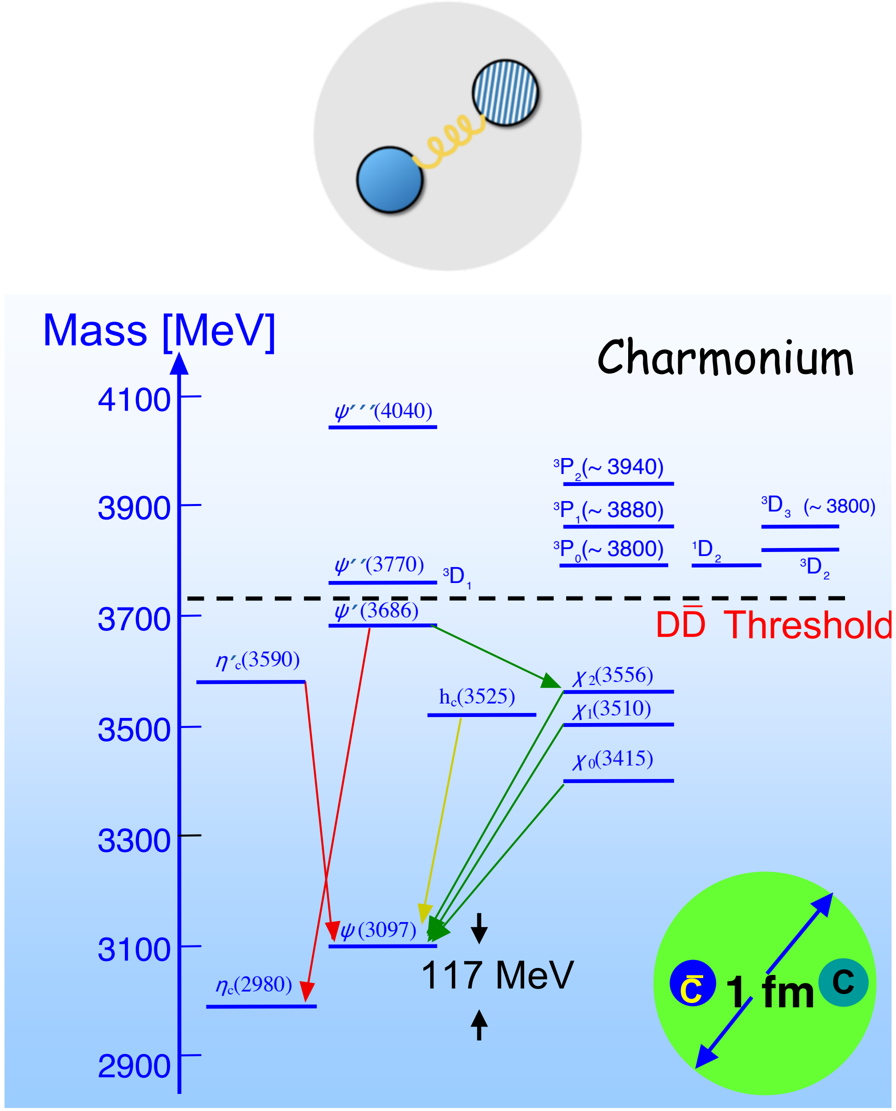
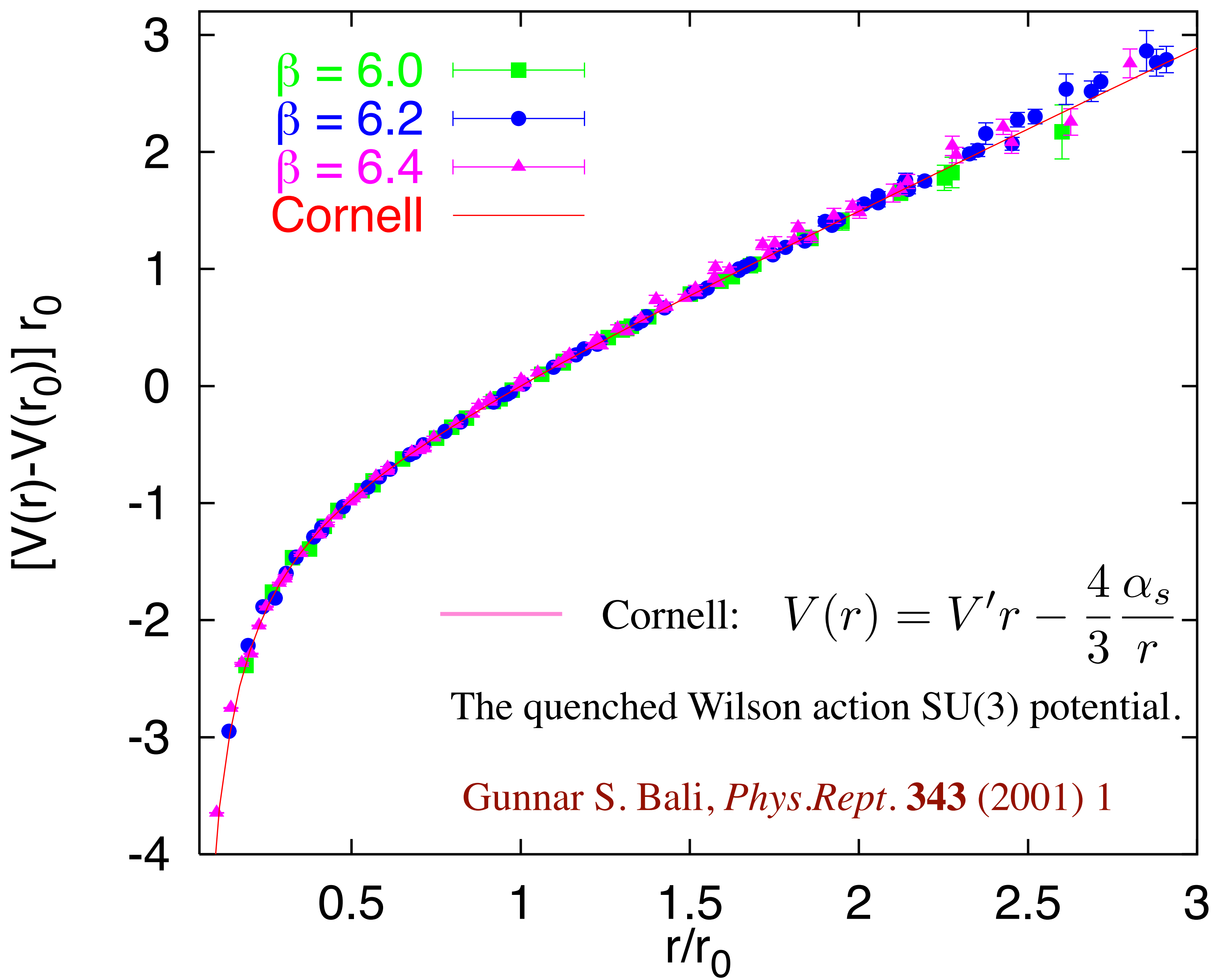
QCD: $b\bar{b}$, $c\bar{c}$ quarkonia



$$V(r) = V'r - \frac{4}{3}\frac{\alpha_s}{r}$$

Lattice QCD agrees with the Cornell potential



Light quarks: π, ρ, N,...

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Valence quantum numbers

$n^{2s+1}\ell_J$	J^{PC}	I $= 1$ $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I $= \frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I $= 0$ f'	I $= 0$ f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
1^1S_0	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'}(958)$	-11.3	-24.5
1^3S_1	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	39.2	36.5
1^1P_1	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}^\dagger$	$\boldsymbol{h_1(1415)}$	$\boldsymbol{h_1(1170)}$		
1^3P_0	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
1^3P_1	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}^\dagger$	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
1^3P_2	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
1^1D_2	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}^\dagger$	$\boldsymbol{\eta_2(1870)}$	$\boldsymbol{\eta_2(1645)}$		
1^3D_1	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}^\ddagger$		$\boldsymbol{\omega(1650)}$		
1^3D_2	2^{--}		$\boldsymbol{K_2(1820)}^\dagger$				
1^3D_3	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	31.8	30.8
1^3F_4	4^{++}	$\boldsymbol{a_4(1970)}$	$\boldsymbol{K_4^*(2045)}$	$f_4(2300)$	$\boldsymbol{f_4(2050)}$		
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
2^1S_0	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$		
2^3S_1	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}^\ddagger$	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		
2^3P_1	1^{++}	$\boldsymbol{a_1(1640)}$					
2^3P_2	2^{++}	$\boldsymbol{a_2(1700)}$	$K_2^*(1980)$	$\boldsymbol{f_2(1950)}$	$f_2(1640)$		

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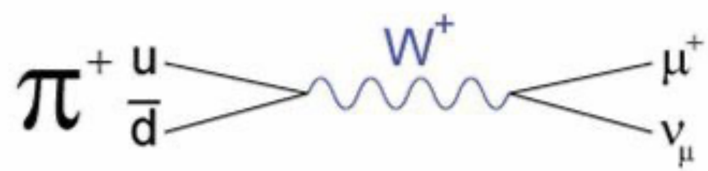
2^3P_1	1^{++}	$a_1(1640)$					
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$		

Current quark Fock states

Mesons have a sizeable current $q\bar{q}$ Fock component

E.g., pion decay:



Stan Brodsky

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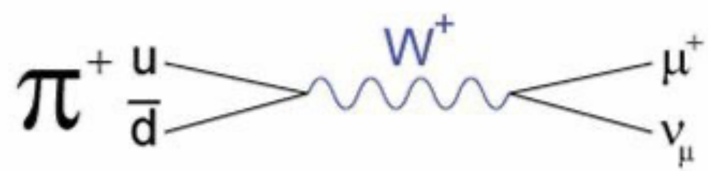
E.g., pion decay:



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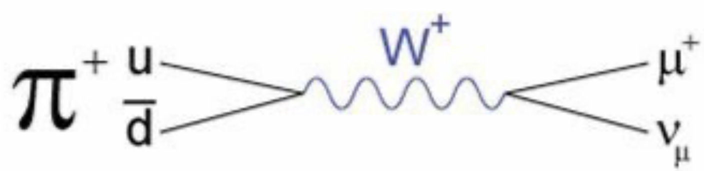

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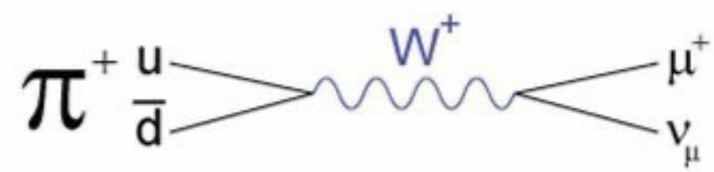

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$\Rightarrow$ $\alpha_s(\Lambda_{QCD})$ is small?

A strategy for bound states

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This requires an instantaneous potential, *c.f.*: $V(r) = -\frac{\alpha}{r}$

… even for relativistic quarks in QCD

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Instantaneous gauge interactions for

$\nabla\cdot\boldsymbol{A}(t,\boldsymbol{x}) = 0$ (Coulomb gauge)

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Bound state calculations generally use Coulomb gauge with Dirac constraints.

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This determines $\nabla\cdot\boldsymbol{E}_L$ in terms of the charge distribution in the state.

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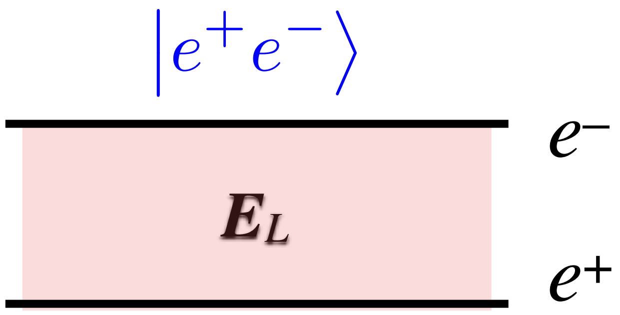
Contrast: In Coulomb gauge A^0 is a quantum field, which creates particles.

Fock state expansion for Positronium in $A^0=0$ gauge

The perturbative expansion in α is chosen to start from the $|e^+e^-\rangle$ Fock state, which is bound by its classical field $\boldsymbol{E}_L$:



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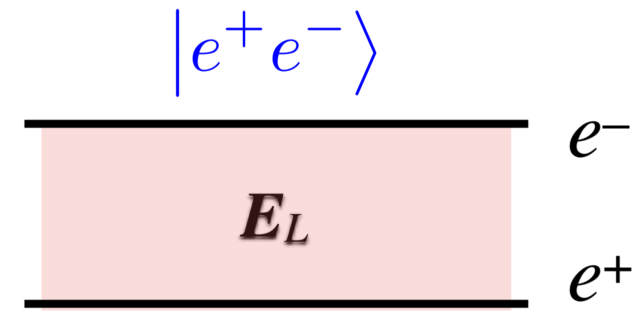
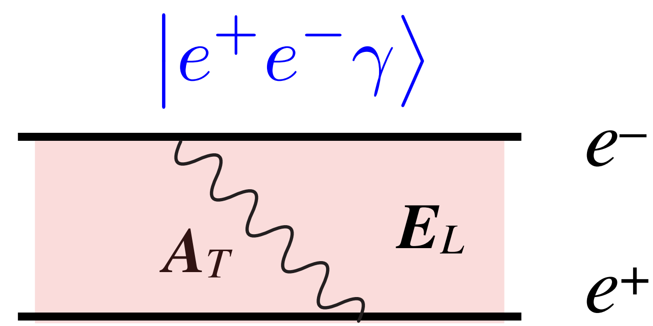


Higher order corrections include states with transverse photons and e^+e^- pairs, as determined by $H_{QED}\,|e^+e^-\rangle$



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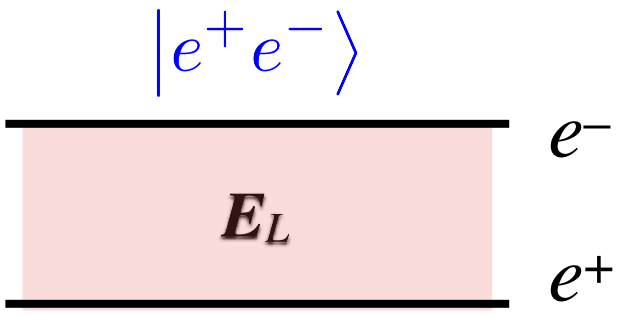

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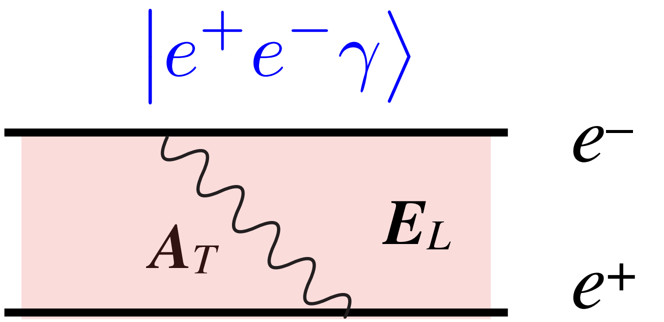


Each Fock component of the bound state includes its particular instantaneous $\boldsymbol{E}_L$ field.

This Fock expansion is valid in any frame, and is formally exact at $O(\alpha^\infty)$.

Positronium in motion: Contraction

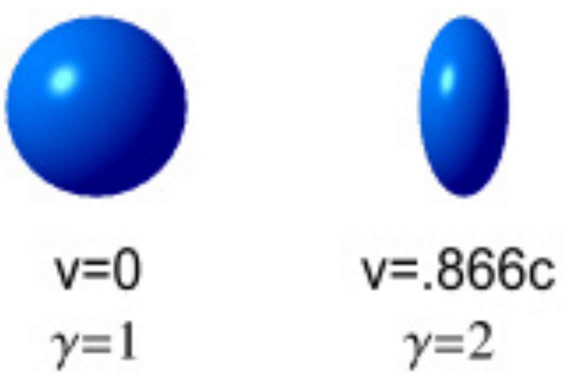
The binding energy in the rest frame ($P = 0$) is $E_b = -\alpha^2 m_e/4 + O(\alpha^4)$

At large momenta P the binding is $\propto 1/P$:

$$\Delta E(P) \equiv \sqrt{P^2 + (2m_e + E_b)^2} - \sqrt{P^2 + 4m_e^2} = \frac{2m_e E_b}{P} + \mathcal{O}\left(\alpha^4\right)$$

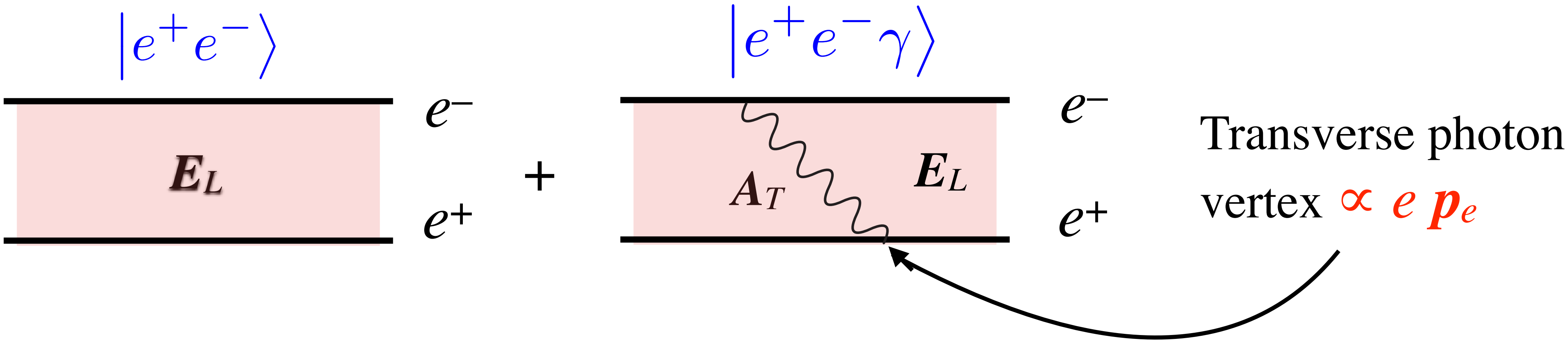
The potential energy $-\alpha/r$ is independent of P for $\boldsymbol{r} \perp \boldsymbol{P}$

Hence the Coulomb potential provides too strong binding



There must be more than contraction going on!

Positronium in motion: Fock expansion



In the rest frame: $\boldsymbol{p}_e \simeq \alpha\, m_e$: transverse photon contribution is $O(\alpha^4)$

For $P > 0$: $\boldsymbol{p}_e \simeq P/2$: transverse photon contribution is leading, $O(\alpha^2)$

The transverse photon exchange cancels the P-independent A^0 contribution, leaving an $O(1/P)$ contribution which agrees with Poincaré invariance.

M. Järvinen, Phys. Rev. **D71** (2005) 085006, PH 2101.06721

Other Fock states do not contribute to the binding energy at $O(\alpha^2)$

QFT gets things right when it is treated correctly

Application to QCD

The classical fields of QED and QCD differ

Global gauge invariance allows a classical gauge field for neutral atoms, but not a color octet gluon field for color singlet hadrons.

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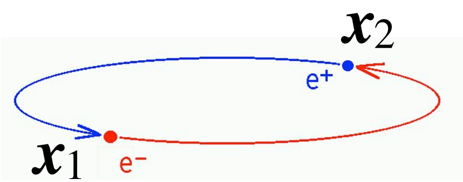

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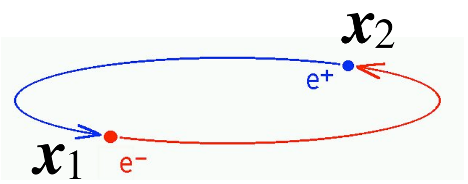

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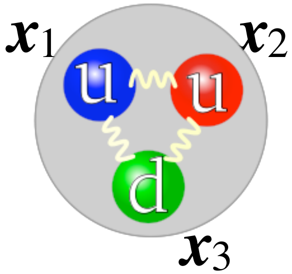

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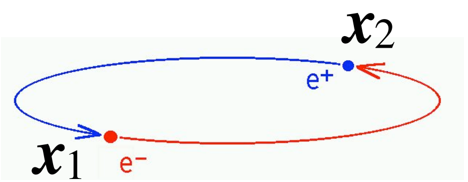

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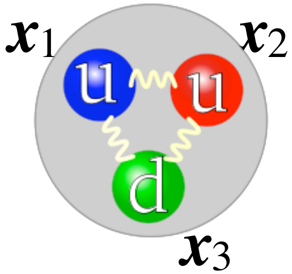

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However:
There is a classical gluon field for each color component C of the proton

$$\boldsymbol{E}_L^a(\boldsymbol{x}, C) \neq 0$$

The blue quark is bound by the $\boldsymbol{E}_L^a(\boldsymbol{x}, C)$ field of the red and green quarks.

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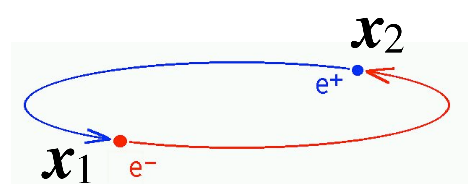
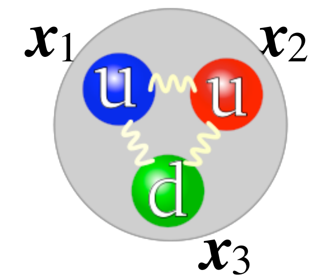
Global gauge invariance allows a classical gauge field for neutral atoms, but not a color octet gluon field for color singlet hadrons.

Positronium (QED)



$$E_L(x) = -\frac{e}{4\pi}\nabla_x\left(\frac{1}{|x-x_1|}-\frac{1}{|x-x_2|}\right)$$

Proton (QCD)



$$E_L^a(x) = 0 \quad \text{for all } x$$

However:

There is a classical gluon field for each color component C of the proton

$$E_L^a(x, C) \neq 0$$

The blue quark is bound by the $E_L^a(x, C)$ field of the red and green quarks.

An external observer sees no field: The gluon field generated by a color singlet state vanishes.

$$\sum_C E_L^a(x, C) = 0$$

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In QED we impose the boundary condition: $\boldsymbol{E}_L(\boldsymbol{x}) \to 0$ for $|\boldsymbol{x}| \to \infty$

In QCD $\boldsymbol{E}_{L,a}(\boldsymbol{x}) \equiv 0$ for (globally) color singlet Fock states.

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The color electric field $\boldsymbol{E}_{L,a}(\boldsymbol{x}) \neq 0$ for each quark color component

Include a homogeneous solution, $\nabla\cdot \boldsymbol{E}_{L,a}(\boldsymbol{x}) = 0$ with $\boldsymbol{E}_{L,a}(\boldsymbol{x}) \neq 0$.
$\boldsymbol{E}_{L,a}(\boldsymbol{x})$ binds each quark color component of a hadron.
The field cancels in the sum over quark colors for singlet states.

Including a homogeneous solution for $E_{L,a}^i$

$$E_{L,a}^i(\boldsymbol{x})\,|phys\rangle = -\partial_i^x \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

where $\mathcal{E}_a(\boldsymbol{y}) = -f_{abc}A_b^i E_c^i(\boldsymbol{y}) + \psi^\dagger T^a \psi(\boldsymbol{y})$ and $\mathcal{E}_a(\boldsymbol{y})\,|0\rangle = 0$

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The gluon field energy density is spatially constant.

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This solution is excluded by the free field BC of Feynman diagrams.

The instantaneous potential from the Hamiltonian

$$E^i_{L,a}(\boldsymbol{x})\,|phys\rangle = -\partial^x_i \int d\boldsymbol{y}\Big[\kappa\,\boldsymbol{x}\cdot\boldsymbol{y} + \frac{g}{4\pi|\boldsymbol{x}-\boldsymbol{y}|}\Big]\mathcal{E}_a(\boldsymbol{y})\,|phys\rangle$$

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$$\mathcal{H}_V \equiv \tfrac{1}{2}\int dx \sum_a \boldsymbol{E}_L^a\cdot\boldsymbol{E}_L^a$$

$$= \int d\boldsymbol{y}d\boldsymbol{z}\Big\{\boldsymbol{y}\cdot\boldsymbol{z}\Big[\tfrac{1}{2}\kappa^2\int d\boldsymbol{x} + g\kappa\Big] + \tfrac{1}{2}\frac{\alpha_s}{|\boldsymbol{y}-\boldsymbol{z}|}\Big\}\mathcal{E}_a(\boldsymbol{y})\mathcal{E}_a(\boldsymbol{z})$$

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The field energy ∝ volume of space is irrelevant only if it is universal.
This relates the normalisation ϰ of all Fock components,
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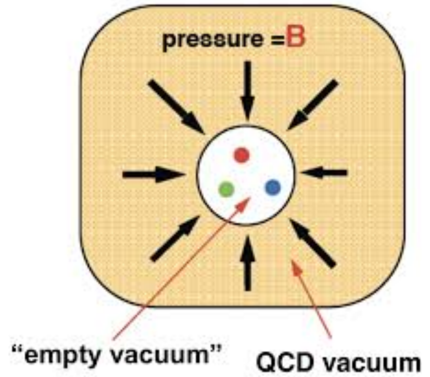
"Bag model without a bag"



Meson $q\bar{q}$ Fock state potential

$$|q(\boldsymbol{x}_1)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_A \bar{\psi}^A(\boldsymbol{x}_1)\,\psi^A(\boldsymbol{x}_2)\,|0\rangle$$ globally color singlet

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$$\mathcal{H}_V\,|q\bar{q}\rangle = V_{q\bar{q}}\,|q\bar{q}\rangle$$

$$V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - C_F\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$$ Cornell potential

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This potential is valid also for relativistic $q\bar{q}$ Fock states, in any frame

Baryon Fock state potential

Baryon: $$|q(\boldsymbol{x}_1)q(\boldsymbol{x}_2)q(\boldsymbol{x}_3)\rangle \equiv \sum_{A,B,C} \epsilon_{ABC}\psi^\dagger_A(\boldsymbol{x}_1)\,\psi^\dagger_B(\boldsymbol{x}_2)\,\psi^\dagger_C(\boldsymbol{x}_3)\,|0\rangle$$

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$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \Lambda^2 d_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) - \frac{2}{3}\,\alpha_s\Big(\frac{1}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}+\frac{1}{|\boldsymbol{x}_2-\boldsymbol{x}_3|}+\frac{1}{|\boldsymbol{x}_3-\boldsymbol{x}_1|}\Big)$$

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When two of the quarks coincide the potential reduces to the $q\bar{q}$ potential:

$$V_{qqq}(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1-\boldsymbol{x}_2| - \frac{4}{3}\,\frac{\alpha_s}{|\boldsymbol{x}_1-\boldsymbol{x}_2|} = V_{q\bar{q}}(\boldsymbol{x}_1,\boldsymbol{x}_2)$$

Analogous potentials are obtained for any quark and gluon Fock state, such as $q\bar{q}g$ and gg.

$\mathcal{O}\left(\alpha_s^0\right)$ $q\bar{q}$ bound states

An $\mathcal{O}\left(\alpha_s^0\right)$ meson state with $\boldsymbol{P} = 0$ and wave function Φ:

$$|M\rangle = \sum_{A,B;\alpha,\beta}\int d\boldsymbol{x}_1 d\boldsymbol{x}_2\, \bar{\psi}_\alpha^A(t=0,\boldsymbol{x}_1)\delta^{AB}\Phi_{\alpha\beta}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi_\beta^B(t=0,\boldsymbol{x}_2)\,|0\rangle$$

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The (rest frame) bound state condition $H\,|M\rangle = M\,|M\rangle$ gives

$$\left[i\gamma^0\boldsymbol{\gamma}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0\right]\Phi(\boldsymbol{x}) + \Phi(\boldsymbol{x})\left[i\gamma^0\boldsymbol{\gamma}\cdot\overleftarrow{\boldsymbol{\nabla}} - m\gamma^0\right] = \left[M - V(|\boldsymbol{x}|)\right]\Phi(\boldsymbol{x})$$

where $\boldsymbol{x} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and $V(\boldsymbol{x}) = \Lambda^2|\boldsymbol{x}|$

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where $\boldsymbol{x} \equiv \boldsymbol{x}_1 - \boldsymbol{x}_2$ and $V(\boldsymbol{x}) = \Lambda^2|\boldsymbol{x}|$

In the non-relativistic limit ($m \gg \Lambda$) this reduces to the Schrödinger equation.

$\Longrightarrow$ The quarkonium phenomenology with the Cornell potential.

Separation of radial and angular variables

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\}+m\left[\gamma^0,\Phi(\boldsymbol{x})\right]=\left[M-V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

Expanding the 4×4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x})=\sum_i \Gamma_i(\boldsymbol{x})F_i(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

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0^{-+} trajectory $[s=0,\ \ell=j]:$ $-\eta_P=\eta_C=(-1)^j$ $\gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

0^{--} trajectory $[s=1,\ \ell=j]:$ $\eta_P=\eta_C=-(-1)^j$ $\gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \boldsymbol{\alpha}\cdot\boldsymbol{L},\ \gamma^0\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

0^{++} trajectory $[s=1,\ \ell=j\pm1]:$ $\eta_P=\eta_C=+(-1)^j$ $1,\ \boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L},\ \gamma^0\gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

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0^{+-} trajectory [exotic] : $\quad \eta_P=-\eta_C=(-1)^j\quad \gamma^0,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{L}$

$\Rightarrow$ There are no solutions for quantum numbers that would be exotic in the NR quark model (despite the relativistic dynamics)

The BSE gives the radial equations for the $F_i(r)$

(There are two coupled radial equations for the 0^{++} trajectory)

Example: 0^{-+} trajectory wf's at $O(\alpha_s^0)$

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

$\eta_P = (-1)^{j+1}$

$\eta_C = (-1)^j$

Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\frac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

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Local normalizability at $r = 0$ and at $V(r) = M$ (!) determines the discrete M

C.f.: Dirac eq.: Has continuous spectrum for a linear potential

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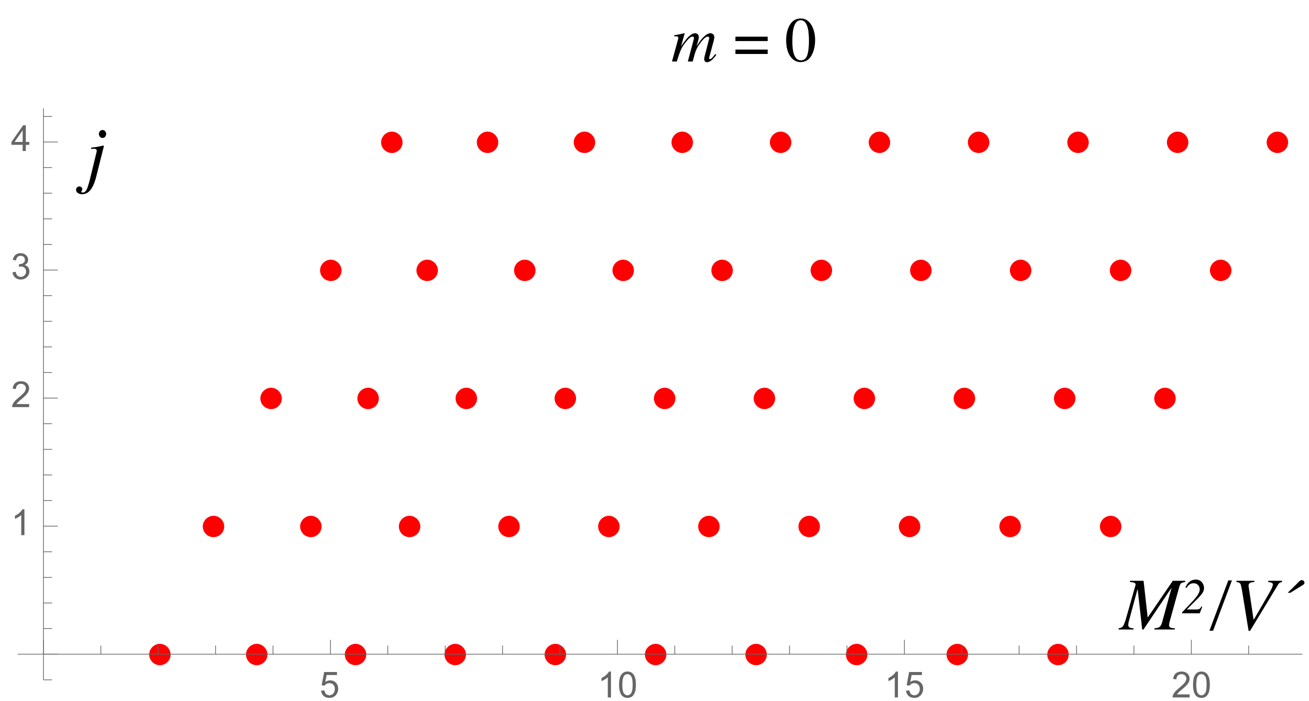
Mass spectrum:

Linear Regge trajectories with daughters

Spectrum similar to dual models



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Temporal gauge ($A^0 = 0$) is advantageous for equal-time bound states

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Many features of hadrons thus obtained look promising & intriguing

PH 2109.06257

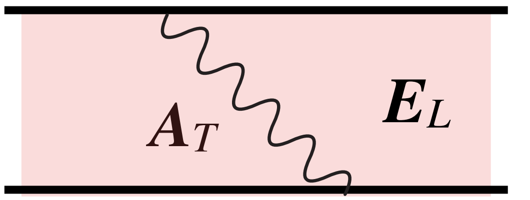
Special thanks to Matti Järvinen, for valuable advice

PH 2101.06721v2

Back-up slides

The $qg\bar{q}$ potential

A $q\bar{q}$ state, with the emission of a transverse gluon:



$$|q(\boldsymbol{x}_1)g(\boldsymbol{x}_g)\bar{q}(\boldsymbol{x}_2)\rangle \equiv \sum_{A,B,b} \bar{\psi}_A(\boldsymbol{x}_1)\, A_b^j(\boldsymbol{x}_g) T_{AB}^b \psi_B(\boldsymbol{x}_2)\,|0\rangle$$

$$V_{qgq}^{(0)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \frac{\Lambda^2}{\sqrt{C_F}}\, d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) \qquad \text{(universal } \Lambda\text{)}$$

$$d_{qgq}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) \equiv \sqrt{\tfrac{1}{4}(N - 2/N)(\boldsymbol{x}_1 - \boldsymbol{x}_2)^2 + N(\boldsymbol{x}_g - \tfrac{1}{2}\boldsymbol{x}_1 - \tfrac{1}{2}\boldsymbol{x}_2)^2}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1, \boldsymbol{x}_g, \boldsymbol{x}_2) = \tfrac{1}{2}\,\alpha_s\Big[\frac{1}{N}\,\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} - N\Big(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_g|} + \frac{1}{|\boldsymbol{x}_2 - \boldsymbol{x}_g|}\Big)\Big]$$

When q and g coincide:

$$V_{qgq}^{(0)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = \Lambda^2|\boldsymbol{x}_1 - \boldsymbol{x}_2| = V_{q\bar{q}}^{(0)}$$

$$V_{qgq}^{(1)}(\boldsymbol{x}_1 = \boldsymbol{x}_g, \boldsymbol{x}_2) = V_{q\bar{q}}^{(1)}$$

The gg potential

A "glueball" component: $$|g(\boldsymbol{x}_1)g(\boldsymbol{x}_2)\rangle \equiv \sum_a A_a^i(\boldsymbol{x}_1)\, A_a^j(\boldsymbol{x}_2)\,|0\rangle$$

has the potential $$V_{gg} = \sqrt{\frac{N}{C_F}}\,\Lambda^2\,|\boldsymbol{x}_1 - \boldsymbol{x}_2| - N\,\frac{\alpha_s}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|}$$

This agrees with the qgq̄ potential where the quarks coincide:

$$V_{gg}(\boldsymbol{x}, \boldsymbol{x}_g) = V_{qg\bar{q}}(\boldsymbol{x}, \boldsymbol{x}_g, \boldsymbol{x})$$

It is straightforward to work out the instantaneous potential for any Fock state.

Strongly bound Positronium in QED_2 ($D = 1+1$)

In QED_2 the spectrum can be determined both for weak ($e/m \ll 1$) and strong ($e/m \gg 1$) coupling

S. Coleman, Annals Phys. **101** (1976) 239

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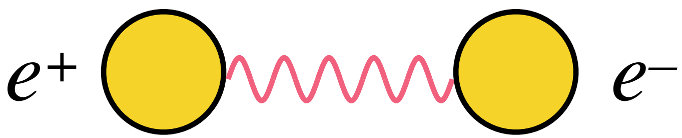
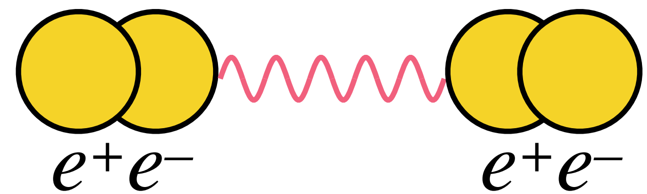
$e/m \ll 1$



Bound states of weakly interacting fermions

$e/m \gg 1$



Bound states of weakly interacting bosons

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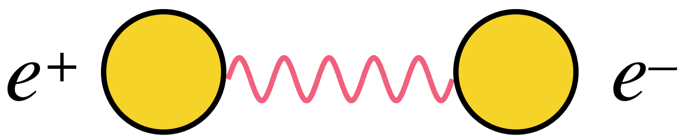
$e/m << 1$



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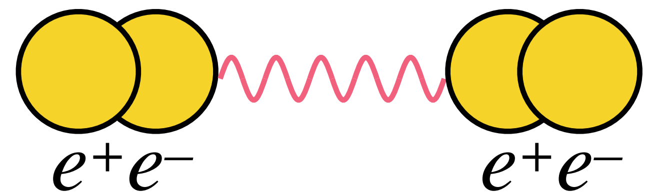

Bound states of weakly interacting bosons

For $e/m \to \infty$ QED_2 describes a non-interacting, pointlike boson field.

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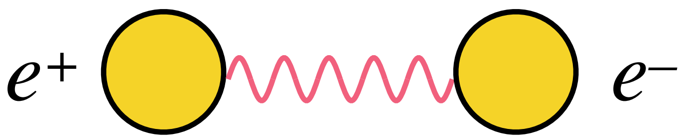
$e/m \ll 1$



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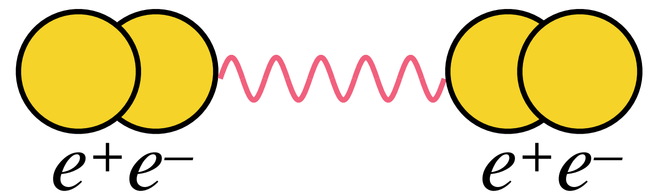

Bound states of weakly interacting bosons

For $e/m \to \infty$ QED_2 describes a non-interacting, pointlike boson field.

Paradox: The hadron spectrum suggests weakly bound valence quarks, yet the light quarks are strongly bound (relativistic).