Remarks on bound states in perturbation theory[1]

Paul Hoyer (11 March 2019)

Hadrons are often said to be non-perturbative bound states of QCD. This assumes that we know the properties of perturbative bound states. However, even Positronia (the e^+e^- atoms of QED) are not perturbative in the usual sense:

- There is no Positronium pole in $e^+e^- \to e^+e^-$ Feynman diagrams of any order in α. The exact scattering amplitude does have such poles.
- Bound state wave functions cannot be approximated by low orders in α. The solutions of the Schrödinger equation are exponential in α.
- Binding energies do have a perturbative expansion. The hyperfine splitting of Positronium is at $\mathcal{O}\left(\alpha^7 \log \alpha\right)$ consistent with accurate measurements.

Hadrons have non-perturbative features such as color confinement and spontaneously broken chiral symmetry. In other respects they resemble atoms:

- The properties of heavy quarkonia are well described using the Schrödinger equation with a linear confining (Cornell) potential.
- Hadron quantum numbers are determined by their valence quarks ($q\bar{q}$, qqq).
- Hadrons have narrow and selective decay widths (the OZI rule).

These and other remarkable facts motivate me to consider whether perturbation theory is relevant for a description of hadrons as QCD bound states. Confinement and chiral symmetry breaking must then appear already at lowest order.

Positronium calculations use a non-relativistic expansion of the action (NRQED), and choose solutions of the Schrödinger equation as "lowest order" bound states. What is the (or is there an) analogous choice in QCD?

QED bound states are largely ignored in field theory textbooks. This hampers progress also in QCD. Issues which can be resolved for atoms include:

- The Schrödinger equation is postulated in Introductory Quantum Mechanics. How can it be derived from the underlying theory of QED?
- What is the Schrödinger wave function of a Positronium atom in motion? Does it Lorentz contract?
- The Dirac equation defines relativistic bound state wave functions, which describe states with more than a single fermion constituent. How does one determine the Fock components of the Dirac states?

A thorough understanding of the above and other issues of QED bound states are required before we may conclude whether perturbative methods are applicable to hadrons. My recent research on bound state issues are linked on this home page.

[1]Papers and references related to these remarks are given on `http://www.helsinki.fi/~hoyer`