L9. Photon-Baryon Fluid

- If we ignore the gravitational redshift effects (which have $\int_{\eta_*}^{\eta_0}$ even in the δ-function approximation), the sources for the line-of-sight integrals are where the photons last scattered. In the δ-function approximation this becomes a snapshot of the last scattering surface $\eta = \eta_*$. At that time, photons were interacting with the baryons, making the transition from behaving as a single baryon-photon fluid to streaming freely. Thus the fluid approximation is beginning to break at this time (photon diffusion).

- Therefore the most prominent features on the C_ℓ spectra, at least from scalar perturbations, can be understood in terms of the behaviour of the baryon-photon fluid and photon diffusion.

- Before photon decoupling, photons and baryons were tightly coupled, in other words

$$\underline{a n_e \sigma_T \text{ was very large}}$$

Thus the behaviour of the photon-baryon fluid can be studied using the tight-coupling approximation, where the eqs. are expanded in terms of powers of $(a n_e \sigma_T)^{-1}$ and the higher powers are dropped.

- Consider scalar perturbations ($m=0$).

The baryon perturbation eqs (7.26 and 7.37) are

$$\delta_b' = -kv_b + 3\psi' \quad (1)$$

$$v_b' = -\mathcal{H}v_b + k\phi + \frac{a n_e \sigma_T}{R}(\Theta_1^0 - v_b) \quad (2)$$

where $R \equiv \dfrac{3\bar{\rho}_b}{4\bar{\rho}_\gamma} \propto a(\eta)$. (3)

The lowest multipoles of the photon perturbation eqs are

$$\Theta_0^{0\prime} = -\frac{1}{3}k\Theta_1^0 + \psi' \quad (4)$$

$$\Theta_1^{0\prime} = k\Theta_0^0 - \frac{2}{5}\Theta_2^0 + k\phi + a n_e \sigma_T (v_b - \Theta_1^0) \quad (5)$$

$$\Theta_2^{0\prime} = -\frac{3}{7}k\Theta_3^0 + \frac{2}{3}k\Theta_1^0 - a n_e \sigma_T \left(\frac{9}{10}\Theta_2^0 + \frac{\sqrt{6}}{10}E_2^0\right) \quad (6)$$

- In the tight coupling limit the large $-\frac{9}{10}a n_e \sigma_T \Theta_2^0$ term in the $\Theta_2^{0\prime}$ equation keeps the quadrupole Θ_2^0 small. The same holds for the higher multipoles Θ_ℓ^0 because of the loss term $-a n_e \sigma_T \Theta_\ell^0$. Thus we can initially drop Θ_2^0 and the higher multipoles.

- Rewrite (2) as

$$v_b' + \mathcal{H}v_b - k\phi = a n_e \sigma_T \cdot \frac{1}{R}(\Theta_1^0 - v_b)$$

$$\Rightarrow \quad v_b = \Theta_1^0 - \underbrace{\frac{R}{a n_e \sigma_T}}_{\text{small}}(v_b' + \mathcal{H}v_b - k\phi) \quad (7)$$

Thus in the tight-coupling limit v_b is close to Θ_1^0.

To 0^{th} order, $\quad v_b \simeq \Theta_1^0 \quad (8)$

To 1^{st} order, $\quad v_b \simeq \Theta_1^0 - \dfrac{R}{a n_e \sigma_T}\left(\Theta_1^{0\prime} + \mathcal{H}\Theta_1^0 - k\phi\right) \quad (9)$

where we applied the approximation (8) to the second term of Eq. (7)

- After dropping Θ_2^0, Eq. (5) is

$$\Theta_1^{0\prime} \simeq k\Theta_0^0 + k\phi + \underbrace{a n_e \sigma_T (V_b - \Theta_1^0)}_{\underset{(9)}{\simeq} -R(\Theta_1^{0\prime} + \mathcal{H}\Theta_1^0 - k\phi)}$$

$$\Rightarrow \quad (1+R)\,\Theta_1^{0\prime} + \mathcal{H}R\,\Theta_1^0 \simeq k\Theta_0^0 + (1+R)\,k\phi \qquad (10)$$

Since $R \propto a \Rightarrow \frac{R'}{R} = \frac{a'}{a} = \mathcal{H} \Rightarrow \mathcal{H}R = R'$ (11)

we can rewrite (10) as

$$\left[(1+R)\Theta_1^0\right]' = k\left[\Theta_0^0 + (1+R)\phi\right] \qquad (12)$$

- Multiplying (4) by $1+R$ and derivating we get

$$\left[(1+R)\,\Theta_0^{0\prime}\right]' = -\tfrac{1}{3}k\underbrace{\left[(1+R)\Theta_1^0\right]'}_{k\left[\Theta_0^0 + (1+R)\phi\right] \text{ by } (12)} + \left[(1+R)\psi'\right]'$$

$$\Rightarrow \quad \left[(1+R)\Theta_0^{0\prime}\right]' + \tfrac{1}{3}k^2\,\Theta_0^0 = -\tfrac{1}{3}k^2(1+R)\phi + \left[(1+R)\psi'\right]' \qquad (13)$$

This equation describes forced (by the metric perturbations) and damped (by baryons which come in via R) oscillations of the photon density perturbation $\delta_\gamma = 4\Theta_0^0$.

- If we temporarily drop the metric forcing, i.e., set $\phi = \psi = 0$, the solutions are (exercise)

$$\Theta_0^0 = A(1+R)^{-1/4}\cos(ks+\varphi)$$
$$\Theta_1^0 = \sqrt{3}\,A(1+R)^{-3/4}\sin(ks+\varphi) \qquad (14)$$

where

$$s(\eta) \equiv \int^{\eta} \frac{d\eta}{\sqrt{3(1+R)}} \qquad (15)$$

is the sound horizon, and A and φ are constants (amplitude and phase shift).

- The effect of the metric perturbations is, roughly, to shift the center value of the oscillation by $-(1+R)\phi$. (16)

- Those oscillations are called acoustic oscillations. The photon-baryon fluid density perturbations oscillate as standing waves, under the forces of gravity and baryon pressure. Those oscillations are reflected also at the higher scalar multipoles Θ_l^0 and E_l^0.

- The dominant contribution to the scalar C_l^{TT} spectrum comes from the effective temperature $\Theta_0^0+\phi$ near the time of photon decoupling (see Eqs. (4.8) and (8.1)). Due to (16), this quantity oscillates around $-R\phi$.

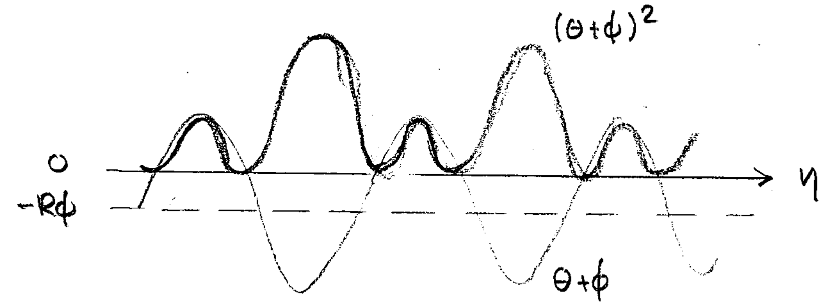

Fig. 1

As a function of time η, the square $(\Theta_0^0+\phi)^2$ then has alternate high and low peaks (see Figure, somewhat exaggerated). Since C_l^{TT} comes from a "snapshot" at $\eta=\eta_*$, where the different k modes are at different oscillation phases, and the $j_l(k\eta_0-k\eta_*)$ give the dominant contribution from different k to different l, these peaks at $(\Theta_0^0+\phi)^2(\eta)$ of Fig. 1 get translated into the "acoustic peaks" as a function of l in the C_l^{TT} spectrum.

- The corresponding oscillations in Θ_2^0 and E_2^0 lead to similar acoustic peaks in the C_l^{EE} spectrum. Because of the different structure of the j_l and ϵ_l^0 radial functions, the peaks in the C_l^{EE} spectrum appear at different l than in the C_l^{TT} spectrum.

- Obviously we get these acoustic peaks in the cross spectrum C_l^{TE} also, located at intermediate (between C_l^{TT} and C_l^{EE}) l positions.

- OK, time runs out. The photon-baryon fluid and other phenomenology of the C_ℓ^{TT}, C_ℓ^{TE}, C_ℓ^{EE}, C_ℓ^{BB} for scalar, vector, and tensor perturbations should be discussed much more extensively, and this section should actually have started a whole new Chapter on these issues.
- If you want to learn more, you could try Section IV of Hu & White, whose beginning I had followed here. (Note the different conventions for ϕ and ψ).
- The case of scalar perturbations, Θ_ℓ^m, and C_ℓ^{TT} is discussed qualitatively in my Cosmology II lecture notes, Chapter 12; and more exactly in my CMB Physics 2004 lecture notes (Chapter A; note that $\Theta_\ell \equiv \frac{1}{2\ell+1}\Theta_\ell^0$.)

The Master's thesis (pro gradu) of Reijo Keskitalo (2005) provides a detailed study.