L7.4 Tensor Perturbations

- Of the energy tensor perturbations, only the anisotropic stress Π_{ij} has a tensor part (CDT §9.1). Thus only photons and neutrinos are involved. Their evolution is given by the photon Boltzmann hierarchy, Eq. (1.1), for $\Theta_L^{\pm 2}$, $E_L^{\pm 2}$, $B_L^{\pm 2}$, and the neutrino Boltzmann hierarchy, Eq. (22), for $N_L^{\pm 2}$.

- The evolution of the tensor metric perturbations is given by the Einstein equation (T1.11)

$$E_{ij}^{T''} + 2\mathcal{H} E_{ij}^{T'} + k^2 E_{ij}^T = 8\pi G a^2 \cdot \bar{p}\, \Pi_{ij}^T \tag{38}$$

In the crd. system where $\hat{z} = \hat{k}$, only the

$$E_{11}^T = -E_{22}^T \equiv \tfrac{1}{2} h_+ \quad \text{and} \quad E_{12}^T = E_{21}^T \equiv \tfrac{1}{2} h_\times \tag{39}$$

components of E_{ij}^T are nonzero. Thus we rewrite (38) as

$$\begin{aligned} h_+'' + 2\mathcal{H} h_+' + k^2 h_+ &= 16\pi G a^2 \cdot \bar{p}\, \Pi_{11}^T \\ h_\times'' + 2\mathcal{H} h_\times' + k^2 h_\times &= 16\pi G a^2 \cdot \bar{p}\, \Pi_{12}^T \end{aligned} \tag{40}$$

- In Section F7.3 we defined $$h^{(\pm 2)} \equiv -\frac{1}{\sqrt{6}} (h_+ \mp i h_\times) \tag{41}$$

$$\Rightarrow \quad h^{(\pm 2)''} + 2\mathcal{H} h^{(\pm 2)'} + k^2 h^{(\pm 2)} = -\frac{16\pi G a^2 \bar{p}}{\sqrt{6}} \left(\Pi_{11}^T \mp i \Pi_{12}^T \right) \tag{42}$$

From Eq. (20) we have that

$$\bar{p} \left(\Pi_{11}^T \mp i \Pi_{12}^T \right) = -\frac{4\sqrt{6}}{5} \left(\bar{p}_\gamma \Theta_2^{\pm 2} + \bar{p}_\nu N_2^{\pm 2} \right) \tag{43}$$

so that finally

$$\boxed{h^{(\pm 2)''} + 2\mathcal{H} h^{(\pm 2)'} + k^2 h^{(\pm 2)} = 16\pi G a^2 \cdot \frac{4}{5} \left(\bar{p}_\gamma \Theta_2^{\pm 2} + \bar{p}_\nu N_2^{\pm 2} \right)} \tag{44}$$

and we have the complete set of perturbation eqs for tensor perturbations.