L7.3 Scalar Perturbations

- At the end of CPT §18 we had arrived at the scalar ($m=0$) perturbation eqs:

$$\delta_c' = -kv_c + 3\psi' \tag{24}$$

$$v_c' = -\mathcal{H}v_c + k\phi \tag{25}$$

$$\delta_b' = -kv_b + 3\psi' \tag{26}$$

$$v_b' = -\mathcal{H}v_b + k\phi + \text{collision term} \tag{27}$$

$$\delta_\gamma' = -\tfrac{4}{3}kv_\gamma + 4\psi' \tag{28}$$

$$v_\gamma' = \tfrac{1}{4}k\delta_\gamma - \tfrac{1}{6}k\Pi_\gamma + k\phi + \text{collision term} \tag{29}$$

$$\delta_\nu' = -\tfrac{4}{3}kv_\nu + 4\psi' \tag{30}$$

$$v_\nu' = \tfrac{1}{4}k\delta_\nu - \tfrac{1}{6}k\Pi_\nu + k\phi \tag{31}$$

We also need the Einstein equations for the metric perturbations ϕ and ψ. In CPT §10 we had 4 Einstein eqs for them, but we need only 2, since we have also used the energy-momentum continuity eqs (above) for the fluid components. We pick CPT Eq. (144)

$$3\mathcal{H}(\psi' + \mathcal{H}\phi) - \nabla^2\psi = -4\pi G a^2 \delta\rho$$

and CPT Eq. (150)

$$k^2(\psi_{\vec{k}} - \phi_{\vec{k}}) = 8\pi G a^2 \bar{p}\Pi_{\vec{k}}$$

Converting also the first one to Fourier space, and expressing the sources $\delta\rho$ and $\Sigma = \bar{p}\Pi$ in terms of components, they become

$$3\mathcal{H}(\psi' + \mathcal{H}\phi) + k^2\psi = -4\pi G a^2(\bar{\rho}_b\delta_b + \bar{\rho}_c\delta_c + \bar{\rho}_\gamma\delta_\gamma + \bar{\rho}_\nu\delta_\nu) \tag{32}$$

$$k^2(\psi - \phi) = 8\pi G a^2(\bar{p}_\gamma\Pi_\gamma + \bar{p}_\nu\Pi_\nu) \tag{33}$$

(baryons and CDM can be neglected in the second eq. since their pressure is so small).

- From Eqs. (12, 13, 20) we have

$$\delta_\gamma = 4\Theta_0^0$$
$$v_\gamma = \Theta_1^0 \qquad (34)$$
$$\Pi_\gamma = \frac{12}{5}\Theta_2^0$$

so we can rewrite Eqs (28, 29) as

$$\Theta_0^{0\,\prime} = -\frac{1}{3}k\Theta_1^0 + \psi' \qquad (35)$$
$$\Theta_1^{0\,\prime} = k\Theta_0^0 - \frac{2}{5}\Theta_2^0 + k\phi + \text{collision term}$$

and comparing to Eq. (1.1) we see that the collision term in question is

$$-an_e\sigma_T\Theta_1^0 + S_1^0 - k\phi = an_e\sigma_T(v_b - v_\gamma) \qquad (36)$$

- The collision terms in Eqs. (27) and (29) represent momentum exchange between photons and baryons. Momentum conservation in collisions requires that

$$(\bar\rho_b + \bar p_b)v_b' = -(\bar\rho_\gamma + \bar p_\gamma)v_\gamma' \qquad \text{(collision term only!)}$$

$$\Rightarrow \quad \text{baryon collision term} = -\underbrace{\frac{\bar\rho_\gamma + \bar p_\gamma}{\bar\rho_b + \bar p_b}}_{\frac{4}{3}\frac{\bar\rho_\gamma}{\bar\rho_b}} \cdot \text{photon collision term}$$

Thus we can complete Eq. (27) as

$$v_b' = -\mathcal{H}v_b + k\phi + \frac{4}{3}\frac{\bar\rho_\gamma}{\bar\rho_b}an_e\sigma_T(v_\gamma - v_b) \qquad (37)$$

- The relations (34) apply also to neutrinos, and allow us to connect the multipole Boltzmann hierarchies (1.1) and (7.21) with the fluid and Einstein eqs (24-37). Thus we have the complete set of perturbation eqs. for scalar perturbations.