L7.2 Neutrino Multipoles

- For CMB physics, we can approximate neutrinos as massless, if $m_\nu \ll T_{eq} = \mathcal{O}(1\,\text{eV})$, the temperature of matter-radiation equality, since neutrinos affect the CMB only through their gravitational effect.

- We can then represent the neutrino distribution function in terms of a brightness function $N(\eta, x^i; \hat{n})$, just like photons:

$$f_\nu(\eta, x^i, q, \hat{n}) = \frac{g_\nu/(2\pi)^3}{\exp\left\{\dfrac{q}{T_\nu(\eta)\,[1+N(\eta,x^i;\hat{n})]}\right\} + 1} \tag{21}$$

($+1$ ← fermions)

(The effect of mass is to make the brightness function energy dependent, and things become more complicated.)

- Massless neutrinos free stream and redshift just like photons; but suffer no collisions (we consider times well after ν decoupling only). Therefore the question of possible neutrino polarization (I know nothing about it) is irrelevant for CMB physics, since polarization does not affect the energy tensor perturbations (only the brightness function is involved), and, in the absence of collisions, it cannot affect the evolution of the brightness function. the following Boltzmann hierarchy for neutrinos:

$$N_l^{m\,\prime} = \frac{\sqrt{l^2-m^2}}{2l-1}\,k N_{l-1}^m - \frac{\sqrt{(l+1)^2-m^2}}{2l+3}\,k N_{l+1}^m + {}^{(\nu)}S_l^m \tag{22}$$

where

$${}^{(\nu)}S_0^0 = \psi' \qquad {}^{(\nu)}S_1^{\pm1} = B^{(\pm1)\prime} \qquad {}^{(\nu)}S_2^{\pm2} = -h^{(\pm2)\prime} \tag{23}$$

$${}^{(\nu)}S_1^0 = k\phi$$