L7. The Full Set of Equations

- Let's now collect the full set of perturbation eqs.[*] that need to be evolved to get the CMB C_ℓ. We have assumed the universe contains 4 components, b, c, γ, ν, that have perturbations.

- The eqs. for photon perturbations are the Θ_ℓ^m, E_ℓ^m, B_ℓ^m Boltzmann hierarchy of Eq. (1.1). However, they involve the baryon velocity perturbations $v_b^{(m)}$; and the metric perturbations $\phi, \psi, B^{(\pm1)}, h^{(\pm2)}$, whose equations are the Einstein eqs. The Einstein eqs. in turn involve the energy tensor perturbations $\delta_{(i)}, \vec{V}_{(i)}, \Pi_{ij(i)}$ of all 4 components (i).

- The C_ℓ computation problem can be split into two parts:

  1) Calculate the transfer functions for a given background universe
  2) Calculate the C_ℓ from the transfer functions and primordial power spectra according to the Eqs. in Section L6.4.

- The perturbation eqs. are used to calculate the transfer functions. We start with initial condition $r_{\vec{k}}^m(0) = 1$ and evolve all the perturbation eqs to arrive at the final $\Theta_{\ell\vec{k}}^m(\eta_0)$, $E_{\ell\vec{k}}^m(\eta_0)$, $B_{\ell\vec{k}}^m(\eta_0)$ which are then just the transfer functions $\Theta_\ell^m(\eta_0, k)$, $E_\ell^m(\eta_0, k)$, $B_\ell^m(\eta_0, k)$ for the scale k and the scalar/vector/tensor mode $m = 0, 1, 2$. In this calculation all the other perturbation quantities then represent just transfer functions $\phi(\eta, k)$, $\psi(\eta, k)$, $B^{(1)}(\eta, k)$, $h^{(2)}(\eta, k)$, $\delta_b(\eta, k)$, $v_b^{(m)}(\eta, k)$, etc.; which give the linear relations between the primordial perturbations $r_{\vec{k}}^m(0)$ and the actual quantities, e.g., $\phi_{\vec{k}}(\eta) = \phi(\eta, k) r_{\vec{k}}^0(0)$. (1)

*) For lack of time, we'll skip the case of vector perturbations