L6.3 Transfer Functions

- The multipoles $\Theta^m_{L\vec{k}}(\eta_0)$, $E^m_{L\vec{k}}(\eta_0)$, $B^m_{L\vec{k}}(\eta_0)$ result from primordial perturbations $r^m_{\vec{k}}(0)$ through linear evolution which does not mix different $\vec{k}, m$ modes. Because of the linearity of the evolution, they are proportional to the initial values $r^m_{\vec{k}}(0)$ (twice as big $r^m_{\vec{k}}(0)$ would result in twice as big $\Theta^m_{L\vec{k}}(\eta_0)$ etc.):

$$\Theta^m_{L\vec{k}}(\eta_0) \propto r^m_{\vec{k}}(0)$$

$$E^m_{L\vec{k}}(\eta_0) \propto r^m_{\vec{k}}(0)$$

$$B^m_{L\vec{k}}(\eta_0) \propto r^m_{\vec{k}}(0)$$

The proportionality constants are called transfer functions. They depend on L, m, and the magnitude k of $\vec{k}$, but not on the direction of $\vec{k}$, since only the magnitude appears in the Boltzmann equations. We use the same symbols to denote the transfer functions and the corresponding multipole quantities:

$$\boxed{\begin{aligned} \Theta^m_{L\vec{k}}(\eta_0) &\equiv \Theta^m_L(\eta_0,k)\, r^m_{\vec{k}}(0) \\ E^m_{L\vec{k}}(\eta_0) &\equiv E^m_L(\eta_0,k)\, r^m_{\vec{k}}(0) \\ B^m_{L\vec{k}}(\eta_0) &\equiv B^m_L(\eta_0,k)\, r^m_{\vec{k}}(0) \end{aligned}} \qquad (25)$$

That is, $\Theta^m_L(\eta_0,k)$ is the value of $\Theta^m_{L\vec{k}}(\eta_0)$ that would result from $r^m_{\vec{k}}(0) = 1$, etc. The transfer functions depend also on the cosmological parameters that describe the evolution of the background universe.

- Inspection of Eq. (11) shows that the evolution of $\{\Theta^m_L, E^m_L, B^m_L\}^{\infty}_{L=0}$ is the same as that of $\{\Theta^{-m}_L, E^{-m}_L, -B^{-m}_L\}^{\infty}_{L=0}$.

$$\Rightarrow \boxed{\begin{aligned} \Theta^{-m}_L(\eta_0,k) &= \Theta^m_L(\eta_0,k) \\ E^{-m}_L(\eta_0,k) &= E^m_L(\eta_0,k) \end{aligned} \quad \text{but} \quad B^{-m}_L(\eta_0,k) = -B^m_L(\eta_0,k)} \qquad (26)$$

Note that $\underline{B^0_L(\eta_0,k) = 0}$