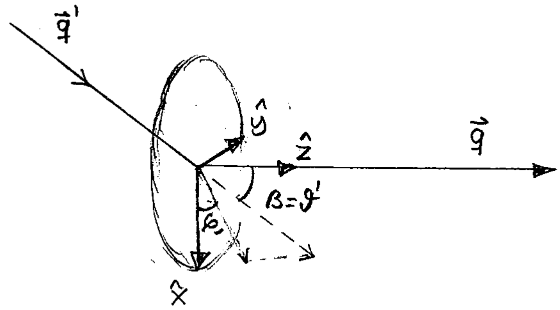
C2. The Case of Unpolarized Incoming Radiation

- Before going to the full analysis of Stokes parameter evolution, consider the simple case of scattering of unpolarized radiation. Since now $Q' = U' = V' = 0$; we don't have to worry about the crd. system for the incoming Stokes parameters. We consider a particular outgoing photon direction $\hat{n}$. For this application we introduce an xyz crd. system, where $\hat{z} \parallel \hat{n}$, and $\hat{x}, \hat{y}$ are the two directions used to define the Stokes parameters for the outgoing photon, $Q \equiv \langle \hat{n}_x \rangle - \langle \hat{n}_y \rangle$ etc.



- If the incoming photon is in the xz plane, then the Stokes parameters of §C1 defined in the $\parallel, \perp$ basis are equal to our xy basis Stokes parameters. Otherwise we get the xy basis Stokes parameters I, Q, U from the $\parallel, \perp$ basis Stokes parameters $\tilde{I}, \tilde{Q}, \tilde{U}$ by rotating the crd. system by the angle $-\varphi'$ where (ϑ', φ') is the direction the incoming photon is going to. Thus we get from (1.16), for $Q(\vec{q}\,') = U(\vec{q}\,') = 0$,

$$\frac{d\tilde{I}(\vec{q})}{dt\,d\Omega'} = n_e \frac{3\sigma_T}{16\pi}(\cos^2\vartheta' + 1)\, I(\vec{q}\,') \qquad \frac{d\tilde{Q}(\vec{q})}{dt\,d\Omega'} = n_e \frac{3\sigma_T}{16\pi}\underbrace{(\cos^2\vartheta' - 1)}_{-\sin^2\vartheta'} I(\vec{q}\,') \tag{1}$$

$$\frac{d\tilde{U}(\vec{q})}{dt\,d\Omega'} = 0$$

and

$$\frac{dI(\vec{q})}{dt\,d\Omega'} = \frac{d\tilde{I}(\vec{q})}{dt\,d\Omega'}$$

$$\frac{dQ(\vec{q})}{dt\,d\Omega'} = \frac{d\tilde{Q}(\vec{q})}{dt\,d\Omega'} \cdot \cos(-2\varphi') + \cancelto{0}{\frac{d\tilde{U}(\vec{q})}{dt\,d\Omega'}} \sin(-2\varphi') \tag{2}$$

$$\frac{dU(\vec{q})}{dt\,d\Omega'} = -\frac{d\tilde{Q}(\vec{q})}{dt\,d\Omega'} \sin(-2\varphi') + \cancelto{0}{\frac{d\tilde{U}(\vec{q})}{dt\,d\Omega'}} \cos(2\varphi')$$

- Now the contributions to the I, Q, U from the different incoming photon directions are all in the same basis, and we can integrate over $d\Omega'$: (we include also the loss terms)

$$\boxed{\begin{aligned} \frac{dI(\vec{q})}{dt} &= -n_e\sigma_T I(\vec{q}) + n_e \frac{3\sigma_T}{16\pi}\int(1+\cos^2\vartheta')\, I(q,\hat{n}')\, d\Omega' && (3) \\ \frac{dQ(\vec{q})}{dt} &= n_e \frac{3\sigma_T}{16\pi}\int(-\sin^2\vartheta' \cos 2\varphi')\, I(q,\hat{n}')\, d\Omega' && (4) \\ \frac{dU(\vec{q})}{dt} &= n_e \frac{3\sigma_T}{16\pi}\int(-\sin^2\vartheta' \sin 2\varphi')\, I(q,\hat{n}')\, d\Omega' && (5) \end{aligned}}$$

- In the case the incoming radiation is also isotropic, $I(q,\hat{n}') = I(q)$, the integral in (3) gives

$$I(q)\int_0^{2\pi} d\varphi' \int_{-1}^{1}(1+\cos^2\vartheta')\, d\cos\vartheta' = I(q)\cdot 2\pi\cdot\frac{8}{3} = \frac{16\pi}{3} I(\vec{q})$$

and the integrals in (4) and (5) give zero, $\int_0^{2\pi}\cos 2\varphi\, d\varphi = \int_0^{2\pi}\sin 2\varphi\, d\varphi = 0$, so we have $dI(\vec{q}) = dQ(\vec{q}) = dU(\vec{q}) = 0$.

$\therefore$ Thomson scattering has <u>no effect on isotropic unpolarized radiation</u>.

- But <u>for anisotropic unpolarized radiation</u>, Thomson scattering

a)	alters the anisotropy
b)	produces linear polarization

- Exercise: Express the results (3–5) in terms of the multipoles $a^T_{\ell m}$ of $I(q,\hat{n}')$, i.e., find which multipoles of the anisotropy contribute to the polarization.

- This result is obvious from the symmetry of the problem; there is no special direction that could stand out for anisotropy or polarization.
Note that the result was derived in the <u>electron rest frame</u>.
For a fluid of electrons, where the electrons have thermal motions, this result generalizes to the <u>rest frame of the electron fluid</u>, where the average electron velocity is zero; since the anisotropy/polarization effects from scattering on individual electrons, average out.
(At least if the electron velocity distribution is isotropic in this frame; i.e. the perfect fluid approximation is valid for them.)