Due on Tuesday, November 27th, by 14:00 o'clock. There are no lectures or exercises on December 6th, due to Independence Day.

1. **Helicity basis.** The helicity basis (of 1-photon space) is defined

$$\begin{aligned} |+\rangle &\equiv \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right) \\ |-\rangle &\equiv \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right) . \end{aligned}$$

   Derive the Stokes operators in terms of the helicity basis for 1-photon space and in terms of the creation and annihilation operators $a_+^\dagger$, $a_+$, $a_-^\dagger$, $a_-$ for many-photon space.

2. **Properties of the reduced density operator.** Show that the 1-particle reduced density operator $\hat{\rho}^{(1)}$ is Hermitian and positive semidefinite.

3. **Polarization of 2-photon state.**

   (a) Consider a general 2-photon (pure) state

$$|\psi\rangle = a_1 e^{i\alpha_1}|20\rangle + a_2 e^{i\alpha_2}|11\rangle + a_3 e^{i\alpha_3}|02\rangle$$

   (expressed in the Fock basis $\{|n_x, n_y\rangle\}$). Find the Stokes parameters $I$, $Q$, $U$, and $V$. Under what conditions do we get $Q = U = V = 0$?

   (b) Construct a 2-photon state with $\varphi$-polarization by applying the creation operator $a_\varphi^\dagger$ on the vacuum state. Find $I$, $Q$, $U$, $V$. Is this state unpolarized, fully polarized, or partially polarized?