Due on Tuesday, October 9th, by 14:00 o'clock. There will be no lecture or exercises on October 17th and 18th.

1. **Ricci tensor for tensor perturbations.** Starting from the Christoffel symbols

$$\begin{aligned}
\Gamma^0_{00} &= \mathcal{H} \\
\Gamma^0_{0i} &= 0 \\
\Gamma^0_{ij} &= \mathcal{H}\left(\delta_{ij} + 2E^T_{ij}\right) + E^{T\prime}_{ij} \\
\Gamma^i_{00} &= 0 \\
\Gamma^i_{0j} &= \mathcal{H}\delta_{ij} + E^{T\prime}_{ij} \\
\Gamma^i_{jk} &= E^T_{ij,k} + E^T_{ik,j} - E^T_{jk,i}
\end{aligned}$$

for the metric

$$ds^2 = a(\eta)^2\left[-d\eta^2 + \delta_{ij}dx^i dx^j + 2E^T_{ij}dx^i dx^j\right] ,$$

find the components of the Ricci tensor R_{ij}.

2. **Tensor perturbations in the vacuum-dominated universe.** Solve the gravitational wave equation

$$h'' + 2\mathcal{H}(\eta)h' + k^2 h = 0$$

for the case of a vacuum-dominated universe,

$$a(\eta) \propto -\frac{1}{\eta}$$

(this particular choice of integration constant for the time coordinate corresponds to η being negative with $\eta \to 0$ as $a \to \infty$ in the infinite future). We want to apply this to the late-time evolution of our universe. Thus do not worry about behavior at the distant past, but assume that initial conditions are set at some finite value of η (the beginning of the vacuum-dominated epoch).

3. **Bardeen potentials.** Show that

$$\begin{aligned}
\Phi &\equiv A + \mathcal{H}(B - E') + (B - E')' \\
\Psi &\equiv D + \tfrac{1}{3}\nabla^2 E - \mathcal{H}(B - E') \;=\; \psi - \mathcal{H}(B - E') \,.
\end{aligned}$$

are invariant under gauge transformations.

4. **Momentum continuity equation.** Derive the continuity equation

$$(\bar\rho + \bar p)(v^N)' \;=\; -(\bar\rho + \bar p)' v^N - 4\mathcal{H}(\bar\rho + \bar p)v^N + \delta p^N + \tfrac{2}{3}\bar p \nabla^2 \Pi + (\bar\rho + \bar p)\Phi$$

for scalar perturbations from $T^\mu_{\nu;\mu} = 0$ in the conformal-Newtonian gauge. Using relations between the background quantities, put it in the form

$$(v^N)' \;=\; -\mathcal{H}(1 - 3w)v^N - \frac{w'}{1+w}v^N + \frac{\delta p^N}{\bar\rho + \bar p} + \frac{2}{3}\frac{w}{1+w}\nabla^2\Pi + \Phi \,.$$