

Due on Tuesday, September 18th, by 14:00 o'clock. Return your homework into the metal "mailbox" marked "CMB Physics" in hall A201. There will be no lecture or exercises on September 26th and 27th.

1. **Gauge transformation of the perturbation of a mixed tensor.** Derive the gauge transformation rules

$$\begin{aligned}\widetilde{\delta A}_0^0 &= \delta A_0^0 - \bar{A}_{0,0}^0 \xi^0 \\ \widetilde{\delta A}_i^0 &= \delta A_i^0 + \frac{1}{3} \xi^0_{,i} \bar{A}_k^k - \xi^0_{,i} \bar{A}_0^0 \\ \widetilde{\delta A}_0^i &= \delta A_0^i + \xi^i_{,0} \bar{A}_0^0 - \frac{1}{3} \xi^i_{,0} \bar{A}_k^k \\ \widetilde{\delta A}_j^i &= \delta A_j^i - \frac{1}{3} \delta_j^i \bar{A}_{k,0}^k \xi^0.\end{aligned}$$

and show that the traceless part of δA_j^i is gauge-invariant.

2. **Gauge transformation of the metric perturbations.** Starting from

$$\widetilde{\delta g}_{\mu\nu} = \delta g_{\mu\nu} + a^2 \left[-\xi^\rho_{,\mu} \eta_{\rho\nu} - \xi^\sigma_{,\nu} \eta_{\mu\sigma} - 2 \frac{a'}{a} \eta_{\mu\nu} \xi^0 \right]. \quad (1)$$

derive the gauge transformation rules for the metric perturbations A , B_i , D , and E_{ij} . Consider then a single Fourier mode in the z direction, and derive the transformation rules for the corresponding Fourier coefficients. Consider each component of B_i and E_{ij} separately and note their relation to the scalar, vector, and tensor parts.

3. **Christoffel symbols for tensor perturbations.** Find the Christoffel symbols $\Gamma_{\mu\nu}^\rho$ for tensor perturbations, i.e., for the metric

$$ds^2 = a(\eta)^2 \left[-d\eta^2 + \delta_{ij} dx^i dx^j + 2E_{ij}^T dx^i dx^j \right]. \quad (2)$$