Chapter 12
Behavioral Heterogeneity Under Approval and Plurality Voting

Aki Lehtinen

12.1 Introduction

Approval voting (AV) has been defended and criticized from many different viewpoints. In this paper, I will concentrate on two topics: preference intensities and strategic behavior. A voter is usually defined as voting sincerely under AV if he or she gives a vote to all candidates standing higher in his or her ranking than the lowest-ranking candidate for whom he or she gives a vote. There are no ‘holes’ in a voter’s approval set.[1] Since this kind of behavior is extremely rare, it has been claimed that approval voting makes strategic voting unnecessary (Brams and Fishburn 1978). On the other hand, Niemi (1984) has argued (see also van Newenhizen and Saari 1988a,b), that even though strategic voting may be rare under AV, even sincere voting may require a considerable amount of strategic thinking under this rule. If *strategic voting* is defined by the fact that a voter gives his or her vote to a candidate who is lower in his or her ranking than some candidate for whom he or she does not vote (see, e.g., Brams and Sanver 2006), I will be studying *strategic behavior* but not *strategic voting* under AV here.

In an earlier paper Lehtinen (2008), I proposed a switch of perspective. Instead of trying to study whether strategic voting or behavior is common or easy under various voting rules, I presented a computer simulation framework for investigating the welfare consequences of strategic behavior under approval and plurality (PV) voting. The utilitarian efficiencies obtained with *Expected Utility voting behavior* (EU behavior) and with *Sincere Voting behavior* (SV behavior) are compared. Under SV behavior all voters are assumed to vote for all those candidates for which the utility exceeds the midpoint of the voter’s utility scale (Merrill 1979; Brams and Fishburn 1983, p. 85; Ballester and Rey-Biel 2007). Under EU behavior voters give their votes to different candidates depending on expected-gain calculations (Merrill

[1] See, e.g., Brams and Fishburn (1978, 1983, p. 29) and Brams and Sanver (2006).

A. Lehtinen
Department of social and moral philosophy, P.O. Box 24, University of Helsinki, 00014 Helsinki, Finland
e-mail: aki.lehtinen@helsinki.fi

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1981a,b). They give a vote to a candidate under EU behavior if the expected gain from doing so is positive (Merrill 1981b; Carter 1990). The distinction between strategic and sincere behavior is thus made according to whether or not voters take their beliefs concerning the winning chances of the candidates into account. They strategize if they take such beliefs into account and they engage in sincere behavior if their actions depend only on their preferences.[2] Under PV voters *vote strategically* if they give their vote to a candidate that they do not consider the best, and sincerely otherwise.

Utilitarian efficiency is defined as the percentage of simulated elections in which the candidate that maximizes the sum of voters' utilities (the utilitarian winner) is selected (e.g., Merrill 1988). The main finding in Lehtinen (2008) was that whether or not voters engage in strategic calculations, AV yields high utilitarian efficiencies and thus often selects candidates with broad public appeal cf. Brams and Fishburn 1983, pp. 135, 171. AV reflects preference intensities rather well even if voters engage in strategic behavior.

It was also shown that strategic voting is beneficial under PV in the sense that utilitarian efficiencies are higher under EU than under SV behavior. I have shown elsewhere that strategic voting is beneficial in many voting rules in that it increases utilitarian efficiency compared to sincere voting (see Lehtinen 2006, 2007a,b). These results mean that from a utilitarian, and thereby welfarist point of view, strategic voting under various voting rules, and strategic behavior under AV, are beneficial. However, the traditional arguments against strategic voting are non-welfarist.[3] One important argument is that 'unequal manipulative skills may lead to destruction of our efforts to design rules with equal treatments of individuals' (Kelly 1988, p. 103). The worry is thus that if some but not all voters engage in strategic manipulation, and if the strategizers are successful in their endeavor, this would be unfair towards the other voters.

In this paper, I will study one aspect of this worry with a welfarist model that allows analyzing whether or not unequal manipulative dispositions in the voting population yield undesirable results. Only one aspect of the worry is analyzed because the model does not specify different manipulative skills but rather just different propensities to manipulate.[4] Voters are assumed to be heterogeneous in the sense that some voter *types* do not engage in strategizing at all. The robustness of approval and plurality voting with respect to behavioral heterogeneity is thus investigated. To the best of my knowledge, this paper provides the first model in which such heterogeneity is explicitly studied.[5]

[2] Although Brams and Fishburn (1983, p. 85) use an expected-utility terminology, their mean utility rule is classified as sincere here.

[3] See Kelly's (1988, p. 103) list of arguments and their critique by Van Hees and Dowding (2007).

[4] Different skills could be studied within the framework presented here by giving some voters better information than others. For the time being, I postpone such an analysis into the future.

[5] I am hoping that someone proves me wrong here. The need for studying heterogeneous behavior in strategic voting is often expressed in conference presentations.

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Strategic voting increases utilitarian efficiency in various voting rules because it allows for expressing preference intensities Lehtinen 2006, 2007b,a. These results depend on the counterbalancing of strategic votes: broadly accepted candidates are likely to obtain many strategic votes and lose few: the strategic votes for a candidate are counterbalanced by strategic desertions for the very same candidate but the utilitarian winner is likely to be on the receiving end of strategic votes. The logic of counterbalancing thus suggests that the beneficial effects of strategic voting may not be very robust with respect to behavioral heterogeneity. In contrast, AV differs from other commonly used voting rules in that it allows for expressing intensity information even with SV behavior (e.g., Brams and Fishburn 2005). When I began this investigation, my intuition was that AV would be fairly robust with respect to behavioral heterogeneity. After all, as voters may express preference intensities under both behavioral assumptions, one would expect AV to yield high utilitarian efficiencies whatever the behavioral assumption, and even if the voting population is behaviorally heterogeneous. However, my intuitions turned out to be completely erroneous. It is indeed AV that is sensitive to behavioral heterogeneity rather than PV! Very roughly, the explanation for poor resistance of AV to behavioral heterogeneity is that strategic behavior dramatically reduces the number of second votes and such reductions do not have a proper counterbalance.

The structure of the paper is the following. Given that the paper is heavily based on my 2008 model, I will only explain its most important features in Sect. 12.2. I refer to this paper for an explanation of the details of the signal-extraction model, an account of interpersonal comparisons in the model, a discussion of reasonable parameter values, and in general for anything about the model that is not concerned with behavioral heterogeneity. Section 12.3 describes the novel feature of the present model: the *mixed behavior* computer simulations *setups*. Simulation results are presented in Sect. 12.4. Section 12.5 presents the conclusions.

12.2 Strategic Behavior Under Approval and Plurality Voting

Let X = {x,y,z} denote the set of candidates (with generic members j, k and m). The six possible types of voters and their preference orderings are presented in Table 12.1. U^i_k denotes voter i's payoff for the k-th best candidate.

Under AV, voters give a vote to any number of candidates. Let $N = 2{,}000$ denote the total number of voters, and let n_j denote the number of voters who prefer candidate j the most. Let n_j^{AV} denote the number of votes candidate j obtains

Table 12.1 Voter types and utilities

Type of voter						
t_1	t_2	t_3	t_4	t_5	t_6	U^i
x	y	z	x	y	z	U^i_1
y	z	x	z	x	y	U^i_2
z	x	y	y	z	x	U^i_3

under sincere behavior under AV, and let n^{AV} denote the total number of votes cast under AV. Let v_x^{PV}, v_y^{PV}, and v_z^{PV} denote the *vote shares* of candidates x, y and z if all voters vote sincerely under PV: $v_j^{PV}=\frac{n_j}{N}$, and let v_x^{AV}, v_y^{AV}, and v_z^{AV} denote similar vote shares under AV ($v_j^{AV} = \frac{n_j^{AV}}{n^{AV}}$). Let $p_{jk}^{i,PV}$=prob(v_j^{PV}=$\mathrm{v}_k^{PV} > \mathrm{v}_m^{PV}$) denote the probability that voter i will be decisive in creating or breaking a *first-place* tie between j and k under PV, i.e., a *pivot probability*. $p_{jk}^{i,AV}$ denotes similar probabilities under AV. The standard way of analyzing strategic behavior in models in which game-theoretical considerations are not taken into account is by way of formulating expected gains for voters.

The expected gain in utility associated with voting for candidate j under AV is Merrill (1979)

$$E_j^i = \sum_{j \neq k} p_{jk}^{i,AV} [U^i(j) - U^i(k)]. \tag{12.1}$$

Voters give a vote to a candidate if the expected gain from doing so is larger than zero (see also Merrill 1981b; Carter 1990). Voters will always give a vote for their most preferred candidate under approval voting (Brams and Fishburn 1978).[6] The conditions for strategic voting under PV can also be deduced from these equations once $p_{jk}^{i,AV}$ are replaced with $p_{jk}^{i,PV}$, see McKelvey and Ordeshook (1972). A voter votes for the candidate who offers the highest expected gain.

12.2.1 A Signal Extraction Model for the Pivot Probabilities

Voters' beliefs are derived by combining methods of computing pivot probabilities (Hoffman 1982; Cranor 1996) with a signal-extraction model. The voters are assumed to obtain an informative but not entirely reliable signal concerning the popularity of the candidates. They compute pivot probabilities on the basis of these signals and their confidence in the quality of those signals. The idea is thus to characterize the beliefs in terms of the *reliability* of the signals and voters' *confidence* in them.

Let v_j denote a generic vote share. Voters obtain perturbed signals about vote shares:

$$\mathcal{S}_j = v_j + \rho R_i, \tag{12.2}$$

where R_i denotes a standard normal random variable, and ρ is a scaling factor that reflects the *reliability* of the signals ($\rho \in [0.005, 0.013]$).[7] The signals thus contain information concerning the real preference profile and noise. The former is modelled through the vote shares v_j. Note that these are vote shares that would come about if *everyone* engaged in sincere behavior rather than vote shares that come about when

[6] Three-way ties are ignored here.

[7] I provide arguments for why such values are reasonable in Lehtinen (2008).

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some or all voters engage in strategic behavior. The vote shares are different under AV and PV because voters may give sincere second votes under AV.

Let $s^i_{\max}$ denote the predicted vote share (i.e., a signal) of the candidate who i expects to obtain the most votes, and let $s^i_{min(j,k)}$ denote the predicted vote share of j or k, whichever i predicts to receive fewer votes. I show in Lehtinen (2008) that the pivot probabilities p^i_{jk} are given by the standard normal distribution function Φ:

$$p^i_{jk} = 2\Phi\left(\frac{i_p - s^i_{\max}}{\sigma}\right), \qquad (12.3)$$

where i_p is a parameter derived from the various signals which describes the closeness of the race and σ is the voter's confidence in his or her signal.[8] Very roughly, the idea is that the closer the predicted vote share (i.e., the signal) for the candidate in question is to the predicted vote share of the perceived winner, the higher the pivot probability. Voters are assumed to construct a probability distribution around their signal. $i_p = \frac{\left(s^i_{\max}\right)^2 - \left(s^i_{\min(j,k)}\right)^2}{2(s_{\max} - s_{\min(j,k)})}$ is the intersection point of densities for the perceived winner and the candidate in question. The distance between this intersection point and the signal for the perceived winner, $i_p - s^i_{\max}$, determine how close the race between the two candidates is perceived to be by voter i.

I refer to my 2008 paper for a detailed explanation of the technical aspects of the model. For the purposes of this paper, it is sufficient to realize that the signal-extraction framework allows modeling beliefs that range from highly accurate to highly inaccurate, and at the same time taking voters' confidence in the quality of their information into account.

12.3 Simulation and Mixed Behavior Setups

A setup is a combination of assumptions used in a set of $G = 1{,}000$ simulated elections. In each simulated election, a profile (U^1,$\mathrm{U}^2, \ldots, \mathrm{U}^N$) of individual utilities is generated. Under PV, the sincere vote shares of the various candidates are computed from this utility profile by ordering the utilities for the three candidates, and by counting how many voters most prefer each candidate. Under AV the sincere vote shares are computed by counting the number of voters for whom the utility lies above the midpoint of the utility scale. The voters then obtain three signals concerning the profile (one for each candidate) according to (12.2), and formulate their pivot probabilities using (12.3). They then use (12.1) to compute the expected gains, and vote accordingly. The winner is then determined and compared to the utilitarian winner.

[8] The confidences are usually assumed to be the same for all voters.

Expected utility setups differ with respect to the reliability of voters' signals (ρ), their confidence in the signals (σ), and the degree of correlation between voter types and preference intensities (C) (see the next paragraph). In *uniform setups* voters' utilities are drawn from a uniform distribution on [0,1],[9] while in *setups with intensity correlation* voter types 3 and 5 have systematically higher and types 1 and 6 systematically lower preference intensities for their second-best candidates x and y respectively. These setups are identical to the corresponding uniform setups with respect to all parameters except voters' preference intensities. In order to generate setups with a correlation between this parameter and voter types without affecting the interpersonal comparisons or the preference orderings, the individual utilities were derived as follows.

U_1, U_2, and U_3 were first generated from the uniform distribution on [0,1] for each voter. U_1 and U_3 were then used for defining the voter's utility scale as the $[U_3, U_1]$ interval. A voter's utility for his or her middle candidate U_2 is referred to as the *intensity*. A *standardized intensity*, $\widetilde{U}_2$ expresses what a voter's utility for his or her second-best candidate would be if the scale was the [0,1] interval. These standardized second-best utilities are referred to as *intra*personal *intensities*. The relationship between the standardized intra-personal utility and the original scale of utility is given by

$$\widetilde{U}_2 = 1 - \frac{U_1 - U_2}{U_1 - U_3}. \qquad (12.4)$$

In setups with an intensity correlation, these standardized intensities were multiplied by a parameter C, $0.5 < C \leq 1$ for those who put y second (voter types 1 and 6) so that the new correlated intensities $\widetilde{U}_2^{C,1}$ and $\widetilde{U}_2^{C,6}$ were given by

$$\widetilde{U}_2^{C} = C\widetilde{U}_2. \qquad (12.5)$$

In order to compensate for the decreases in utility for voter types 1 and 6, the intensities for voters of types 3 and 5 (i.e., for x) were given by

$$\widetilde{U}_2^{C} = 1 - C\widetilde{U}_2. \qquad (12.6)$$

These adjustments made the average utilities for x higher and the average utilities for y lower than in the uniform setups, while keeping the overall average utility fixed.[10] In uniform setups, $C = 1$. C thus denotes the *degree of correlation* between preference intensities and voter types.

These standardized intensities were then scaled back into the original $[U_3, U_1]$ utility scale. Let U_2^* denote a voter's correlated intensity expressed in terms of the original $[U_3, U_1]$ scale. U_2^* is given by:

$$U_2^* = U_3 + \widetilde{U}_2^{C}(U_1 - U_3). \qquad (12.7)$$

[9] The simulations were thus based on the so-called impartial anonymous culture assumption.

[10] Note that the utility for the second-best candidate in uniform setups is $1 - \widetilde{U}_2^{C}$ rather than $\widetilde{U}_2^{C}$. Since $\widetilde{U}_2^{C}$ is drawn from a uniform distribution on [0,1], it does not matter which one is used.

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In *pure behavior setups* (PBS) all voters engage in the same kind of behavior: either EU or SV behavior. In *mixed behavior setups* (MBS) some voters engage in SV behavior and some in EU behavior. The simplest MBS is one in which voters who engage in SV behavior are randomly selected from the set of all voters.

More interesting results are likely if only some voter *types* engage in EU behavior, or if only some voter types engage in SV behavior. In *abstaining setups* all voters except those of two particular types engage in EU behavior, and these abstaining types engage in SV behavior. Let $A_R(st)$ denote a setup in which voters of types s and t engage in SV behavior, and the rest engage in EU behavior under voting rule R. Similarly, in *engaging setups* all voters except those of two particular types engage in SV behavior, and these two types engage in EU behavior. A setup in which only types s and t engage in EU behavior is denoted $E_R(st)$.

12.4 Simulation Results

12.4.1 Non-systematic Behavioral Heterogeneity

The simulations were run with 0.005, 0.009, and 0.013 for both σ and ρ. The results will be shown only for the setups in which $\rho = \sigma$.[11]

A setup in which one-half of all voters were randomly selected to engage in EU behavior, and the rest in SV behavior was tried. Figures 12.1 and 12.2 show utilitarian efficiencies under AV and PV, respectively, when the probability of any given voter to engage in EU behavior is 0.5. UE_{SV} and UE_{EU} stand for utilitarian efficiency under SV- and EU behavior, respectively. Let EA_R(random) denote such a setup. Let us say that behavioral heterogeneity is *systematic* if there are systematic differences between the different voter types with regard to behavioral dispositions, and *non-systematic* otherwise. The setups in this section thus concern non-systematic behavioral heterogeneity.

It is easy to see from these figures that strategic behavior under AV and strategic voting through EU behavior under PV yield reasonably high utilitarian efficiencies. They are higher under AV, and particularly so under SV behavior. The reason for this is rather simple. Candidate x is practically always the utilitarian winner in setups in which correlation between intensities and voter types is high (C is small), but because the voting population is generated with the impartial anonymous culture (IAC), under PV it is selected only in one-third of the simulated elections under SV behavior. Under AV, however, voters are able to express preference intensities also under SV behavior, and the utilitarian efficiencies are correspondingly higher. These setups, while they may depict real-world elections in a realistic way, are not very interesting because the results simply reflect the relationships that hold under the pure behavior setups, but in a mitigated form.

[11] The full sets of data are available from the author on request.

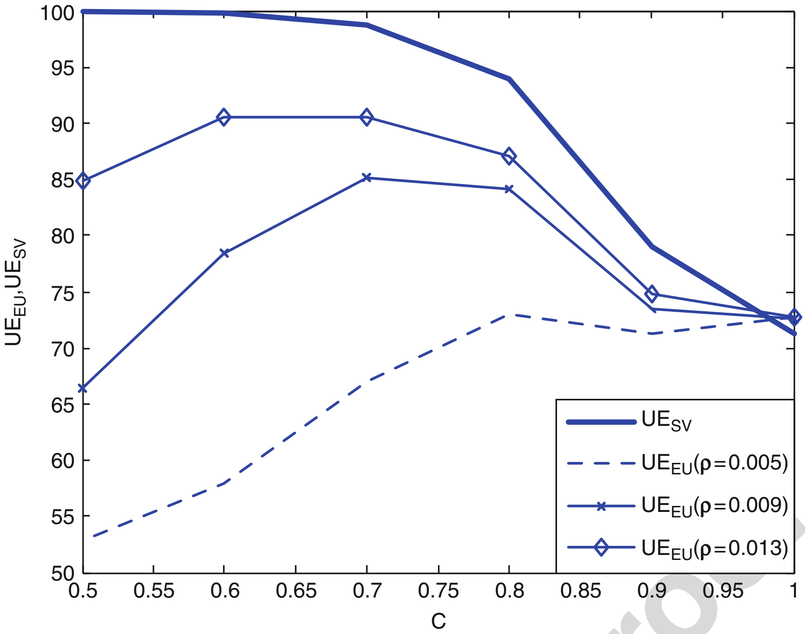

Fig. 12.1 Utilitarian efficiencies under EA_{AV}(random)

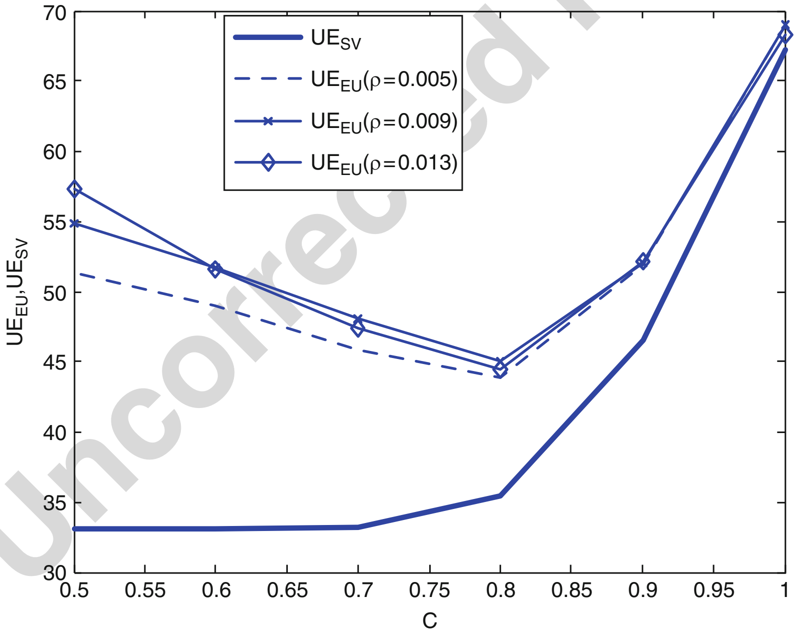

Fig. 12.2 Utilitarian efficiencies under EA_{PV}(random)

12.4.2 Systematic Behavioral Heterogeneity

12.4.2.1 Engaging Setups: Plurality Voting

The investigated setups were chosen in such a way as to provide the maximum amount of understanding on how various different heterogeneities affect the utilitarian efficiencies. In most setups only two illustrative voter types were selected to engage in SV behavior or EU behavior. The setups discussed below are not very realistic in that *all* voters within each voter type are assumed to engage either in EU or in SV behavior. It is highly likely that reality is much more complex in this respect. As the model is based on non-cooperative behavior, it is not assumed that there is a coordinating agent who could enforce one or the other behavioral assumption within a voter type.

The logic of counterbalancing suggests that the utilitarian efficiencies should be lower under most MBSs than under PBSs because these setups are constructed in such a way that the counterbalance is systematically removed. In most MBS's the utilitarian efficiencies are indeed lower than in the corresponding pure behavior setups.

Let us start by looking at PV. Figures 12.3, 12.4, and 12.5 show the results when two voter types only engage in EU behavior and the rest in SV behavior. In what

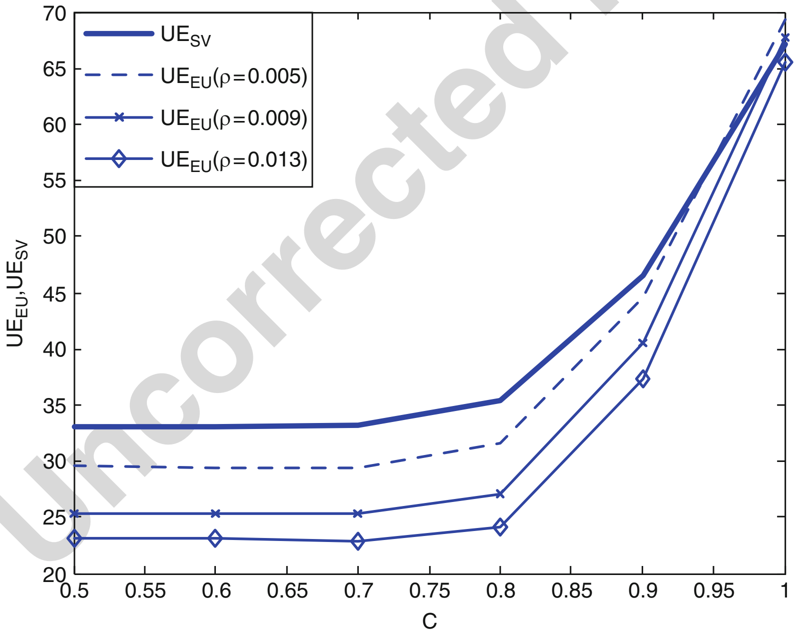

Fig. 12.3 $E_{PV}(14)$

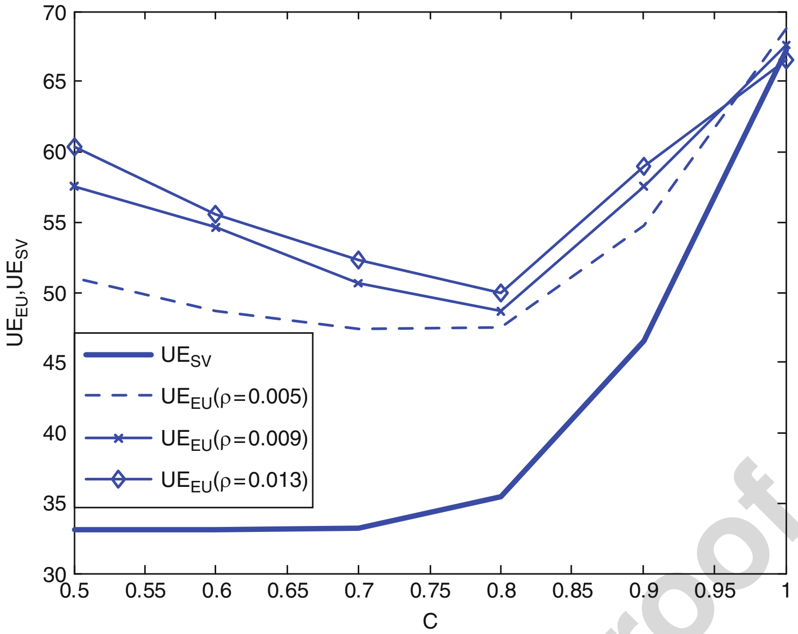

Fig. 12.4 $E_{PV}(25)$

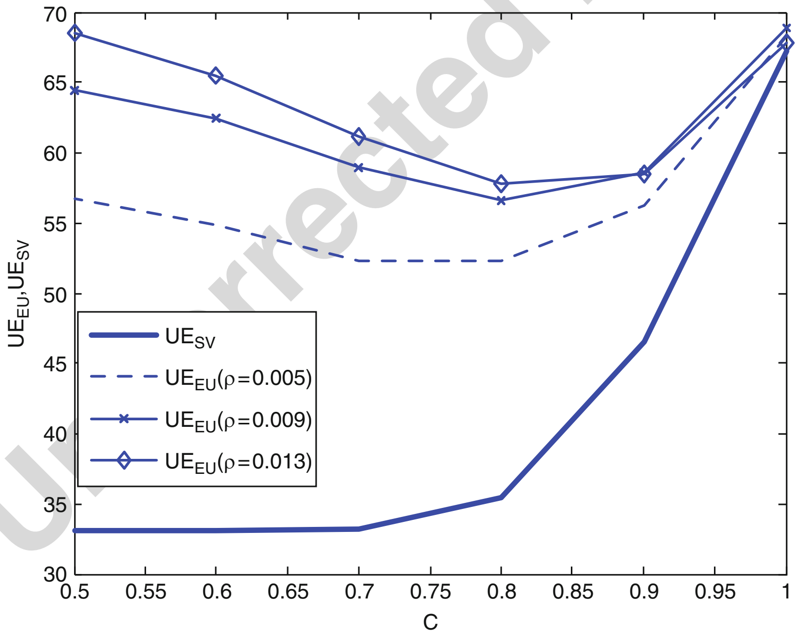

Fig. 12.5 $E_{PV}(36)$

follows, the figure titles include only the name of the setup: all results concern utilitarian efficiencies.

Strategic voting becomes more welfare-increasing under $E_{PV}(36)$ than under E_{PV}(random), remains roughly the same under $E_{PV}(25)$, and it becomes welfare-diminishing under $E_{PV}(14)$. Explaining these findings is easy once the logic of counterbalancing is invoked. First, under $E_{PV}(14)$ only voters who prefer x the most engage in strategic voting. But x is usually the utilitarian winner in setups with strong correlation. Welfare-increasing strategic voting is thus theoretically possible only in those $E_{PV}(14)$ setups in which the correlation is not very high (i.e., C is close to one), and in which x is not the utilitarian winner. In all other setups strategic voting can only be harmful because it may only *decrease* the probability that the utilitarian winner wins. Second, under the $E_{PV}(36)$ setups there is a proper counterbalance: even though voters of type 6 may vote strategically for y, they do so much more seldom than voters of type 3 vote for x. Utilitarian efficiencies are higher than under the pure behavior setups because the 'wrong' kind of counterbalance is removed. Note that from the point of view of utilitarian efficiency, it is more important that there are not too many voters who vote strategically for z than those who vote strategically for y. This is because there may often be enough strategic votes for z to make it win, but y is usually the loser in any case. This also explains why utilitarian efficiencies are somewhat lower under the $E_{PV}(25)$ than the $E_{PV}(36)$ setup. Here strategic votes for z rather than for y counterbalance those for x.

Figures 12.6, 12.7, and 12.8 show the findings from the setups in which voter types who engage in strategic behavior consider the same candidate second-best.

As one might expect by now, the highest utilitarian efficiencies come from the $E_{PV}(35)$ setup where x is the only candidate to obtain strategic votes in the first place, and the worst from $E_{PV}(16)$ where y is the only candidate in this position.

Note that even though strategic voting is welfare-diminishing in some setups, the results shown thus far have been rather supportive of PV. If the main worry about strategic voting is that it benefits one particular group at the expense of everyone else, then the results show that this worry is mainly not warranted. In the $E_{PV}(14)$, the strategic voters hurt mostly themselves by their actions! They prefer x the most, but their strategic voting makes it less likely that x will be selected. It is thus clear that if they were to have perfect information about the behavioral propensities of the different voter types, they would switch to SV behavior. In a word, their strategic voting is not model-consistent because if voters knew that they are the only ones who engage in strategic behavior, they would realize that they have no incentive to act according to strategic behavior as it is specified in the model.[12] Another way to approach the issue is to note that since the signals depend on voters' preferences but not on their behavioral propensities, they give a systematically misleading picture of the winning chances of the various candidates.[13] I do not attempt to provide an

[12] Model-consistency is also known as the rational expectations hypothesis (Muth 1961).

[13] However, note that if type-1 voters vote strategically for y, and it emerges as winner, their prior beliefs are corroborated by the outcome!

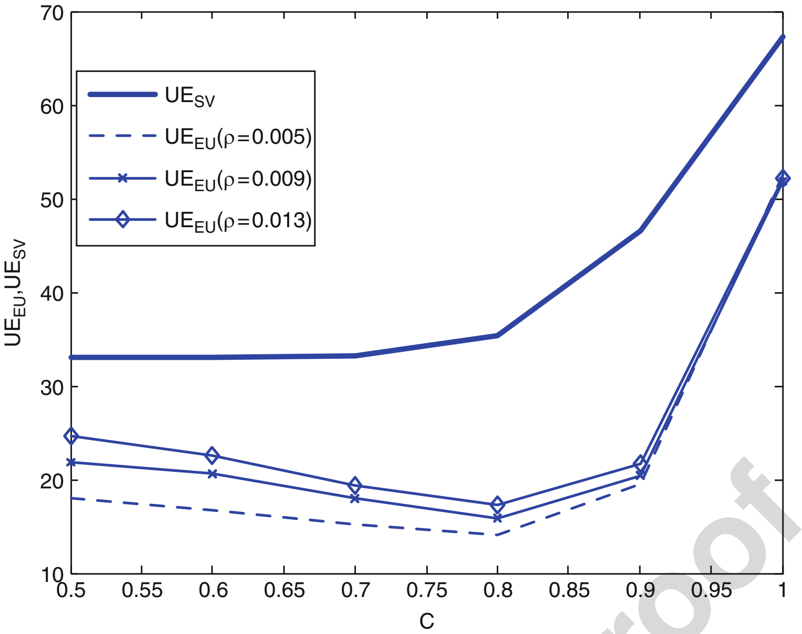

Fig. 12.6 $E_{PV}(16)$

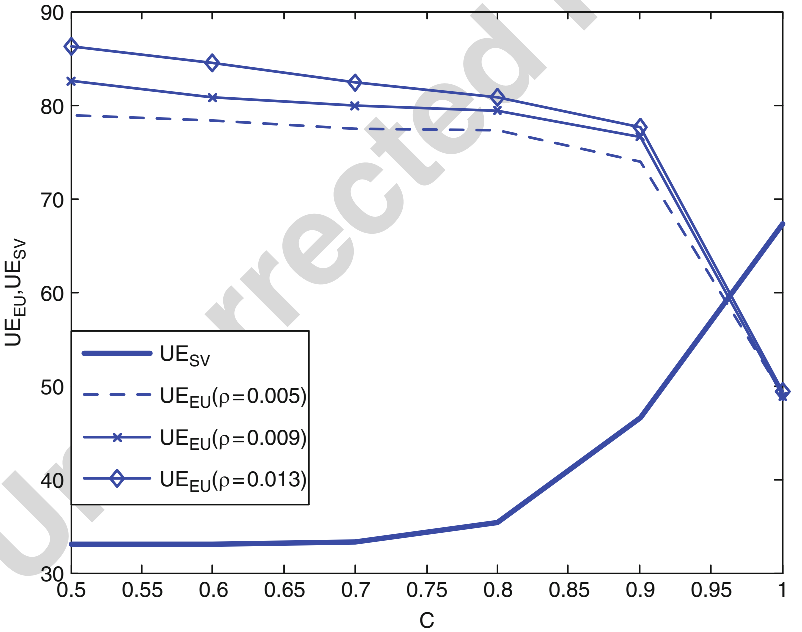

Fig. 12.7 $E_{PV}(35)$

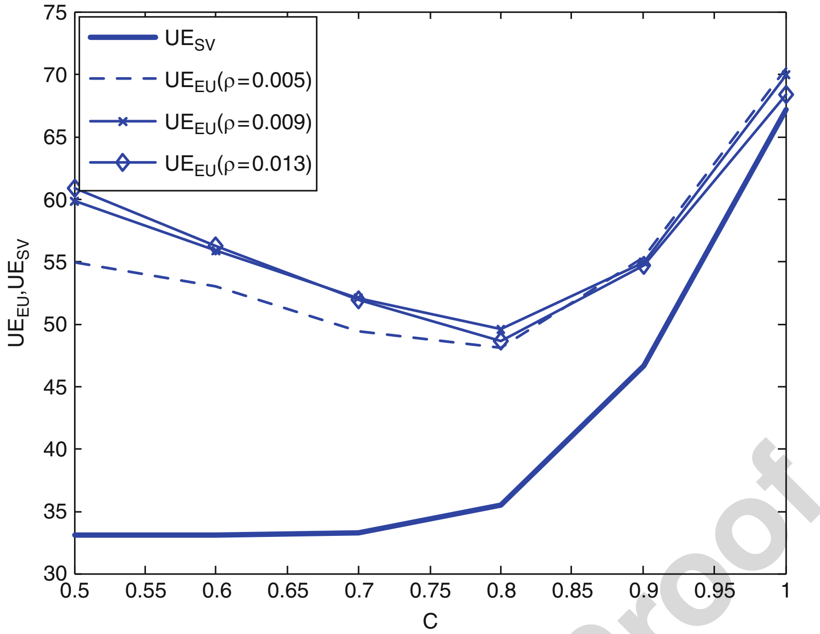

Fig. 12.8 $E_{PV}(24)$

account in which the behavioral propensities are taken into account in a formal way in this paper. $E_{PV}(16)$ causes more concern than $E_{PV}(14)$ because it is not always the same candidate who loses the strategic votes. Nevertheless, even in this setup the outcomes are usually better under the pure behavior SV setup for the very types that engage in strategic voting, and they have an incentive to switch into sincere behavior. It is inevitable that someone must lose as a result of strategic behavior, but the results show that under an utilitarian evaluation, strategic voting is welfare-decreasing only when it harms the strategisers themselves.

12.4.2.2 Engaging Setups: Approval Voting

Let us now see what happens under AV in engaging setups. Figures 12.9, 12.10, and 12.11 show the utilitarian efficiencies under AV.

The utilitarian efficiencies are completely different from those under PV: they are highest in E(14) setups, and lowest under E(36) setups. In other words, strategic behavior under AV yields low utilitarian efficiencies precisely when strategic voting is particularly welfare-increasing under PV, and vice versa.

The key to understanding these results lies in the difference in the number of voters who give a second vote under SV behavior and under EU behavior (cf. Saari 2001). Many voters give second votes under SV behavior. Under the uniform setups

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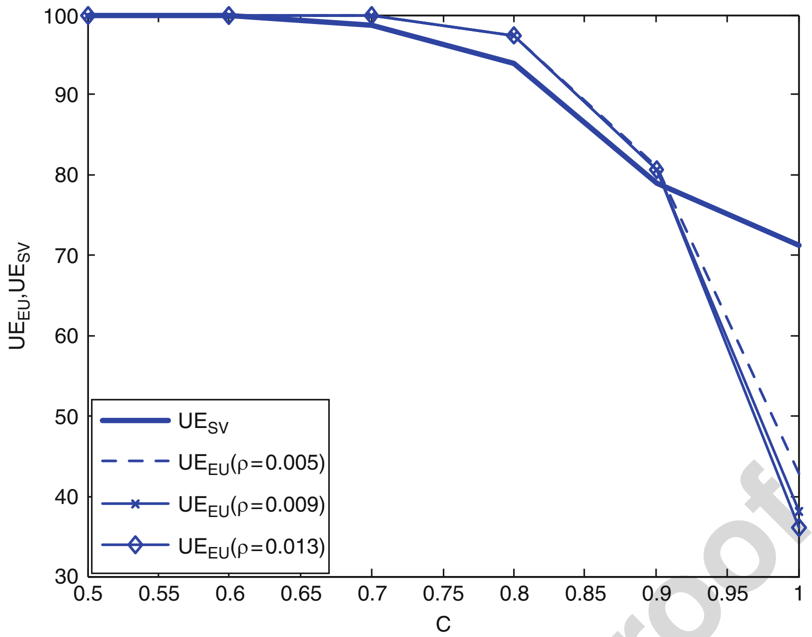

Fig. 12.9 $E_{AV}(14)$

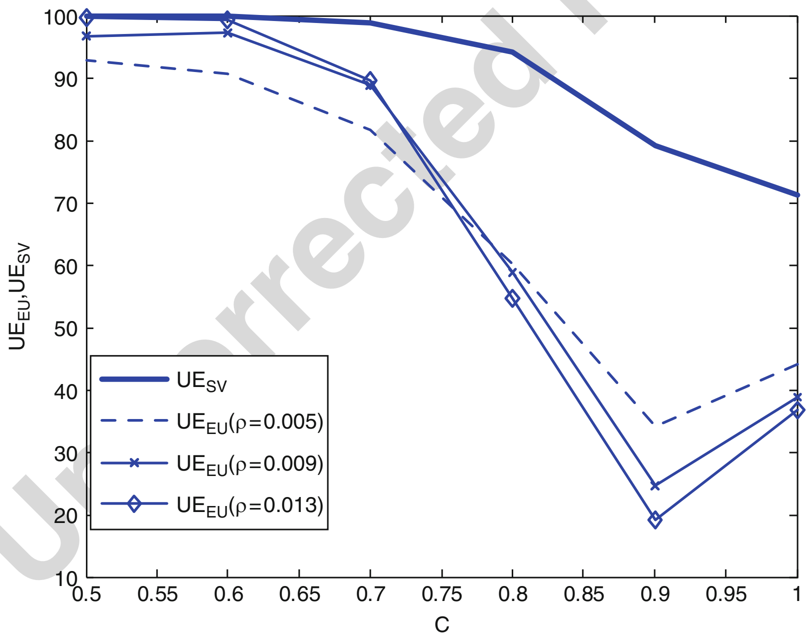

Fig. 12.10 $E_{AV}(25)$

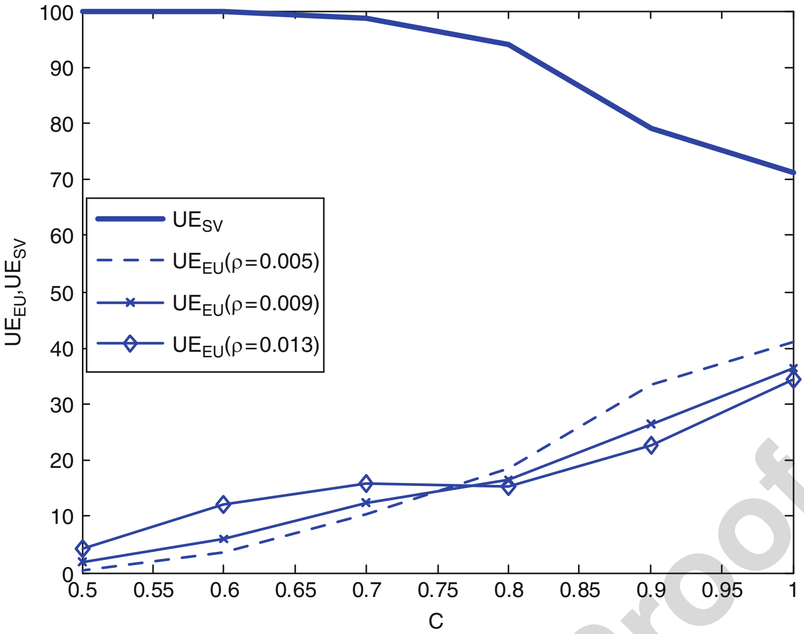

Fig. 12.11 $E_{AV}(36)$

exactly one-half of each voter type do so. Under EU behavior voters continue to give second votes, but they do so much more rarely. This reduction in the second votes is the main consequence of strategic behavior under AV. In pure behavior setups the utilitarian efficiencies are rather high because counterbalancing still ensures that the utilitarian winners obtain more second votes than the other candidates. However, in the E(36) setups, although x obtains more second votes from type-3 voters than y obtains from type-6 voters, what really matters is the dramatic reduction in second votes for x (compared to SV behavior), together with the fact that z obtains all the second votes it does under SV behavior. z is thus almost always the winner in these setups. In E(25) setups the counterbalancing is rectified by the fact that the reduced number of second votes from type-5 voters is counterbalanced by the reduced number of second votes for z from type-2 voters. It is thus more important that the reduced number of second votes for the utilitarian winner are counterbalanced by a similar reduction for the *second-best* candidate (in utilitarian terms) than the *worst*. The reason for this is that in the engaging setups there are still four voter types who give different amounts of second votes, and counterbalancing among these second votes is more important than counterbalancing among the strategically determined second votes.

Although the findings seem to support AV superficially, the setups in which strategic behavior is welfare-diminishing are in fact more worrisome than under PV. Consider, for example, $E_{AV}(36)$. This is a setup in which those who prefer z the most are the only ones to engage in strategic behavior. They give much fewer second votes to x (and y) than under sincere behavior. As a consequence, their best candidate z often wins. Unlike in the $E_{PV}(14)$ and in the $E_{PV}(16)$ setups, upon learning the behavioral differences between the voter types, they would not have an incentive to switch into SV behavior. $E_{AV}(36)$ is thus a setup in which the one group of voters is indeed able to inflict harm on others by strategising: if they acted sincerely, the results would be better for the whole electorate.

12.4.2.3 Abstaining Setups

Figures 12.12, 12.13, and 12.14 show utilitarian efficiencies under setups in which two voter types abstain from strategic behavior. As expected, $A_{PV}(35)$ exemplifies a catastrophe because the only voter types to abstain from strategic behavior are those that may vote strategically for x under EU behavior. But why are efficiencies higher under $A_{PV}(24)$ than under $A_{PV}(16)$? The reason is again that strategic votes for the *second-best* candidate are more likely to lower utilitarian efficiency than those for the worst candidate, because the worst candidate rarely wins the election anyway. Thus, under $A_{PV}(24)$ those who might vote strategically for z refrain from doing so but under $A_{PV}(16)$ such voters would have voted strategically for y. Figures 12.15, 12.16, and 12.17 show the corresponding results under AV.

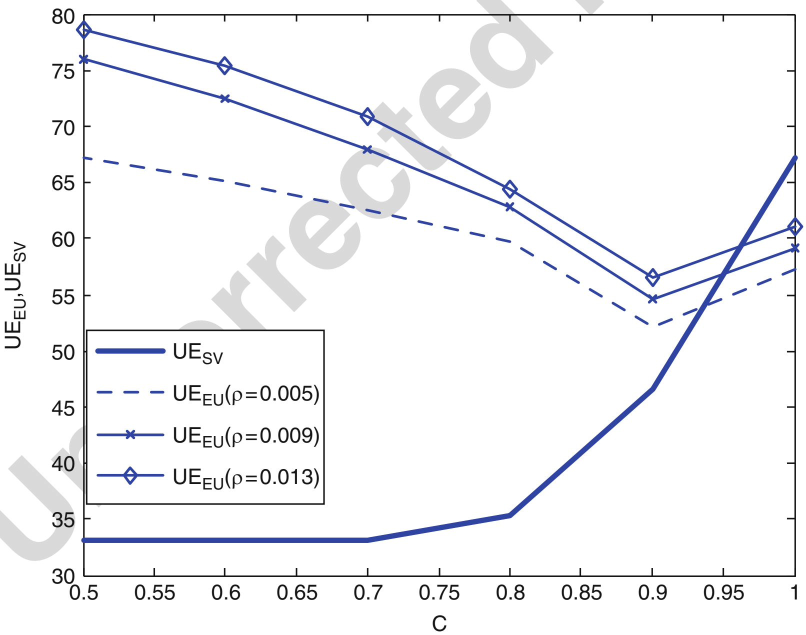

Fig. 12.12 $A_{PV}(24)$

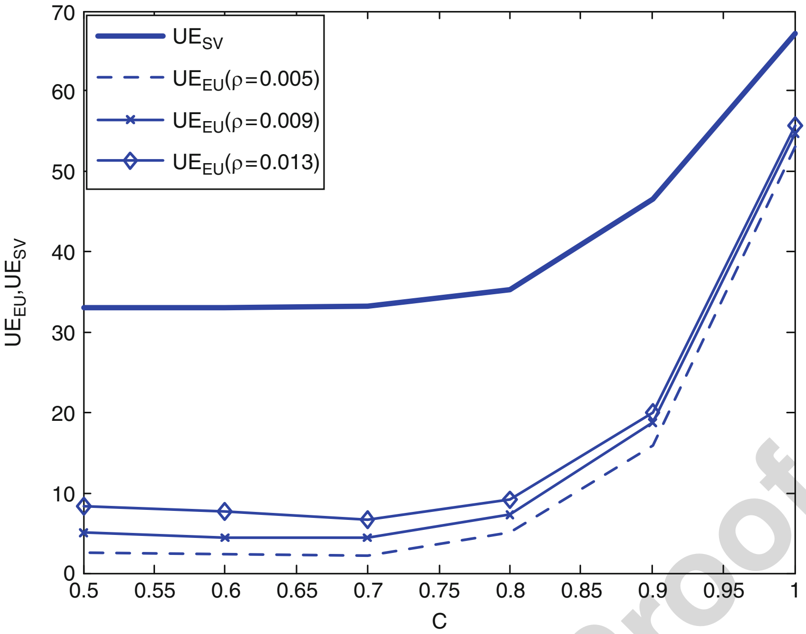

Fig. 12.13 $A_{PV}(35)$

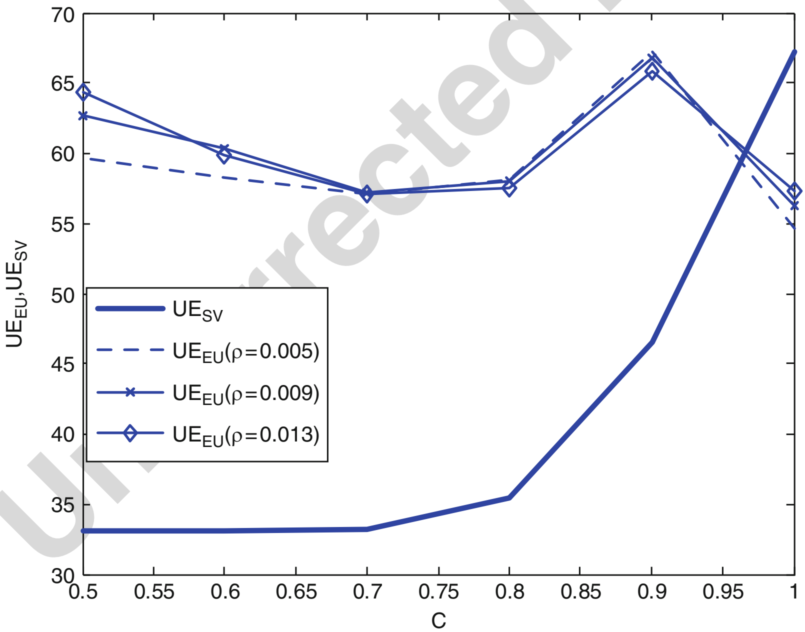

Fig. 12.14 $A_{PV}(16)$

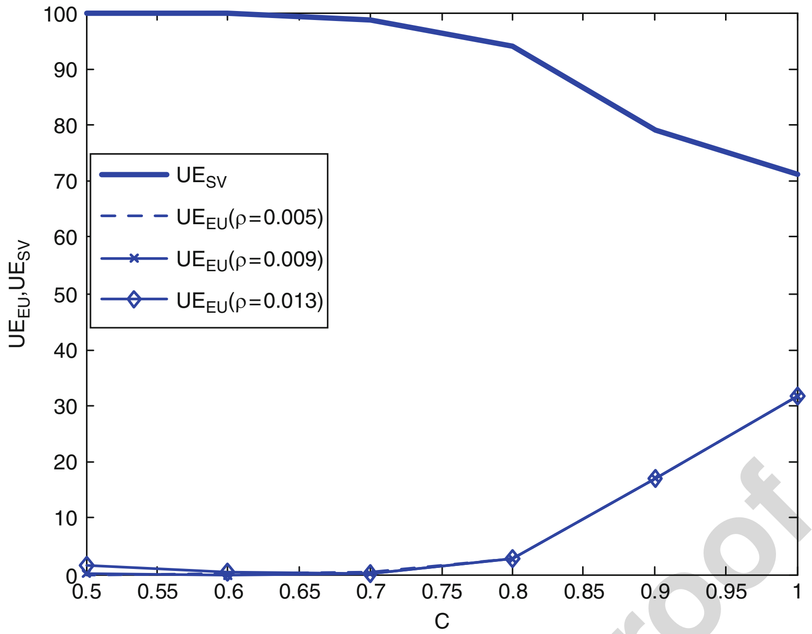

Fig. 12.15 $A_{AV}(24)$

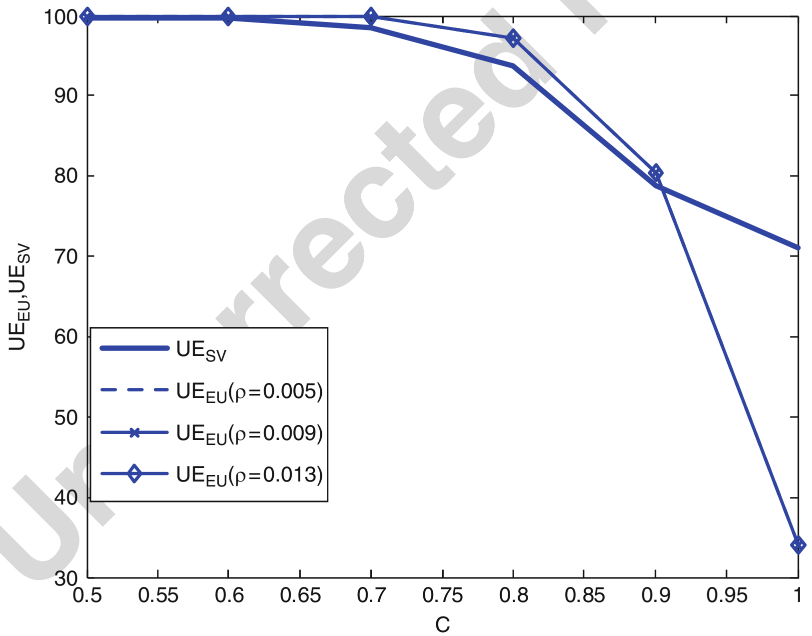

Fig. 12.16 A_{AV} (35)

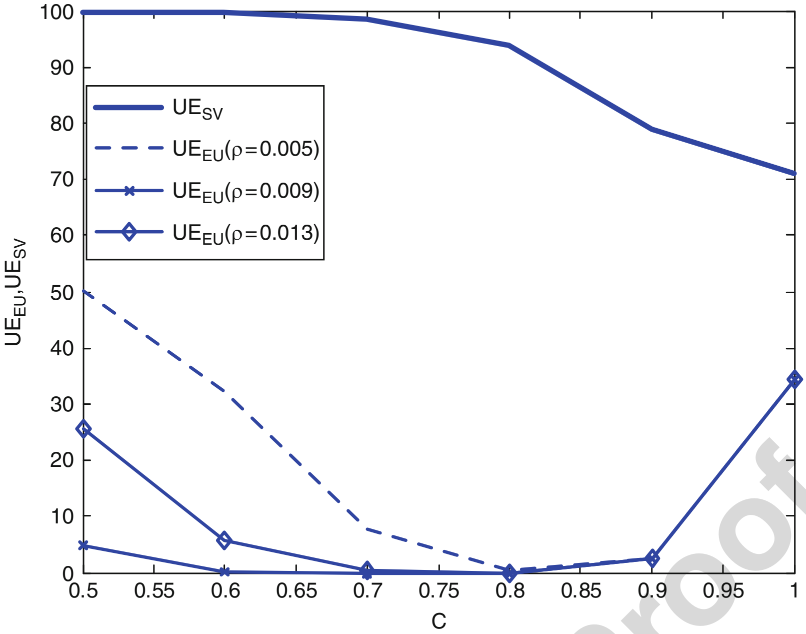

Fig. 12.17 $A_{AV}(16)$

The utilitarian efficiencies are now very low except in $A_{AV}(35)$ where those who put x second refrain from strategic behavior and give plenty of second votes for x. Note that $A_{AV}(24)$ and $A_{AV}(16)$ are setups in which those who refrain from strategic behavior consider the *same* candidate second-best. Hence, if they abstain from strategising, this will often result in the victory of their second-best candidate. The utilitarian efficiencies are low in setups where that second-best alternative is not the utilitarian winner. Furthermore, the efficiencies are lower under $A_{AV}(24)$ than under $A_{AV}(16)$ because x is able to win some elections even when those who put y second give their sincere second votes for it, but x has no chance against z because there are more of those who give their sincere second votes to z under $A_{AV}(24)$ than those who give their sincere second votes to y under $A_{AV}(16)$.

Let us now look at setups in which those who refrain from strategic behavior consider the same candidate best. Figures 12.18, 12.19, and 12.20 show utilitarian efficiencies in $A_{AV}(14)$, $A_{AV}(25)$, and $A_{AV}(36)$ setups.

It seems clear that utilitarian efficiencies remain high if at least some voter types give sincere second votes to x, but if the only types that abstain from strategic behavior put x first, then utilitarian efficiencies are understandably very low because y and z obtain a large number of sincere second votes from type-1 and type-4 voters, and the strategic second votes from the other voter types are not a sufficient counterbalance to these sincere votes.

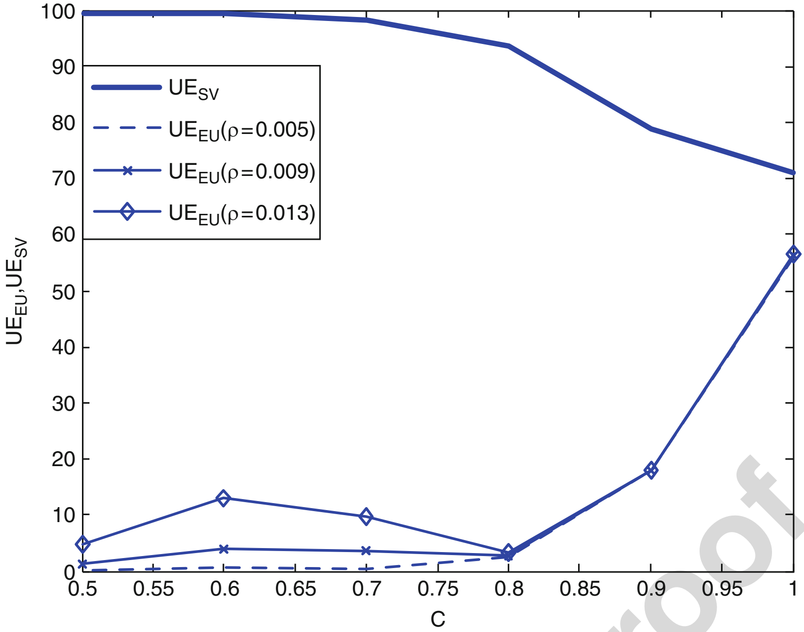

Fig. 12.18 $A_{AV}(14)$

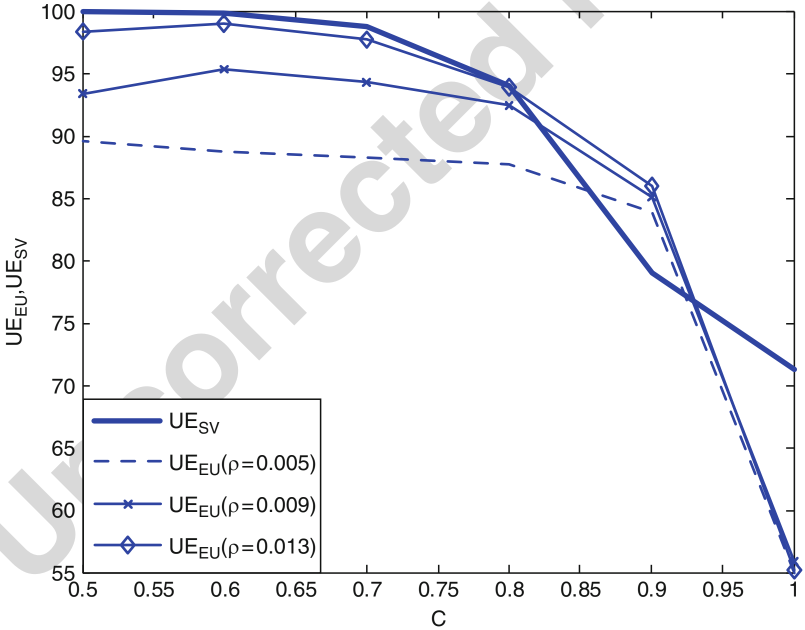

Fig. 12.19 $A_{AV}(25)$

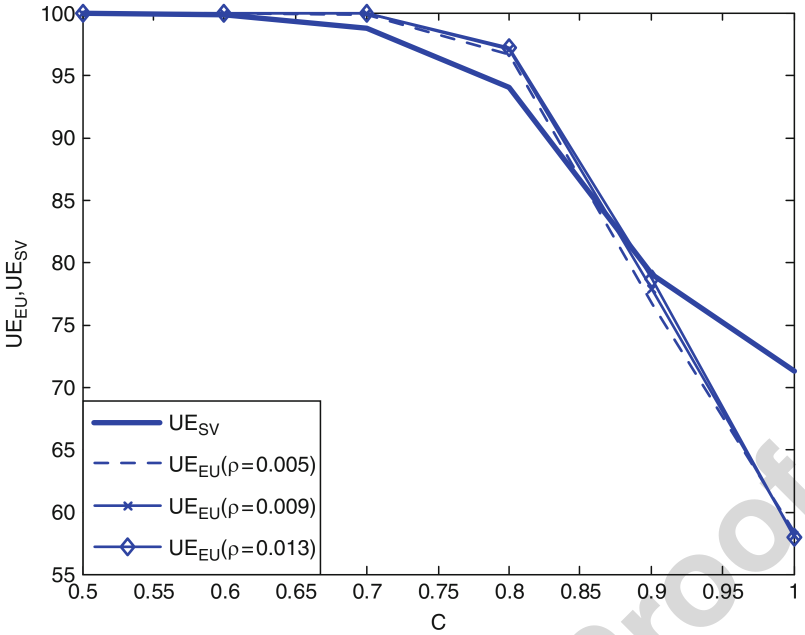

Fig. 12.20 $A_{AV}(36)$

12.4.3 A Comparison

The previous findings have provided detailed information concerning how the different combinations of behavioral assumptions matter for utilitarian efficiency. It may be somewhat difficult to derive an overall judgement concerning the two rules on the basis of them. In order to provide an explicit comparison, setups in which two randomly selected voter types engage in EU behavior were investigated. Let E_R(random) denote such a setup under voting rule R. Figures 12.21 and 12.22 show the findings from such setups.

The utilitarian efficiencies remain somewhat higher under PV than under AV. Perhaps the most important aspect of these results is that, on average, strategic voting remains welfare-increasing even in setups with the most extreme kind of behavioral heterogeneity. A simulation was run also for the case in which two randomly selected voter types abstained from strategic behavior, and the rest engaged in EU behavior. The results were highly similar to those in Figs. 12.21 and 12.22, and will thus not be shown here.

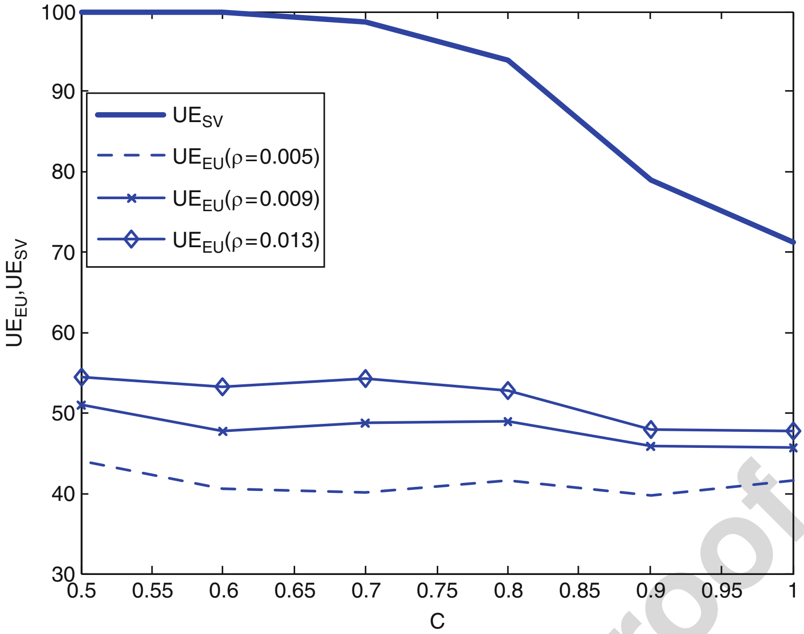

Fig. 12.21 E_{AV}(random)

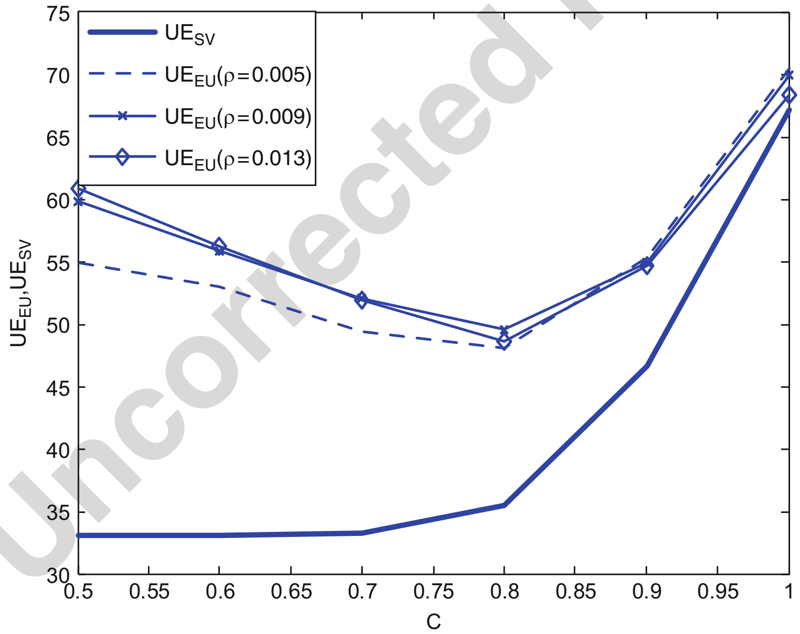

Fig. 12.22 E_{PV}(random)

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12.4.3.1 The Consequences of Intensity Information in the Signals

As explained in detail in Lehtinen (2008), all the simulations discussed thus far are unrealistic for two reasons. First, it is psychologically unrealistic to assume that voters engage in strategic voting if they consider the second-best candidate almost as bad as the worst one. Second, given that the signals already contain some information on the preference intensities under AV but not under PV, the previous setups are likely to yield lower utilitarian efficiencies for PV than for AV. To rectify these weaknesses in the model, voters were also assumed to obtain some intensity information under PV, and to vote strategically only if their intensity exceeds a threshold-level τ. As in Lehtinen (2008), the threshold was assumed to be rather low: $\tau = 0.2$.

Let U denote the sum of utility for all candidates, and U(j) the sum of utility for candidate j. Let $\lambda \in [0.1]$ denote the relative share of intensity information in the signals. A *composite signal* consists of a combination of preference and intensity information, and a random term:

$$S_{i,j} = \lambda v_j + (1 - \lambda)\frac{U(j)}{U} + \rho R_i, \qquad (12.8)$$

where R_i and ρ have the same interpretations as before. When $\lambda = 1$, the pivot probabilities are based only on information on preference orderings under PV. The findings from simulations with full information are shown in Figs. 12.23 and 12.24.

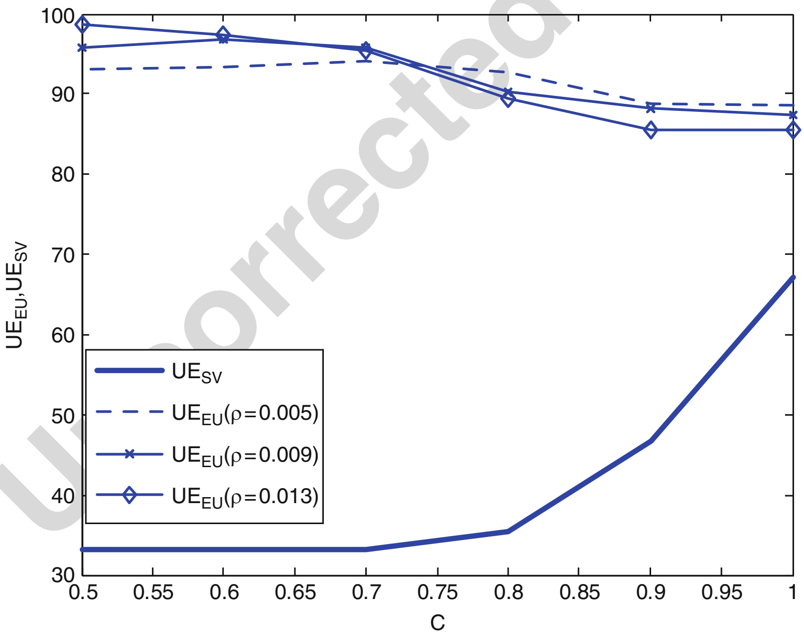

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Fig. 12.23 E_{PV}(random, $\tau = 0.2, \lambda = 0$)

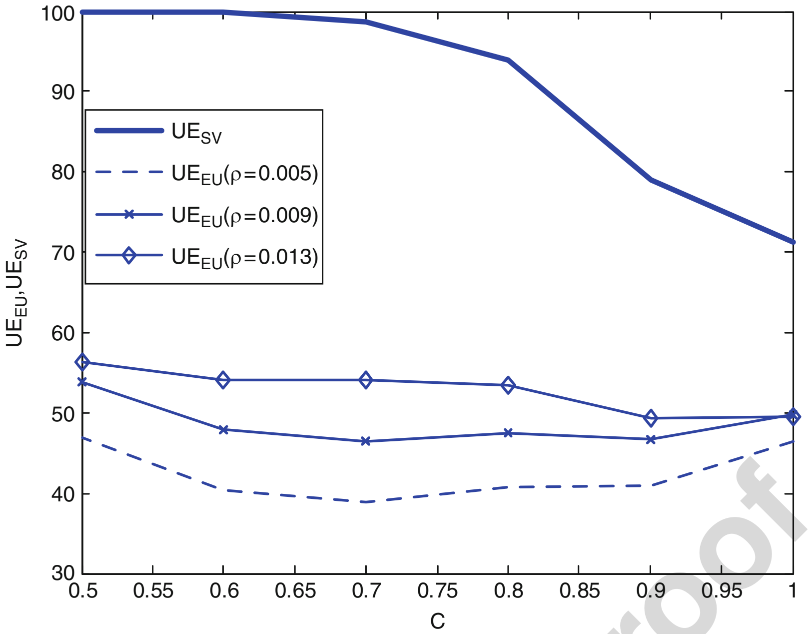

Fig. 12.24 $\mathrm{E}_{AV}(\text{random}, \tau = 0.2, \lambda = 0)$

Under PV the utilitarian efficiencies are considerably higher in setups with full intensity information ($\lambda = 0$), but full intensity information is not all that important under AV: the utilitarian efficiencies remain relatively low.

12.5 Conclusions

As expected, utilitarian efficiencies are lower in the mixed behavior setups than in pure behavior setups. The results depend heavily on which voter types engage in strategic and sincere behavior. Strategic voting and strategic behavior continue to be welfare-increasing in many mixed behavior setups, but in some cases strategic behavior leads to a catastrophe.

The findings are somewhat surprising. AV is much more sensitive to behavioral heterogeneity than PV. The main reason is that under the standard specification of sincere behavior, many second votes are given under AV. Strategic behavior decreases the number of second votes dramatically, and if only some voter types abstain from giving second votes, the reduction in these second votes is often sufficient to change the winner. If the reduction in second votes concerns the utilitarian winner, it is often not selected. Even though there is counterbalancing among the strategically given second votes, this does not matter so much because the difference in the number of sincere and strategic second votes trumps the counterbalancing among the strategic second votes.

When strategic voting is welfare-diminishing under PV, the voter types that engage in it typically obtain a worse outcome for themselves than they would have obtained under the pure behavior SV setup. As such voters would not have an incentive to continue to vote strategically if they knew that they are the only ones to do so, it does not seem very likely that such strategic voting will be found in the real world: strategic voting is only welfare-diminishing under PV when voters who engage in it do not act in a model-consistent fashion. The worry that some particular groups would be able to benefit from strategic voting at the expense of everyone else thus really has to be formulated in a non-welfarist way: when particular groups benefit from strategic voting, they typically increase the overall welfare at the same time.

The consequences of behavioral heterogeneity are usually exactly the opposite in the two voting rules: when EU behavior is welfare-increasing in a mixed behavior setup under PV, it is welfare-decreasing under AV, and vice versa. It is then not surprising that when strategic behavior is welfare-diminishing under AV, the voter types that engage in it typically obtain a better outcome for themselves than they would have obtained under the pure behavior SV setup. This means that those voters really have an incentive to engage in strategic behavior. The worry about unequal manipulative propensities thus turned out to be an argument against AV.

The findings concerning the comparison of AV and PV can be summarized as follows. AV yields higher utilitarian efficiencies than PV when there is no behavioral heterogeneity or when heterogeneity is of the non-systematic type. PV is much more resistant to systematic heterogeneity, particularly if voters obtain perturbed information on preference intensities. An overall judgment concerning preference intensities and strategic behavior in the two rules depends on the relative magnitude between the various parameters. Given that there seems to be no particular reason why behavioral heterogeneity should be of the systematic type, it may be that the findings reported here are not so devastating for AV after all. An empirical investigation concerning behavioral heterogeneity might provide a fuller picture.

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