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ORIGINAL RESEARCH

Imagination and fiction in modelling; an epistemic critique

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Abstract
This paper criticises the Waltonian fictional view for misconstruing the role of imagination in scientific modelling, and for failing to provide an adequate account of the epistemology of modelling. It is argued that the central notions of prescribed imagination, props and principles of generation should be abandoned because they confuse more than clarify the philosophy of modelling. It is argued that the epistemic credibility of inferences in modelling require that results are explicitly derived with model descriptions in a proof, but since prescribed imagination does not guarantee that any actual inferences are made, principles of indirect generation cannot determine the content of a model. An alternative epistemic account of model content is proposed, and it is shown that the Waltonian fictional view does not have resources to accommodate the practices in which such content is determined. This critique highlights the limitations of using concepts from fiction theory in scientific modelling, suggesting that the Waltonian fictional view obscures rather than clarifies modelling practices.

Keywords Imagination · Fiction · Modelling epistemology · Props · Principles of generation · Modelling practice · Model content

1 Introduction

A prominent fictional account of scientific modelling posits that model descriptions are to be interpreted as props that prescribe imagining fictional truths. Let us call it the *Waltonian fictional view* (Poznic, 2016) because it relies on three central concepts from Kendall Walton's (1990) theory of fiction: the *props*, the *principles of generation* and *prescribed imagination*.

The proponents present the process of modelling as follows. The modeller first writes down the 'original' model descriptions (e.g., Salis & Frigg, 2020), and then

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derives implied fictional truths about the model via the Principles of Indirect Generation (PIGs). Laws and general principles are such principles in scientific modelling (e.g., Frigg, 2010a, p. 120). They are claimed to be important in the epistemology of modelling because model descriptions are not sufficient by themselves for making such inferences, and learning about the model system (i.e., the imagined system in the modeller's head) requires combining the PIGs with the model descriptions in imagination.

The Waltonian fictional interpretations of PIGs capture several aspects of modelling, including the use of inferences after initial model descriptions are available, the potential involvement of laws or principles in these inferences, the application of rules in making inferences, the implicit nature of resources required for some such inferences, possible interdisciplinary differences in model-based inferences, and the possibility that the content of the inferences may be the same for everyone. Indeed, these aspects can all be observed in modelling to some extent.

Proponents of the Waltonian fictional view claim that PIGs provide two philosophical benefits: determining the content of a model and accounting for the practice of modelling. However, I argue that they fall short in both regards. Attempting to capture multiple aspects of modelling with a single concept borrowed from fields outside modelling, such as art and literature, leads to inconsistencies even after modifications.

Roman Frigg and Fiora Salis, who have advocated for the Waltonian fictional epistemology, may only have intended to provide a partial account that focussed on answering the question: 'How do we learn what is true in a model?' Answering this question provides an account of model content. Here they draw directly from Walton's account of truth in fiction: Model content is established by combining model descriptions (props) with laws and principles (PIGs).

I present four criticisms of this account of model content. Before outlining the criticisms, it is important to highlight their significance, particularly for those who might view the issue of determining model content as trivial. Weisberg (2013, p. 57) criticised the fictional views by arguing that if imagination determines model content, there would inevitably be intersubjective disagreement about that content. In response, proponents of the Waltonian fictional view began emphasising that prescribed imagination does not lead to such disagreement, unlike actual imagination. If my criticisms are successful, given that they are independent from Weisberg's, the rationale for adhering to prescribed imagination and thereby the Waltonian fictional account should disappear.

First, I argue that a set of model descriptions cannot be called 'a model' if it does not allow for making inferences without introducing additional assumptions. Mathematical model results are accepted only if there exists a proof formulated with explicit model descriptions. Thus, laws and principles must be incorporated into the model descriptions. If they are, the only learning that is possible after writing down the model descriptions is deducing the mathematical consequences. However, prescribed imagination cannot take part in making such inferences because it does not guarantee that actual inferences are made. A criterion of model content is only useful if the content is guaranteed to be known when the criterion is employed, yet the Waltonian fictional account fails to meet this requirement insofar as the PIGs merely prescribe imagination.

The second criticism highlights that model development typically involves combining further model descriptions rather than integrating laws and principles with existing descriptions. However, viewing model descriptions as props for imagination cannot elucidate such model development because there are no rules governing how such modifications should be conducted. They require creative rather than prescribed imagination.

The third and the fourth criticism demonstrate that the Waltonian fictional view fails to account for the practices with which modellers actually determine the epistemically relevant model content. The third criticism starts by positing that the epistemically relevant model content is not reducible to the deductive closure of a model because some assumptions used in formulating a model may not be necessary for it. To find out which assumptions are necessary, modellers de-idealise or generalise models, or resort to robustness analysis. Such exercises enable comparisons between several models in a model family, which in turn allow determining the epistemically relevant model content. I propose that the *epistemically relevant model content* is established with such activities, thus providing an alternative account of model content. However, the central tenet of taking model descriptions as props for prescribed imagination makes the Waltonian fictional view incapable of making sense of model-target comparisons,[1] and comparisons between models,[2] thereby making it incapable of accounting for how the relevant model content is actually determined.

The fourth criticism focusses on the timing of prescribed imagination within the model-building process. Because the model descriptions are interpreted as props for the imagination, it is supposedly employed only *after* the model descriptions have already been written down. This implies that prescribed imagination cannot play a role in the *process* of determining the content of a model. It can only be used after the model has already been built.

This aspect of props is also crucial for another criticism of the modelling practice. Imagination is epistemically relevant for coming up with the right kind of model descriptions in the first place, but what is needed is creative rather than constrained imagination. I propose that there are three kinds of content of actual imagination: About the target system, about which model descriptions could be used to describe the target system, and about the model system. The problem here is that the content identified by the Waltonian fictional account, imagination about a model system, cannot change without a change in the other kinds because prescribed imagination presupposes that model descriptions are already available. Consequently, prescribed imagination is epistemically inert. In contrast, developing models via trying different

[1] They have developed a way of responding to this charge, namely, resorting to ad hoc principles of generation (e.g., Toon 2012, 47, Salis 2020a, 2021b, Frigg 2021). I argue in Sect. 5 that this strategy does not successfully account for epistemically relevant model-target comparisons.

[2] According to Walton (1990, 410) any comparison between two fictions (which, Walton himself would interpret as model descriptions because he interprets a written novel as a fiction) leads to unauthorized games. Given that unauthorized games are not governed by rules, one is free to imagine whatever one likes. I thus presume that the Waltonian fictionalists agree that Walton's theory does not provide any resources for understanding such comparisons.

model descriptions requires creative imagination. I thus argue that the Waltonian fictional view misidentifies the role of imagination in modelling insofar as it commits itself to prescribed imagination.

In summary, these criticisms collectively argue that the Waltonian fictional view is inadequate in explaining how modellers determine the relevant content of models, and in accounting for several important features of the practice of modelling.

Different fictional accounts of modelling are affected in different ways by my criticism, depending on how central the PIGs and the interpretation of model descriptions as props are. The indirect fiction view of Frigg, Salis and Nguyen is the most affected, and the direct fiction view (esp. Levy, 2015) is affected a little differently because it requires interpreting the target rather than the model descriptions as a prop. Godfrey-Smith (2006, 2009, 2020), Thomasson (2020), and Thomson-Jones (2020) do not employ the Waltonian framework at all, and these fictional accounts of modelling are unaffected. However, my arguments are not intended to support these non-Waltonian fictional accounts of modelling. Neither do I aim to criticise Walton's account itself, insofar as it is not applied to modelling (see, however Levy, 2020).

I might be taken to task to provide an alternative account of models. However, according to Frigg and Nguyen (2021c), the Waltonian fictional view is primarily a view about ontology, but I am trying to solve epistemic rather than ontological problems (see Friend, 2020). Epistemology of modelling does not necessarily need an ontological account in the first place, but rather an account of model (family) identity and content. I give a rough sketch of such identity in this paper, but given space limitations, I relegate the development of a full account into another paper draft. Identifying the Waltonian fictional view with ontology is not self-evident, however, because it is perfectly possible to discuss imagination in modelling without taking any stance on the ontology of models. This is why it would be inappropriate to require this paper to provide an account of the ontology of models.

The structure of the paper is as follows. Section 2 briefly describes the Waltonian fictional view. Section 3 argues that PIGs do not explain how one arrives at truths about the model if they are interpreted as contents of prescribed imagination. Section 4 argues that the concept of imagination constrained by model descriptions as props provides an inadequate account of the role of imagination in modelling. Section 5 shows why the Waltonian view cannot make sense of model-target and model-model comparisons.

2 The Waltonian fictional view of modelling

Walton's (1990) basic idea is that the attitude of consumers of fiction is that of pretence; one make-believes that what is said or implied in a fictional text, for example, is to be imagined. It is common to distinguish between fiction from non-fiction by the different attitude to truth: What is to be imagined may be true, but if it is not, that does not diminish the value of fiction qua fiction. Walton calls a proposition a *fictional truth* if it is to be imagined in some context (p. 38). Fictional propositions are *to be* imagined—whether they are in fact imagined.

A prop induces a prescription to imagine something. What is to be imagined is constrained by *principles of generation* together with the prop. A principle of generation

is in force in some context, if it is understood in that context that, given such-and-such circumstances, so and so is to be imagined. The *direct* principles of generation are (implicit or explicit) rules that determine what readers should imagine, given the properties of the prop. The *indirect* ones require some *inferences* from the props and the direct principles. In *authorized* games, the props have the function of prescribing imaginings. In *unofficial* games (p. 406), for example when children pretend that a tree stump is a bear, the principles of generation are 'ad hoc' or conventional.

Walton (1990, Ch. 4) discusses at some length two principles of generation, the reality principle and the mutual belief principle. As acknowledged by some Waltonian fictionalists (e.g., Frigg, 2010a), they cannot be applied in scientific modelling because they would lead to unacceptable results.[3] Walton says that 'there is uncertainty and disagreement, in many cases, about what principles of generation are applicable to a given work' (1990, 138). Most principles involving works of art are never explicitly agreed or even formulated, imaginers may be unaware of them (1990, 38), and there is no simple and systematic way of describing how fictional truths are generated. He denies that such principles should be applied to scientific works (1990, 70). He says, for example, that 'there is no understanding to the effect that readers are to believe whatever [Darwin's The Origin of Species] says because it says it.'

The Waltonian fictional view of models comes in a direct (Levy, 2012, 2015; Toon, 2010, 2012) and an indirect version. The latter postulate that modellers' imaginings constitute the 'model system'. The direct view denies the need to postulate such intermediate entities between model descriptions and targets. For our purposes, there are two important differences between them. The direct view takes the model descriptions rather than the imagined model system to be the model, and Levy (2015) takes the target phenomena rather than the model descriptions as the props for the imaginings.[4] The direct view then considers models as 'imaginative descriptions of targets'.

According to the *new fiction account* (Salis 2020a; 2020b; 2020c; 2021a; 2021b; Salis & Frigg, 2020; Salis, Frigg & Nguyen, 2020) the model consists of the model descriptions and their content instead of the model system. The content, in turn, refers to all those propositions that can be derived from the model, including indirect fictional truths that are derived with the PIGs.

When Walton's theory is applied to scientific modelling, the central tenet of the indirect view is that modelling is to be understood as an authorised game of make-believe in which the model descriptions are the props that prescribe imaginings (Frigg, 2010a, 2010b, 2010c, 2021; Frigg & Nguyen, 2016, 2017; Salis, 2016; 2020a; 2020b; 2020c; 2021a; 2021b; Salis & Frigg, 2020; Salis, Frigg & Nguyen, 2020).

The crucial idea of the Waltonian fictional view is that the model descriptions do not include all the epistemically relevant information. The *principles of direct generation* are properties of the prop or linguistic conventions used in interpreting them (Frigg & Salis, 2020; Frigg, 2010a, 2010b). The 'initial' model descriptions only contain a

[3] According to Salis (e.g., 2021a; 2020c; 2020a; 2020b), however, there are interdisciplinary differences in how prevalent the reality principle is. Salis thus seems to think that such principles are indeed used in science.

[4] For Toon the model descriptions are props (2012, p. 40), and they prescribe to imagine *about* the target (2012, p. 56).

sparse mathematical description, but they do not include the results of inferences one may make with them. Models thus do not reveal all their content in model descriptions.

The PIGs ultimately explain why the model descriptions do not already include all the necessary components for making epistemically relevant inferences from models: 'Due to the principles of generation the imagined-object can have properties that have not been written into the original model description, which is why the study of imagined-objects is cognitively relevant' (Frigg & Nguyen, 2016, p. 237). In science the PIGs include the laws of nature and general principles prevalent in each scientific discipline, as well as mathematics and logic (Frigg, 2021; Frigg & Nguyen, 2020, p. 122; 2021b). These principles are shared, public and known by the relevant academic community, but may be implicitly expressed. They are rules that govern individual imaginings in such a way that everybody ought to end up with the same imaginings. On the other hand, there may be disciplinary differences in such principles and they may be context-dependent (e.g., Salis 2020b).

Learning from models occurs when the PIGs are used in deriving consequences (indirect fictional truths) from the model descriptions (Frigg, 2021; Frigg & Nguyen, 2020, p. 122; Frigg & Salis, 2020). Frigg (2021) identifies the epistemology of modelling with finding out what is true and false in a model. Salis (2021a) claims that 'the model system is an imaginary object that is manipulated and developed *in the imagination* by drawing certain *inferences* according to the principles of generation [italics added]'.

The Waltonian fictionalists emphasise that imagination is *constrained* in that imaginers are taken to follow rules, namely, the principles of generation, in their imagination. Such imagination is to be contrasted with *creative* imagination.

Given that I do not focus on ontology, I will not discuss whether the model system is to be considered as concrete or abstract, and whether to take a realist or an anti-realist stance on its existence.

3 Principles of indirect generation and model content

Consider now the central idea that the principles of generation confer the imagined-object properties that have not been written into the 'original' model description. The relevant model identity (or model content), according to the Waltonian fictional view, is given by the model descriptions supplemented with the implied truths inferred with the help of PIGs (e.g., Salis 2021b). More precisely, prescribed imagination with PIGs determines the content of a model. Since the PIGs consist in laws and principles that are known by modellers but may be implicit in the modelling practice, the Waltonian fictional view accounts for how the same model descriptions may entail different contents in different disciplines, 'in imagination'. I agree with some further aspects of this account. For example, there are 'models' in physics that can be supplemented with different laws, there are epistemically relevant aspects of modelling that are implicit, and models are often developed further after a first version is presented.

In this section, however, I will argue that the above account of model content is problematic for several reasons. As background to the discussion, recall that Weisberg (2013, p. 57) argued that it is not a problem if different people use different principles

of generation in reading literature, but it is a problem in scientific modelling because different participants would end up with different views about what the model says (see also Levy, 2013). In response, the Waltonian fictionalists have tended to emphasise the difference between prescribed and actual imagination, arguing that the former rather than the latter determines the content of models. This move confers uniqueness to the content of imagination because, what is to be imagined need not depend on the vicissitudes of what modellers actually happen to imagine. Friend (2020) puts the point about content prescribed by imagination as follows. 'Part of the specification of the Lotka-Volterra model is a set of equations. It will be fictional in any version of the model that the model system conforms to these equations, whether anyone actually works out the mathematical conclusions or not.'

Knuuttila (2017, 2020) argued that one does not have epistemic access to model systems: One does not have access to PIGs because they are in each modellers' head. In response, Salis (2020b, p. 470) claims that 'whenever two modelers disagree on the content of a particular model, they can solve their dispute on the basis of the objective and intersubjectively available symbols together with their interpretation.' I take this to be tantamount to conceding Knuuttila's point: The intersubjectively available symbols and their interpretation are nothing other than the model descriptions. Salis (2020b) also challenges Knuuttila's presupposition that different scholars have different imagination and thus different model systems. Knuuttila could retort that even if one had epistemic access to the PIGs, and even if everyone had the same content of imagination, it does not matter because the explicitly derived results and the model descriptions used in deriving them determine what is epistemically relevant about a model. But even this could be challenged by the Waltonian fictionalists by arguing that if one is only considering the content of a model, it should depend only on what the PIGs prescribe to imagine, not what modellers actually imagine, and hence whether or not one has access to modellers' actual imagination is irrelevant.

I will now provide three arguments against the way in which the content of a model is determined according to the Waltonian fictional view. (1) First, the Waltonian fictional account is not able to account for model content in many sciences because they do not use laws and principles in the first place. (2) Second, PIGs cannot determine the content of a model because prescribed imagination cannot make explicit inferences for us, but model results are only accepted, and should only be accepted by the modelling community, if they are presented with an explicit proof. (3) Third, the Waltonian fictional view is not able to account for how the relevant content of models is actually determined because it does not have resources to account for why model identity is typically determined for a whole family of models rather than for a single one.

(1) The limited applicability of interpreting PIGs as laws and principles derives from the fact that the view is physics-centred. In that context, applying principles on some set of model descriptions that physicists call a model makes some sense. Frigg (2010b) presents the example of deriving the elliptical orbits of the earth around the sun, using Newton's model of the sun-earth system and classical mechanics. His epistemic point is to say that the elliptical orbit was not written into the model descriptions (which include the gravity law), and that one needs Newton's force equation ($F = ma$) to derive it (see also Frigg & Nguyen, 2016, Frigg & Nguyen 2020 use this example throughout the book). This is indeed a case in which the entity that physicists recognise as a 'model'

does not yet yield all the relevant results. Initial conditions must be set to derive the consequences from the 'initial' model descriptions. Adding them to the model does not mean introducing new assumptions because the model descriptions may specify the domains of the variables, and thus implicitly contain the initial conditions. Note, however, that filling in the initial conditions in a game of make-believe (or otherwise in one's imagination) does not bring about the *inferences* required to derive the result unless they are represented as model descriptions.

Insofar as a given PIG prescribes exactly one inference such that interpersonal differences in what is inferred are impossible, an alternative way to express what is fictional in a model is just to say that the deductive closure of the model descriptions is what is fictional in the model. Thus, the deductive closure could be the content of the model. But then, why not simply make this claim, rather than argue that modellers are engaged in a game of make-believe in which they combine model descriptions with PIGs in imagination? According to the Waltonian fictionalists, it is because a given set of model descriptions may give rise to several different results if different PIGs are combined with them (e.g., Frigg & Nguyen, 2020, p. 123). This is correct: Given that Newton's model supplemented with different (classical or quantum mechanical) assumptions gives rise to different consequences, the different deductive closures constitute epistemically relevant differences, and thereby they should be counted as different models.

However, in the social sciences, biology, economics, and many other special sciences, it is not possible to distinguish between principles or laws and the rest of the model descriptions. If there are any things that could be called general principles in these sciences (say, maximising expected utility), they are employed in formulating the model descriptions in the first place.[5] What are called 'laws' (e.g., the 'law of demand') are *derived from* model descriptions rather than combined with them. What follows is that it is not possible to distinguish between an incomplete 'initial model' and the PIGs that complement it in the way that the Waltonian fictionalists claim.

(2) Just like the Waltonian fictionalists claim, manipulating a model after modellers have come up with an 'initial' model description is epistemically important. The problem with the Waltonian fictional view is that it obscures the nature of such manipulations by insisting that PIGs are needed for deriving results because model descriptions are not sufficient for such inferences. While a first version of a model often does not reveal all its properties, the further manipulation of the model typically involves introducing new model descriptions instead of introducing laws and principles.

I will now argue that if the model descriptions are not sufficient for deriving the model-results, then modellers should not (and they do not) view such a collection of model descriptions as a 'model' at all. Instead, modelling results must be derived by literally using the model descriptions. To see why, suppose a modeller has come up with a set of model descriptions and claims in a conference presentation that one can derive an interesting result when those descriptions are combined with well-known laws and principles in imagination. The audience is likely to think that the modeller does not really know or respect the rules of academic behaviour. Claiming that one can

[5] Giere (2004) argues that this is also the case in the natural sciences.

obtain a result by making some further inferences simply does not count as a scientific result. Let me give further arguments for this.

If a protomodel is incomplete in this way, new model descriptions must be added. Since new model descriptions entail new commitments, modellers do not accept model-results unless they are explicitly expressed with model descriptions: there can be no implicit results known by a community of modellers, and there can be no model-results that are not demonstrated with a proof that uses the model descriptions. It is instructive to discuss another model as an illustration of these considerations.

Morgan (2012, Ch. 5) discusses the development of Edgeworth's box. Sugden (2013, p. 112–3) describes it as follows in his review of Morgan's book.

> As understood by most economists, this is a two-dimensional diagram that can describe endowments and preferences in a two-person exchange economy. It does not specify what those individuals actually do, or what counts as an equilibrium solution. To derive any conclusions about the pattern of trade in the model, one has to add additional assumptions, for example that trade takes place at market-clearing prices, or that 'blocked' trades do not take place. If one treats the Edgeworth Box diagram as 'the model', the way it is used in economics fits Morgan's schema; adding assumptions about individual behaviour or specifying a solution concept can then be characterised as 'manipulation'. But my sense is that economic theorists would be more inclined to say that, until those components are added, the Edgeworth Box is not a fully-specified model; it is only an extremely useful modelling component.

Sugden challenges Morgan's distinction between the model and its manipulation by pointing out that what could be called a 'model' is only a component or a protomodel if it does not contain all the required resources for making the relevant inferences. The reason why protomodels are not good enough is also expressed by Sugden: Making the relevant inferences requires further assumptions, and those assumptions may well be problematic for the epistemic credibility of the inferences. This is why all the assumptions should be as explicit as possible.

Model content ought to be determined, and is actually determined, by making inferences with model descriptions. A model is always based on a set of assumptions, some of which are false, and when modellers think about the epistemically relevant model identity, they think of model results as conditioned on the idealisations that inferring the results requires: The epistemic credibility of the results depends on whether one can show that the idealisations are not responsible for the results. This is why modellers think of models as the following kinds of proof: If one makes such and such commitments, one can get such and such results. The commitments must be at least surveyable because otherwise modellers do not know which assumptions the result depend on. To put it differently, given that the results depend on idealisations, there is typically some epistemic uncertainty concerning whether the result could also be derived if the model descriptions included different idealisations or if they were to be removed altogether. It follows that modellers should not accept model results unless they are explicitly demonstrated with a proof. This is because the proof makes most of the assumptions explicit and the rest of them at least potentially discoverable.

There is thus a good epistemic reason for why proofs are required in science: Modelling is not well described by logical omniscience. If it were, there would be no reason to require proofs for mathematical models, and the Waltonian fictional epistemology would not stumble on the epistemic need for explicit proofs. We now have an epistemically pregnant argument for why claiming that one can obtain a result by making some further inferences in imagination (prescribed or not) does not count as a scientific result. To reiterate, modellers do not accept results, and they should not accept results, unless one shows which assumptions are used in deriving them. This is ultimately the reason why results derived in imagination (prescribed or not) are not accepted. It also explains and justifies why I claimed that promising that results can be derived is not accepted in scientific practice, and that protomodels are not called models in the first place.

The Edgeworth box has been further developed after Edgeworth's 1881 initial diagrams. The Waltonian fictionalists argue that model manipulation is a matter of applying PIGs to the original model descriptions. The inferences that one makes are supposedly done by following rules that prescribe what one should imagine. But as Morgan's (Ch. 3) historical account of this model makes clear, the development was a matter of creating *further* diagrammatic and mathematical resources rather than applying rules. This is an obvious consequence of the fact that this model was not developed by applying laws and principles. Historically, the advances required making new assumptions, and thus creative rather than constrained imagination.

As a matter of fact, it does not even make sense to ask what 'Edgeworth's box' prescribes us to imagine because there are several different model-versions that each prescribe a different content for imagination. For example, unlike the modern versions, his original graph did not even include a box, and thereby the assumption that the endowments of the two persons are given. Are we to imagine that the endowments are given or not? The differences in the versions have epistemically relevant consequences. But since the different versions differ in idealisations and typically also in some substantive assumptions rather than just laws and principles, the Waltonian fictional view is not able to explain why modellers develop such families of models and compare their details across the model versions.

We can now draw the consequences from this analysis of protomodels and model development. The problem with the idea that prescribed imagination determines the content of a model is that using it does not allow us to *know* what the content of the model is: Had the mathematical conclusions of the Lotka-Volterra model not been worked out, the scientific community should not have accepted the idea that the Lotka-Volterra model system is consistent with the mathematical conclusions of the equations, because there would not have been any proof that they actually have such consequences.

We can now also better understand why the Waltonian fictional view gives a misleading account of model content from the point of view of epistemology: since laws and principles are epistemically secure, even though they also introduce assumptions, those assumptions are so harmless that they do not raise epistemic concerns. In Newton's model the assumptions introduced by Newton's second law are known to be epistemically secure for the model at hand, but this is not the case with models more

generally. Adding laws and principles to initial model descriptions is thus epistemically different from adding further or different idealisations because the latter but not the former may change the epistemic acceptability of the inferences. The problem is thus that conceptualising PIGs as laws gives rise to the misleading idea that model manipulation after an 'initial' set of model descriptions are written down no longer involves epistemic difficulties and thereby to ignoring the reasons why proofs are required in modelling.

Inferences *prescribed* by the PIGs are not actual inferences, but nobody knows what the content of a model is if no actual inferences are made (recall what Friend said about the Lotka-Volterra model, as quoted above). The Waltonian fictional view would thus provide a criterion for the content of a model, but it could never be used to determine the content of a model if the prescribed imagination does not need to mesh with actual imagination, and if the actual inferences from model descriptions to results are never made. Hence, claiming that the content of a model is determined by prescribed imagination in a game of make-believe is not acceptable because prescribed imagination does not make inferences for us. Consequently, emphasising the difference between what one is prescribed to imagine, and what is imagined, does not solve the problem of inter-scientist variation if one must be able to determine the content of a model so that the whole scientific community knows what it is. Emphasising prescribed imagination merely obfuscates the fact that proofs and thereby actual inferences are necessary for epistemic reasons.

On the other hand, if PIGs concern actual imagination, and if they are allowed to have interpersonal differences like in Walton's account about literature and art, it becomes impossible to make sense of the epistemic uses of the PIGs for inference-making because then Weisberg's inter-scientist disagreement rears its ugly head again. Furthermore, make-believe cannot be flexible and allow the freedom of choosing between different principles (Salis & Frigg, 2020) if it is to determine the content of a model.

In conclusion, the requirement of explicit proof entails that, when modellers use laws and principles used in constructing a model, they must be included in the model descriptions in the first place. Indeed, Toon (2012, p. 40) puts his view precisely this way: 'The prepared descriptions and theoretical laws that the scientists write down still act as props in games of make-believe.' Frigg and Nguyen (2020, p. 187) also include the principles of generation into the model descriptions in one of their publications:

> We explore the behaviour of a fictional model by finding out what follows from the basic model assumptions stated in D_X when combined with a set of principles of generation. To do so we use the principles in a certain formulation. Let D_G be a description of these principles, which […] should be considered to be part of the model-description.

Given that their reason for proposing such a solution is to make sense of the fact that models have an internal dynamic, I presume that they mean that PIGs should also be included in the model descriptions. If all principles of generation belong to model descriptions, then the laws and the general principles should also be props for the imagination. However, instead of adopting this view, they claim that only part of the model descriptions (viz., D_X) are props for the imagination.

Frigg and Nguyen (ibid.) present the idea of including the principles of generation into the model descriptions as part of their account of model representation (DEKI) rather than as an explication of the Waltonian fictional view. This makes it difficult to determine the status of this diversion of their more usual line of argument.

If the well-considered view of Waltonian fictionalists is to accept the idea that laws and principles must be included in the model descriptions, they face a several obvious problems. Namely, the epistemology of modelling understood in terms of PIGs is no longer tenable: There is only mathematics to apply when the model descriptions are written down, but deducing the mathematical consequences is not done in imagination. Consequently, model content cannot be determined by joining PIGs to model descriptions in a game of make-believe. Making this move also entails that the model system can no longer be different from the model descriptions, and that the model content is determined solely by model descriptions. However, if this is so, there is no need to invoke make-believe or imagination for the purpose of determining model content in the first place.

The Waltonian fictionalists claim that there is something to discover after the initial model descriptions are written down because the PIGs are often implicitly expressed. I agree with the epistemic importance of these activities, and with the existence of implicit contents, but not that they can usefully be described by the PIGs. Consider now any model which is complete enough to be capable of generating inferences. To derive the results, one still must apply mathematics, or run a computer simulation, or construct a physical artefact and let it run its course. This much is true about the Waltonian fictional view. Mathematical rules qualify as something that one must add to the model descriptions to make epistemically relevant inferences. However, applying mathematics does not have anything to do with imagination, constrained or creative, because solving a model is simply a matter of deductively deriving results from the model descriptions.[6]

(3) I will now formulate an account of epistemically relevant model identity and content. Trying new model descriptions is crucial for the epistemically relevant model identity and content. However, insofar as the Waltonian fictional view is committed to the importance of prescribed imagination in determining model content, it not only ignores such modelling activities, it is also unable to account for how the epistemically relevant model content is actually determined in many cases. The relevant model

[6] Although this paper is concerned with mathematical models, inference-making via PIGs is equally problematic with simulations and material models. There are two possible ways of cashing out the relationship between models and model descriptions for material models. If the material artifact is the model, then the written blueprints for constructing it could be model descriptions (written design specifications for the Reber plan, instructions for building a Phillips-Newlyn machine etc.). Alternatively, unlike with theoretical models, the material artifact could be both the model and the model description. The first option is problematic for the Waltonian fictional view because it would then be possible to have models without model descriptions: It is possible to build something like a Phillips-Newlyn machine by whatever basic macroeconomic knowledge one happens to have, without ever writing down any kind of plan of the exact physical makeup. Although the aforementioned cases had such design specifications, the possibility of constructing material models without model descriptions raises a problem: if a written or drawn specification does not exist, what is supposed to be the prop to imagine about? Moreover, even if one existed, no amount of imagination, constrained or creative, will bring about the physical artifact. If the physical artifact is both the model and the model description, there is a similar problem: imagination, constrained or not, does not bring about the results in physical artifacts, some physical processes do.

content is typically determined by a comparison of several model versions in a family of models, but constrained imagination does not provide any resources for understanding why modellers would engage in such comparisons, or the epistemic basis of such comparisons.

Modellers try to see how different combinations of model descriptions give rise to different results. In some cases this gives rise to a *model family*, that is, a set of sets of model descriptions that have some common features (the literature on robustness call it a 'core' structure) as well as some varying model descriptions (Ylikoski & Aydinonat, 2014). This gives rise to two kinds of model identities. On the one hand, the similarities between the model versions and the characteristic consequences of all model versions within a family allow modellers to denote the model, thus helping in everyday communication. These common elements and consequences thus allow them to speak about 'the Schelling model', 'the Lotka-Volterra model', or 'the Edgeworth box' etc., despite the fact that there are model-versions that have epistemically relevant differences. This could be called *referential* model identity because it merely helps communication among modellers. But the differences in the consequences due to different model descriptions are typically epistemically relevant. This is why modellers themselves are much more concerned with *epistemic* model identity. This is a model identity that may be different due to the minutest of differences in model descriptions.

The deductive closure of a set of model descriptions is an important source of information about the relevant epistemic model identity, but it is best not to define the relevant model identity with the deductive closure. The reason is not because this leads to a proliferation of models because the referential model identity is sufficient for making sense of talking about one model that has various versions. The reason is rather that some consequences in the deductive closure, especially the false ones, may ultimately be irrelevant for the relevant model identity and content. This is the case when a family of models is demonstrably robust with respect to some of its idealisations, or when it can be de-idealised or generalised. Then, those idealisations have consequences that count for the deductive closures of some model versions, but the fact that they are shown to be irrelevant for deriving the common model result shows that they are not part of the relevant model content.

Let me take an example to illustrate how adding new model descriptions may be necessary for determining the epistemically relevant model content, and to show how the relevant content does not necessarily reduce to the deductive closure.

Some implicit assumptions and their consequences can only be discovered by examining the joint implications of several functional forms. For example, the so called constant elasticity of substitution (CES) utility function $V\left(\sum_{i=0}^{n} x_i^{\rho}\right)^{1/\rho}$, when combined with the rest of the assumptions[7] in Krugman's (1980) model of international trade, has the implication that, when trade expands as two countries open their borders to trade, the firm size cannot change, and all the increase in production has to occur through an increase in the number of firms. This is unrealistic because empirical data

[7] These other assumptions are needed to apply the theory of monopolistic competition to international trade: the existence of different countries, relative size of those countries etc. Furthermore, all the models include the assumption of increasing returns to scale. Needless to say, none of these assumptions is a law or a principle; no PIGs are involved.

indicate that part of the increase in production in such cases occurs by an increase in the size of the existing firms (Neary, 2004). When Dixit and Stiglitz (1977) proposed the CES function for a context in which there is no international trade, they did not know that it has such consequences. How, then, do we know that this consequence can be attributed to the CES function and not some other parts of Krugman's, 1980 model? It is because Krugman (1979) provided another model of international trade which is otherwise almost identical to the 1980 model, but used the following utility function: $\left(\mathrm{V} = \sum_{i=0}^{n} v(x_i)\right.$, $v\prime > 0$, $v'' < 0)$. This function does not have the aforementioned consequence when combined with the rest of the model. Finding out about the existence or the relevance of implicit assumptions thus requires introducing new model descriptions and thereby further assumptions.

The fact that new assumptions and functional forms must be introduced in such model-explorations means that new models are generated. There are literally thousands of different versions of the Dixit-Stiglitz model. They are different models for epistemic purposes because their consequences differ in epistemically relevant ways. Even when the relevant consequences are the same, their sameness is epistemically relevant because it means that the model is robust.

To find out whether idealised assumptions affect the model result, one examines whether the result still obtains if one uses different idealisations. If the results are robust with respect to such a change, then we learn that the idealisations were not needed. If they are not robust, more is learned about which assumptions were responsible for the result (Kuorikoski et al., 2010). Here, the fact that Krugman's 1979 model can derive the central result of the model family, namely, that the existence of international trade in similar commodities, can be explained with a combination of monopolistic competition and increasing returns, tells us that fixed firm size entailed by the 1980 model is not a part of the relevant content of the model family.

These considerations show that the relevant unit of epistemic evaluation is not an individual model but rather a set of models. This is a severe problem for the Waltonian fictional view. When two models share some but not all components, it only allows noting that one of them prescribes one set of imaginings, and the other a different set, but it provides no resources to understand why there is an epistemic benefit in comparing sets of similar model descriptions. The Waltonian account is thus silent about the most important epistemic practices in modelling, practices that are crucial for determining the epistemically relevant content of models.

The root of problems (2) and (3) is that, while the Waltonian fictional view rightly identifies many aspects of modelling, the notion of PIGs includes several different features that cannot be reasonably analysed with one concept. The straightjacket forces one to misdescribe the actual modelling practices. The same notion cannot account for inference-making after some model descriptions have been written down, for implicitness in such rule-governed inference-making, and for interdisciplinary differences in the rules, while at the same time guaranteeing that every modeller makes the same inferences and understands the model in the same way 'in imagination'. Resorting to prescribed imagination provides a solution to the problem of intersubjective disagreement only by creating new problems that are equally serious.

Let me finally discuss the scope of the above criticisms. Some of those who have espoused a fictional account of modelling might agree that models (or 'model systems') have content that goes beyond model descriptions without however committing to the idea that PIGs provide the missing content (e.g., Godfrey-Smith, Thomasson, Thomson-Jones). If they were to also claim that epistemically relevant inferences cannot be made without supplementing the model descriptions with something, I would disagree with them too.

However, it is possible to limit the fiction view in such a way that it merely concerns the phenomenology of modelling; when modellers think about their models and their targets, they add properties to them in their actual imagination. If the model system consists of such actually imagined properties together with model descriptions, then one can account for the face-value practice by appealing to the fact that the modellers can distinguish the model system and the model descriptions in their actual imagination. I endorse this insofar as imagination is taken to be relevant only for a phenomenological account of the practice but not for epistemically relevant inferences.

I am not concerned with the issue of what it is that could be taken to represent something in modelling. It is consistent with this stance to endorse the idea that modellers actually imagine that targets have properties not mentioned in the model descriptions. They probably do. However, agreeing with this does not commit me to the existence of model systems (an indirect view) nor deny the possibility of them (a direct view). More generally, imagination is relevant in modelling but including it into an account of modelling does not require a fictional account (as exemplified by e.g., Morgan, 2004, 2012).

According to Hausman (2023, p. 173), acknowledging the relevance of make-believe and imagination for the phenomenology of modelling does not need to entail any ontological commitments. If it does not, then I am not obliged to defend any particular account of model ontology (abstract structures etc.) in the first place, let alone provide a new account of model ontology. It is perfectly acceptable to discuss and study the role of imagination in scientific practice without taking any stance on model ontology. This is indeed what I intend to promote.

4 Imagination and model construction

Frigg and Nguyen claim that the activity of manipulating assumptions, relying on background theories, and learning about what the different posits entail, 'is naturally analysed as being involved in a game of make-believe, and in doing so an imagined-object is explored' (2020, p. 188). They also argue that modelling involves 'playing with assumptions and considering different options', and that the 'fiction view of models utilizes fiction as imagination to understand this practice' (Frigg & Nguyen, 2021c, p. 138). Salis (2020b) claims that constrained imagination can be used in constructing the model. These claims indicate that the Waltonian fictionalists take make-believe with prescribed imagination to also provide an account of modelling practice.

In this section, I argue against these claims. Prescribed imagination cannot be used in constructing models, even in principle, if the model descriptions are supposed to be

props for prescribed imagination. More specifically, I argue that the imagined model system cannot change in an epistemically relevant way if imagination is prescribed by model descriptions as in the indirect fictional view, nor if imagination is prescribed by the target system as in the direct fictional view. In the indirect view, the reason is that when prescribed imagination is supposedly used in modelling, the model must already have been constructed because prescribed imagination presupposes that the right kind of model descriptions are already available. The direct view does not presuppose that the model descriptions are already available, but it faces a different but related problem: we don't know what the target prescribes us to imagine before the model is actually constructed by writing the model descriptions.

Consider what the critics of fictional accounts in modelling concede is correct about them. Weisberg (2013, p. 51) credits the fictional position for being able to make sense of a practice in which the modellers start with a mental picture of a target phenomenon, and then proceed to write a model description (equations) on the basis of this picture. More generally, he (2007, p. 217) argues that modelling involves the interplay of imagination and model descriptions because the latter may need to be refined in response to failing to correspond with the former. One must find a tractable set of equations that manage to describe the target adequately. Model building is in an important sense a constructive activity (Sugden, 2000). Giere writes 'It is widely assumed that a work of fiction is a creation of human imagination. I think the same is true of scientific models. So, ontologically, scientific models and works of fiction are on a par. They are both imaginary constructs' (Giere, 2009, p. 249).

It is premature to take these concessions as supporting the Waltonian fictional view because the critics hail a view that is importantly different from that expressed by the proponents. But it is not merely different. I will now argue that the Waltonian fictional view cannot even accommodate the reflective comparison of imagination and model descriptions that Weisberg takes to be the most important insight of the fictional view.

As Weisberg argues, when creating a new model, a modeller must first formulate a mental image[8] about what she wants to represent with the model. If Weisberg is right about the role of imagination in modelling—and I think he is right (see also Morgan, 2004, 2012)—then the model descriptions can only be written down after the modeller has already imagined them. Imagination is thus necessary for modellers because they must find model descriptions that describe the phenomena in the right way. After having decided which phenomena they would like to account for, they figure out whether there are mathematical tools that could be applied to them. If modelling requires putting various different kinds of resources together, the mathematics may require some 'moulding'; figuring out whether the formal descriptions or their interpretations need to be modified when they are used in a new context (see Boumans, 1999). Imagination is thus needed for deciding how the phenomena could be represented with model descriptions. However, the Waltonian fictional view is unable to make sense of this. To see why, let us consider the contents of modellers' imagination in some more detail.

[8] I do not mean that the mental content has to be imagistic. I agree with the arguments in (Frigg and Salis 2020; Salis 2020a; Salis and Frigg 2020) that the content of the mental images may well be propositional.

There are three kinds of content of actual imagination. One consists of the characteristics of the real target that the model aims to account for: The *imagined target content*. The second concerns the modeller's conception about how such characteristics could be described with model descriptions: The *imagined model descriptions*. Recall that the indirect Waltonian account is based on interpreting model descriptions as props for imagination. The third kind of content of imagination is the model system, and imagination is constrained by what is prescribed by the model descriptions according to the principles of generation: The *imagined model system*. Unlike the two previous kinds, this one presupposes that the model descriptions are already at hand. Furthermore, unlike imagined model descriptions, Waltonian imagination requires imagining the content of a model description, not the description itself.

If there is to be learning from the practice of modelling, the contents of imagination must change in the modelling process. All the different imagined contents might change in modelling, but they change because of different processes. Genuine learning only occurs when the imagined target content changes in a justified and model-based way. It changes when the modeller derives results from the model, or from a similar model, or when her background information changes due to collecting or studying empirical data or studying other similar models. Imagined model descriptions change when the modeller studies the formal tools that could be applied in the model, or learns more about the target.

How, then, does an imagined model system change? Given that changing the model descriptions creates another prop and thereby a new imagined model system, such a change does not change any *given* model system. Thus, this cannot be how the imagined model system changes. According to the Waltonian fictional view, it changes when one employs the PIGs by applying laws or general principles to the model descriptions to derive the content of the model.

The Waltonian fictionalists might claim that if the imagined model system describes the content of a modeller's *actual* imagination, then there is also a fourth kind of content, that which is *prescribed* by the principles of generation. The modeller changes the actually imagined model system by bringing it closer to the prescribed model content by joining the principles of generation with the model descriptions. However, prescribed imagination is irrelevant as a description of modelling practice if its content does not correspond to actual imagination. This is why I only discuss *actual* imagination in this section.

The imagined model system could change if modellers and their audiences 'fill in the gaps' in model descriptions in their imagination (e.g., Friend, 2020). Here we must reject the strict interpretation of PIGs as laws and principles because such filling in the gaps gives some genuine leeway for interpersonal differences in the contents of imagination. For example, the Lotka-Volterra model can be presented as involving sharks and mackerels, or as foxes and rabbits, but presenting it as involving sharks and rabbits is presumably prohibited by the PIGs. However, insofar as such gap-filling PIGs do not affect the mathematical results of a model, they are epistemically irrelevant.

If, however, a model is applied in a new setting, or contextualised into a more specific setting, filling in the gaps does have a role in constraining the actual imagination of modellers. If the application or contextualisation requires new model descriptions,

coming up with them requires creative imagination constrained by the rule-bound filling in the gaps.

Nevertheless, I now argue that epistemically relevant changes in the imagined model system always require a change in the imagined model descriptions. Suppose, then, to the contrary, that filling in the gaps suggests an idea about how to modify a given model, and that this idea also results in new imagined model descriptions. Alas, it is possible that when such imagined model descriptions are written down, they turn out to be intractable, or that putting together the model components yields a result which is not compatible with what is known about the target. For these reasons, what ends up written down as model descriptions is often different from the modeller's imagined model at first. Alternatively, the modeller may not be able to write down the model descriptions at all because the mathematics is intrinsically intractable or just too difficult. In such cases, filling in the gaps does indeed guide modelling, but the results are not epistemically important in the end.

Recall from the previous section that when one 'studies the model system', all the epistemically relevant components are included in the model descriptions and all the relevant inferences and learning are in terms of the model descriptions. Inferences can only be made in an epistemically relevant way by manipulating the model descriptions or by introducing new model descriptions. The former is not a matter of imagination, creative or constrained. While the latter can be guided by constrained imagination via filling in the gaps, it is impossible to come up with new model descriptions without creative imagination because the currently known model descriptions do not carry their possible formal modifications on their sleeves.

Imagination, and thereby make-believe, is not needed *for making inferences* once the model descriptions have been written down, and this point holds irrespective of whether imagination is creative or constrained. The problem with taking model descriptions as props for imagination is thus that it forces one to presuppose that the model descriptions are already in existence when imagining starts, and thus that the imagined model system cannot change in an epistemically relevant way. Emphasising the difference between prescribed and actual imagination merely obfuscates the issue because it helps perpetuate ignoring the question of *when* something is actually imagined.

Let me illustrate the role of creative imagination in generating new model descriptions and applying existing ones with an example already discussed in Sect. 3. When Dixit and Stiglitz wrote the new functional form (i.e., new model description) that describes monopolistic competition in a general equilibrium framework, $\mathrm{U}\ x_0^{1-\mu} V^{\mu}$, $\mathrm{V}\left(\sum_{i=1}^{n} x_i^{\rho}\right)^{1/\rho}$, which appeared in their 1977 paper, they first had to 'hold this functional form in their head'. Put differently, this model description does not emerge on a piece of paper unless there is first a person who imagines that monopolistic competition can be described with such a functional form. New mathematical descriptions may also emerge without anybody imagining them if they are the result of derivation from model descriptions. However, these new mathematical descriptions are results rather than model descriptions.

When a model description, which is already known from previous publications, is applied in a different context and/or modified, modellers need to understand what the

model descriptions are committed to, and what their consequences are. Krugman had to learn about the properties of the Dixit-Stiglitz model in one way or another in order to apply it. But when he (1980) applied the Dixit-Stiglitz model into international trade by using the simplified form $V\left(\sum_{i=0}^{n} x_i^{\rho}\right)^{1/\rho}$, he no longer needed to creatively imagine that this functional form can be used for modelling monopolistic competition because Dixit and Stiglitz had already explained how a relevantly similar form does so. Here it is important that Krugman's functional form can be derived from Dixit and Stiglitz' form by setting μ=1. In my (Lehtinen, 2022) words, the Dixit-Stiglitz model has more expressive power than Krugman's model (concerning the general equilibrium properties of the former). Krugman merely needed to creatively imagine that the Dixit-Stiglitz form could be applied in the context of international trade, and then, after finding out that the original form is not tractable, simplify it with a degenerate version. In contrast, as soon as a model modification or application requires more rather than less expressive power, creative imagination is needed also for coming up with the new model descriptions.

Consider what modellers do when their model is ready enough to be presented to scientific audiences or published. Suppose thus that a modeller thinks that no further model descriptions are needed because she is sufficiently satisfied with the ones she has already written down. Furthermore, she has already derived some interesting results from the model.

What she now needs to do is communicate what the model contributes to an audience of other modellers and scientists. To succeed in this, she may need to explain how the different model components are to be interpreted (the assignment), and why certain features of the target were modelled with certain model descriptions. It may be useful to present things as if the model descriptions gave rise to a model system that the audience can compare to the reality. The model will be presented with sentences like 'imagine a population of…' or 'consider an economy with…' Such expressions invite the audience to imagine a model system described as if it were like the one given in the model descriptions. Although the audience can be said to need imagination in understanding the model, *this* constrained imagination is epistemically irrelevant in that the model no longer reveals any new results at this stage, and imagination is only needed by the academic community (cf. Levy, 2020). Its content may or may not correspond to the content of the modeller's imagination when she developed the model. There are thus two acts of imagination, one that the modeller engages in before writing the model descriptions, and one that the audience and the further developers engage in when they are already available.

We are now in a position to provide a second criticism of the Waltonian fictional model-content determination. It is logically impossible for constrained imagination to take part in determining the content of model descriptions if those very same model descriptions are supposed to be props for constrained imagination. I have argued above that while prescribed imagination via filling in the gaps can be a part of the process of determining the content of further models, new model descriptions can only be created with creative imagination. To put this differently, constrained imagination can take part in the modelling process, but it can never take part in what is ultimately epistemically relevant. Filling in the gaps is thus surely a good description of the

folk phenomenology of modelling, but it is highly misleading if it is taken to carry any epistemic weight. One can use prescribed imagination for thinking about the properties of the model when it has already been constructed. In the previous section I argued that the Waltonian fictional epistemology cannot be cogent because one cannot separate model descriptions from PIGs while making epistemically relevant inferences. These two criticisms of the Waltonian fictional epistemology of model content are independent of each other.

Levy's (2015) version of the direct fictional view is affected by the problem of constrained imagination in a different way because the prop for the imagination is the target rather than the model description. His account of the content of imagination captures the relevant epistemic issue better than the indirect view: Imagination concerns how a target could be depicted with particular model descriptions.[9]

However, the interpretation of model descriptions as props in a game of make-believe makes it incapable of making sense of external criticism of models (Thomasson, 2020). The direct view succumbs to this criticism too despite the different prop: If one is *prescribed* to imagine that the target is as the model descriptions depict it, this constrained imagination does not allow for an external evaluative perspective to regulate how the target ought to be modelled. In fairness to Levy, he does not emphasise the prescriptive side of Walton's account. If he would be willing to give it up entirely, the account could work: It would then be possible to accommodate the idea that modellers compare the properties of the target with those of the model in their actual imagination.

Levy's view also has some problems with imagination that are not shared with the indirect view. First, if the notion of a prop is interpreted strictly in Waltonian terms, it is unclear how exactly the target could be a prop in an authorised game of make-believe: Knowing what a target prescribes to imagine requires choices concerning what is relevant about the target. However, the targets themselves do not tell us how to choose their relevant properties. Consequently, it is unclear what the target prescribes us to imagine. Creative imagination is needed.

Second, in terms of the contents of imagination, Levy's view forces the imagined content of the target to merge with the imagined content of model descriptions, but this means that it cannot make sense of how modellers reflect on how the model descriptions fit the target—the aspect of modelling that Weisberg touted as an important insight in the fictional view.

Finally, Levy's oft-quoted slogan, 'all we have are targets, imaginatively described', is ambiguous. It means that the model descriptions are somehow providing such imaginative descriptions, and that the targets are as if they were like the model descriptions depict them. The expression 'imaginatively described targets' hides the difference between the modeller's and the audience's imagination, and thereby the fact that a given imaginative act cannot simultaneously be creative and constrained. The act of imagining, by the audience, a target as if it were like the model descriptions present it, is not the same act as the much more complicated creatively imaginative process that led the modeller to the model descriptions.

[9] This is not meant to be taken as a position on whether actual imagination typically concerns concrete objects or imaginary systems.

Another way to look at the problem is that once the model descriptions are written down, it no longer matters what kind of imagination was responsible for generating them, or whether indeed they were generated with imagination at all. They could be generated without any imagination like when a computer algorithm generates the model. If one feels uncomfortable with claiming that the computer-generated model 'imaginatively describes its target', it is because parts of the process of generating the model are known not to have involved imagination.

To sum up this section, the problem with the Waltonian fictional view is that constrained imagination can only be employed when the model descriptions are already at hand, just like a novel is already available when a reader starts imagining by using it as a prop, but at this stage imagination is no longer needed for epistemic purposes. This is why constrained imagination cannot provide a credible account of model construction (pace Salis 2020b). This problem also provides an additional argument against the Waltonian fictional model-content.

Note that different modellers are likely to have different contents of imagination concerning the model descriptions and about the targets. This kind of variation is not epistemically problematic, however. On the contrary, if we are to believe social epistemologists, scientific progress requires differences in what individual modellers imagine about model descriptions and targets. Consequently, seeking recourse in prescribed imagination to respond to Weisberg's intersubjective disagreement argument ends up providing a systematically misleading account of the role of imagination in the practice of modelling by denying the variety in the contents of imagination.

In response to my criticisms, the Waltonian fictionalists might argue that they do not necessarily need to reject the relevance of creative imagination in modelling. Although this is a possible stance, it does not change the fact that being able to account for the practice of modelling could not be an argument in favour of using constrained imagination in understanding modelling.

The face-value practice (Thomson-Jones, 2010) has been taken as an argument for the fictional view, as it implies that the modellers' talk about model systems as though they were real instantiated systems requires filling in the gaps in model descriptions in one's imagination. Constrained imagination does not even account for the face-value practice very well. Recall that this is the 'folk-ontological' fact that modellers describe 'missing systems' in full awareness that they are different from real systems. However, committing to prescribed imagination with laws as PIGs entails that all modellers have the same contents of imagination, and if they must be included in the model descriptions as I have argued, the Waltonian fictional view is no better in accounting for the face-value practice than defining models as sets of model descriptions (as in e.g., Odenbaugh, 2021).

We can now see that Weisberg must have acknowledged the adequacy of a fictional account in accounting for modelling practice merely because he was thinking about actual rather than prescribed imagination. I have tried to show that accounts relying on prescribed imagination cannot be adequate in this way because analysing prescribed imagination without surreptitiously presupposing actual imagination leads to problems in accounting for modelling practices.

The motivation for clinging to prescribed imagination derives from the Waltonian fictionalists' urge to respond to Weisberg's intersubjective disagreement criticism.

However, if prescribed imagination cannot determine the content of models for reasons independent from Weisberg's criticism, namely the ones discussed in the previous section, why not dispense with it altogether if responding to Weisberg's criticism is no longer called for? After all, Walton's own description of the application of PIGs allows for intersubjective disagreement even though the practice as a whole is rule-governed. Some concepts from Walton's theory could be used after dropping the shackles. For example, modellers' imagination about model descriptions is probably governed by the reality principle at least to some degree, but relying on it allows for intersubjective disagreement.

5 Model-target and inter-model comparisons

The Waltonian fictionalists have generally admitted that Walton's theory does not provide a convincing account of the relationship between the fictional truths in the model and the claims about the target: Comparing a model either with another model or with a real-world object involves elements that are not part of the authorised game of make-believe and hence are not covered by it (Frigg, 2021; Frigg & Nguyen, 2021c, p. 143). Yet Frigg (2021) opts to run with Walton's suggestion that we devise an unauthorised game of make-believe to make such comparisons.

Toon urges us to make inferences from the model to the system by taking what is fictional in the model to be true of the system (2012, 67). For example, we are to imagine that the prepared description and equation of motion about the pendulum are true of the system (p. 46). In other words, here Toon is arguing that one extends the fictional attitude also into the interpretation of the model in terms of the target. Ad hoc principles of generation may thus consist in simply proposing a convention to treat the model properties as being true about the target. Salis (2020a, pp. 76–7) presents an example couched in terms of ad hoc principles of generation. Here the ad hoc rule is given by combining the prescriptions given by the model descriptions with those given by prescriptions to imagine determined by the real target. This proposal is vulnerable to the same criticism as Levy's account of taking the target as a prop for the imagination; we don't know what it prescribes because we have to choose which properties are relevant. However, such ad hoc principles of generation are acceptable if the principles of generation can exhibit interpersonal differences (in which case they are rule-governed in a very weak sense). Given that the content of model-world comparisons (in an unofficial game of make-believe) is the object of imagination rather than belief (Salis 2020a), such comparisons are not intended to carry any epistemic weight. As a description of modelling practice, the ad hoc move is acceptable.

I have two objections to this move to unofficial games and ad hoc principles. First, one of the primary motivations for the Waltonian fictional view is to make sense of the face-value practice where the model descriptions are known not to be satisfied by any real systems (e.g., Friend, 2020 puts it in this way). Salis and Frigg (2020) argue that when scientists invite us to suppose something, they typically invite us to imagine something without any commitment to its truth. I agree with this, but one cannot eat one's cake and have it too: it is not acceptable to adopt such an attitude when one

considers which characteristics of the model may be claimed to be true about the target.

What Salis (e.g., 2019, 2020a) calls 'direct attributions' are interpreted in terms of truth. However, since there are no clear rules for when a modeller is supposed to play a given game of make-believe (Thomasson, 2020), it is impossible to tell when one is supposed to take model-target comparisons as bona fide claims that are intended to have epistemic weight: The Waltonian fictional view does not provide criteria for which fictionally true claims in the model are to be interpreted as direct attributions.

The Waltonian fictionalists might appeal to the idea that there are different stages in model development in response to this criticism: the fictional attitude is taken when the model is developed, and the comparison between the model and the target is only considered when the modellers have worked out the details (e.g., Frigg & Nguyen, 2021c; Frigg & Salis, 2020; Frigg, 2010a). I agree that there are moments in modelling when one may disregard the relationship between the model and the target, such as calculating the results with mathematics. However, such deduction constitutes only a part of model-development, and all the rest of the epistemically relevant activity requires modifying the assumptions. But modifying the assumptions is not and cannot be done without considering whether they correspond to the target.[10] The model descriptions include idealisations and abstractions, but modellers do not propose them with a fictional attitude: they make choices between different model descriptions based on several different criteria, but they do not ignore their truth-values. Furthermore, taking a fictional attitude at an early stage is not compatible with other claims by the Waltonian fictionalists: when modellers figure out which model descriptions to use in the early stage, they cannot be constrained by model descriptions because they do not yet exist.

Second, and much more importantly, from an epistemic point of view, the ad hoc gambit is simply unacceptable. Consider Cartwright (2010):

> How can a result that must occur given characteristics different from those in the target inform conclusions about what will happen in the target? The conclusion is supposed to be guaranteed because it follows deductively from the premises. How does that provide information about what conclusions to expect when the premises are different?

Cartwright's point is that insofar as the assumptions are false, model-results are not epistemically acceptable just because one can derive them. As argued above, demonstrating the robustness of the model results, and de-idealising them provide epistemically relevant information about the conclusions when the 'premises are different'.

The literature on model representation has concentrated on studying 'epistemic representations' (Contessa, 2007). Despite the term, such representations are usually defined in such a way that they allow for learning something about the targets, but not necessarily learning truths about the targets (e.g., Frigg & Nguyen, 2021a). Cartwright's point means that this is setting the bar too low for the epistemology of modelling: Only true assumptions guarantee that one is learning truths about the

[10] Portides (2014) makes a similar claim (see also Poznic 2018; Yablo 2020).

world. Robustness analysis, de-idealising models, and allocating confirmation to individual model components are research activities devoted to evaluating the relevance and truth-values of the assumptions. The fictional account is not able to make sense of these activities because they require epistemically relevant comparisons between the model and the target, and between partly different models. For model-model comparisons, it is difficult even to tell what the ad hoc principles of generation could possibly be, because imagining that what is fictional in a model is true of the target makes no sense if one is comparing two incompatible models about the same target.

6 Conclusions

The Waltonian fictional account identifies the content of a model with Walton's notion of truth in fiction. This account fails even in physics because laws and principles must be part of the model descriptions to begin with. The main reason for this is that modellers do not accept model results without an explicit proof. Hence, while there are certainly implicit elements in modelling, model results cannot be implicit. Since modelling in the special sciences typically does not employ laws and principles, a different account of content is needed. The proposed epistemic notion of content is based on determining the model content by modifying the model descriptions in various ways. The Waltonian fictional account cannot account for these practices in a credible way because the only way of discussing comparisons of models within a model family is using ad hoc principles of generation—an obviously inappropriate concept for this purpose.

Applying the Waltonian view into modelling confuses the role of imagination in modelling because it takes model descriptions as props for the imagination. Requiring imagination to be constrained by the model descriptions and principles of generation makes it impossible to make sense of the epistemically relevant parts of modelling practices: Constructing new models, modifying them, and determining their content. Important learning with models can only occur via changes in model descriptions, but the Waltonian fictional view has no way of accounting for such changes because imagination is supposedly employed only when the model descriptions are already written down.

Earlier critics have urged the proponents to provide a more elaborate account of the principles of generation (Poznic, 2016), or a more extensive epistemology for the Waltonian fictional view more generally (Friend, 2020). I would rather reject it altogether because the account is based on notions borrowed from the theory of fiction that, upon closer analysis, are forced into a Procrustean bed when applied to scientific modelling. In particular, PIGs cannot be simultaneously shared, implicit, agreed, exhibit interdisciplinary differences, and account for epistemically relevant inferences from models. PIGs surely capture various features of modelling, but the problem is precisely that they capture too many features under one concept but there is no concept in modelling practice that corresponds to it.

It would make sense to seek inspiration from a theory of art if the adopted concepts could be applied without major modification into modelling, or if the concepts were successfully used for a very specific purpose. But even the most limited application of

determining the content of a model fails. Since the required modifications are major, it is better to reject the Waltonian fictional view entirely. The Waltonian fictional view does not provide a credible account of the epistemology of modelling because the key concepts of props, PIGs and prescribed imagination obfuscate instead of clarifying what is going on in modelling. Rejecting it removes unnecessary shackles, and prompts philosophers to study the modelling practices themselves.

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