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Strategic Voting and the Degree of Path-Dependence

Aki Lehtinen

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Abstract This paper generalises Enelow (J Polit 43(4):1062–1089, 1981) and Lehtinen's (Theory Decis 63(1):1–40, 2007b) model of strategic voting under amendment agendas by allowing any number of alternatives and any voting order. The generalisation enables studying utilitarian efficiencies in an incomplete information model with a large number of alternatives. Furthermore, it allows for studying how strategic voting affects path-dependence. Strategic voting increases utilitarian efficiency also when there are more than three alternatives. The existence of a Condorcet winner does not guarantee path-independence if the voters engage in strategic voting under incomplete information. A criterion for evaluating path-dependence, the degree of path-dependence, is proposed, and the generalised model is used to study how strategic voting affects it. When there is a Condorcet winner, strategic voting inevitably increases the degree of path-dependence, but when there is no Condorcet winner, strategic voting decreases path-dependence. Computer simulations show, however, that on average it increases the degree of path-dependence.

Keywords Strategic voting · Path-dependence · Amendment agendas

1 Introduction

Social choices are path-dependent if the outcome of voting depends on the order in which the alternatives are presented for consideration. The existence of a Condorcet winner, an alternative that has a majority against all other alternatives, guarantees that social choices are path-independent under amendment agendas if voters act strategi-

A. Lehtinen (✉)
Department of Political and Economic Studies, University of Helsinki,
P.O. Box 24, 00014 Helsinki, Finland
e-mail: aki.lehtinen@helsinki.fi

cally under complete information (McKelvey and Niemi 1978) or if they vote sincerely (Farquharson 1969). For these reasons, path-dependence is commonly thought to be closely connected to the cyclicity of the preference profile. Some authors have even proven results according to which cyclicity is a necessary and sufficient condition for path-dependence (List 2004). However, cyclic preferences are not necessary for path-dependence under incomplete information because the Condorcet winner is not necessarily selected under amendment agendas (Ordeshook and Palfrey 1988). There are thus cases in which social choices are path-dependent even though there is no preference cycle, and strategic voting may exacerbate the problem of path-dependence.

On the other hand, strategic voting may alleviate the problem of path-dependence when there is a preference cycle. The reason for this possibility is related to Lehtinen's (2007b) results on the consequences of strategic voting: it typically increases the chances that the *utilitarian winner*, the alternative with the largest sum of utility, is selected. If voters' preferences cycle over all alternatives, sincere voting yields different outcomes under all agendas that introduce a different alternative at the last round of voting, but strategic voting may lead to fewer different outcomes because it increases the chances that one particular alternative, the utilitarian winner, is selected under many agendas.[1] Strategic voting cannot eliminate path-dependence altogether, however, because the utilitarian winner will inevitably lose against some alternative in the last round of voting unless it also is a Condorcet winner.

Given that strategic voting may both increase and decrease the degree to which social choices are path-dependent, it is natural to ask which tendency is more important. This question is here investigated by comparing the extent to which social choices are path-dependent when they engage in strategic voting and when they vote sincerely. Conducting such an investigation with a formal model requires formulating a criterion for the *degree of path-dependence*, and constructing a model of strategic voting that can be applied to any number of alternatives and any (amendment) agenda.

The analysis of strategic voting is based on a computer simulation framework introduced by Lehtinen (2007b) and a model of incomplete information via signal extraction presented in Lehtinen (2006). The welfare consequences of strategic voting are studied by comparing the utilitarian efficiency under *Expected Utility maximising behaviour* (EU behaviour) and *Sincere Voting behaviour* (SV behaviour). Under the latter behavioural assumption, all voters always vote sincerely, and under the former they vote sincerely or strategically depending on their expected utilities. The incomplete information model is based on the idea that voters obtain a perturbed signal on other voters' preference orderings or utilities. Generalising Lehtinen's model to any number of alternatives also allows investigating whether strategic behaviour increases utilitarian efficiency when there are more than three alternatives. Given that the simulation framework and the signal extraction model are extensively discussed

[1] There is a sense in which the vast literatures on the uncovered set (Miller 1977) and the Banks set (1985) study related questions: they endeavour to determine the set of possible outcomes under agendas when voters are strategic. Given, however, that these approaches are based on complete information, they will not be further discussed here.

and justified in Lehtinen (2006, 2007b), this paper endeavours to justify only those modelling choices that pertain to generalizing the model.[2]

The structure of the paper is the following. Section 2 provides a description of a modification of Enelow's (1981) model, on which Lehtinen's model is based. Section 2.1 shows two examples of path-dependence in order to give some intuition on the logic of the model. The generalisation to any number of alternatives is based on an indexing system (based on 'ordering numbers') for pairwise contests. The details of the indexing system are of interest mainly to those who are themselves interested in constructing similar computer simulations, and its description is relegated to an appendix which is published only online.[3] Since Enelow and Lehtinen's model was limited to three alternatives, voters did not need to take other voters' strategic behaviour into account. Section 3 describes how this can be done by remodelling voters' signals. Section 4 describes how the simulation framework needs to be adjusted so as to be able to study any number of alternatives: Lehtinen's (2007a, 2007b, 2008) main result is that strategic voting behaviour generates higher utilitarian efficiencies than sincere behaviour. These results are most salient when some particular alternatives are commonly considered to be acceptable, i.e., when they have higher average utilities than other alternatives even though the preference profile is created with the anonymous impartial culture assumption. Section 4 shows how to generate such computer simulation setups with more than three alternatives: the utility of one alternative (alternative 1) is increased at the expense of others without changing voters' preference orderings. Section 5 formulates a mathematical expression for the degree of path-dependence. Section 6 shows the simulation results.

2 A Model of Strategic Voting

This section explains how Enelow (1981) and Lehtinen's (2007b) model of strategic voting under amendment agendas can be generalised. Under *an amendment agenda*, two alternatives are put to a majority vote against each other in a first round of voting.[4] The winner of this first contest is then put to vote against the third alternative in a second round, and so on.

Let $X = \{1, 2, 3, \ldots, n\}$ denote the set of available alternatives, U^i voter i's payoff function, and $I = \{1, 2, \ldots, i, \ldots, N\}$ a set of voters. Let us consider the case of four

[2] Although the indexing methods apply with literally any number of alternatives, in practise, given that the model is based on computer simulation, the results can be computed with 8 alternatives at most if one runs all the parameter values in one simulation. Some results are reported with 10 alternatives below. With 11 or more alternatives, the combinatorial explosion becomes unbearable for ordinary supercomputers. For example, there are n!/2 different agendas. The number of different agendas is 2520 with 7 alternatives. With 10 alternatives, there are 3628800 different agendas and with 11, the number is 39916800. With almost 80 million different preference orderings, the matrices seem to become too big for the memory of the supercomputer I am using.

[3] The appendix can be downloaded from http://www.mv.helsinki.fi/home/alehtine/. Upon request, I can provide a reasonably well-documented FORTRAN code. In order to run the simulations, one needs access to a fairly new supercomputer and to IMSL libraries.

[4] See Ordeshook (1986), Ordeshook and Schwartz (1987) and Miller (1995) for a discussion of different agendas.

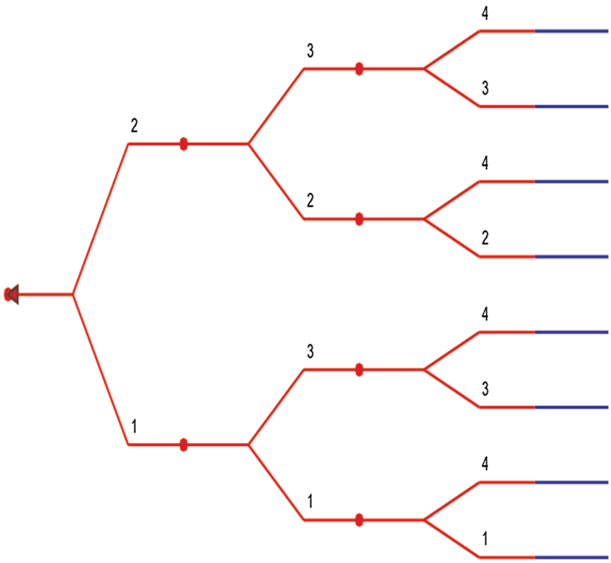

Fig. 1 A voting tree

Table 1 Preference orderings with four alternatives

Voter type	1	2	3	4	5	6	7	8	9	10	11	12
U_1^i	1	2	1	3	3	2	1	2	1	4	4	2
U_2^i	2	1	3	1	2	3	2	1	4	1	2	4
U_3^i	3	3	2	2	1	1	4	4	2	2	1	1
U_4^i	4	4	4	4	4	4	3	3	3	3	3	3
Voter type	13	14	15	16	17	18	19	20	21	22	23	24
U_1^i	1	4	1	3	3	4	4	2	4	3	3	2
U_2^i	4	1	3	1	4	3	2	4	3	4	2	3
U_3^i	3	3	4	4	1	1	3	3	2	2	4	4
U_4^i	2	2	2	2	2	2	1	1	1	1	1	1

alternatives as an example. Figure 1 shows an amendment agenda with four alternatives (Table 1).

In what follows, I will call different voting orders simply *agendas*, and denote the agenda shown in Fig. 1 as (1234). If, say, 3 is first put to a vote against 4, then the winner of this contest against 2, and the winner of this contest against 1, the agenda will be denoted (3421). With four alternatives, there are $4! = 24$ different *types of voters*.

Let p^i_{jk} denote voter i's subjective probability that alternative j beats k ($j, k \in X$) in a pairwise contest. Maximising expected utility implies giving one's vote for that branch in the voting tree that has the greatest expected utility. In the case of three

Table 2 An example of path-dependence with a Condorcet winner

A	B	C
2 (1)	2 (1)	1 (1)
1 (0.9)	1 (0.9)	3 (0.9)
3 (0)	3 (0)	2 (0)

alternatives under agenda (123), voters compare two lotteries $(1, 3; p^i_{13}, 1 - p^i_{13})$ and $(2, 3; p^i_{23}, 1 - p^i_{23})$. A vote is given to the branch of the voting tree containing alternative 1 if

$$p^i_{13}U^i_1 + (1 - p^i_{13})U^i_3 \geq p^i_{23}U^i_2 + (1 - p^i_{23})U^i_3. \tag{1}$$

With four alternatives, in order to obtain an expression for the incentives of voting in the first round, the utilities for the winners in the second round need to be replaced with expected utilities in the third round. Dropping the superscripts denoting the individuals, we see that U_1 must be replaced with $p_{14}U_1 + (1 - p_{14})U_4$, U_2 with $p_{24}U_2 + (1 - p_{24})U_4$, and U_3 with $p_{34}U_3 + (1 - p_{34})U_4$. The condition for voting for the lower branch in the first round with four alternatives is thus

$$\begin{aligned} &p_{13}\,[p_{14}U_1 + (1 - p_{14})U_4] + (1 - p_{13})\,[p_{34}U_3 + (1 - p_{34})U_4] \\ &\quad \geq p_{23}\,[p_{24}U_2 + (1 - p_{24})U_4] + (1 - p_{23})\,[p_{34}U_3 + (1 - p_{34})U_4]\,. \end{aligned} \tag{2}$$

Enumerating all formulas for expected utilities for different branches in a voting tree quickly becomes cumbersome as the number of alternatives increases. Furthermore, since the winner in each pairwise comparison must be found, and this requires computing expected utilities at each node of the voting tree, a general indexing method for denoting the nodes and the corresponding probabilities and expected utilities is needed. *Ordering number*s provide the indexing method.

2.1 Two Examples

Let us now use the model in order to obtain some intuition about path-dependence by way of two examples. Assume that the preferences of three voters A, B, and C, for alternatives 1, 2 and 3 can be described with Table 2.

The numbers in parentheses denote voters' utilities. 1 is the utilitarian winner and 2 the Condorcet winner. Assume first that the agenda is (123). If all voters engage in sincere voting behaviour, the Condorcet winner 2 will beat 1 in the first round and 3 in the second round, and emerges as the final outcome. Suppose now that voters maximise expected utility under incomplete information. Assume that all three voters have identical beliefs such that $p_{23} = 0.7$, and $p_{13} = 0.9$. The voters thus believe that it is likely that 2 beats 3, but even more likely that 1 beats 3 in the last round. These beliefs are fairly 'reasonable' because both 1 and 2 beat 3 if they survive the first round of voting. Furthermore, 1 beats 3 by three votes to zero, and 2 beats 3 by two votes to one. These beliefs are of course essentially plucked out of nowhere,

Table 3 An example of path-dependence

A	B	C
1 (1)	2 (1)	3 (1)
2 (0.9)	3 (0.5)	1 (0.1)
3 (0)	1 (0)	2 (0)

but voters might well have such beliefs if they derived from the model of incomplete information described in the next section.

Voters A and B vote sincerely for 2 in the first round if $U_1 < \frac{p_{23}}{p_{13}}$ (i.e. if $0.9 < \frac{0.7}{0.9} = 0.7778$). Since this is untrue, A and B will vote strategically for 1 in the first round of voting. Voter C has a weakly dominant strategy to vote for 1 in the first round of voting. 1 is thus the outcome because it beats 2 in the first and 3 in the second round. The utilitarian winner 1 is chosen if voters maximise expected utility, but the Condorcet winner 2 is chosen if all voters vote sincerely. We may conclude that a Condorcet winner is not necessarily chosen under amendment agendas. Note, however, that the Condorcet winner is always selected under some agenda because if it enters the voting in the last round, it beats any other alternative.

Cyclic preferences are thus not a necessary condition for path-dependent social choices, but they are sufficient. Consider the payoffs displayed in Table 3. Assume that $p_{12} = 0.55$, $p_{13} = 0.3$ and $p_{23} = 0.6$ for all voters. You may verify that even though alternative 2 would be the outcome under agendas (123) and (132), it could not win under agenda (231) because it faces 1, which has a majority against it, in the second round. Similar reasoning applies under other agendas. If the preferences are cyclic, there is always an agenda in which an alternative is not selected.

The second example also shows how strategic voting may lead to a lower degree of path-dependence than sincere voting. Note that EU behaviour resulted in two different outcomes, whereas sincere voting yielded a different outcome in all three agendas.

3 Extending the Signal Extraction Model

Lehtinen (2007b) presented a model of signal extraction under amendment agendas. Since that model only featured three alternatives, voters did not need to take other voters' strategic behaviour into account. The only question of interest was which of two alternatives will win a pairwise contest in the *last* round of voting in which voters no longer have an incentive to vote strategically. With four or more alternatives, voters also need to take such behaviour in intermediate rounds into account. They are not assumed to have any knowledge on the behavioural propensities of other voters. However, as Lehtinen (2007b) explains, intensively preferred alternatives are likely to obtain most strategic votes. Each individual voter thus can take the consequences of other voters' strategic behaviour into account if they obtain perturbed information on aggregate-level differences in preference intensities. Voters are not assumed to have any knowledge about *individual* preferences or behavioural propensities, but they can nevertheless take into account other voters' strategising if they have such perturbed aggregate-level information. Thus, a natural way of modelling the idea that voters take other voters' behaviour into account is by assuming that they obtain signals concerning the difference in the *sum of utility* between *each pair* of candidates.[5] There are thus two kinds of signals: those that concern the last round in which voters no longer have an incentive to vote strategically, and those that concern the rounds from the second to the penultimate one. The former contain perturbed information on preference orderings whereas the latter contain perturbed information on aggregate preference intensities (i.e. sums of utilities). In order to determine voters' behaviour in the first round, all the probabilities for all voting rounds must be available. Furthermore, there is no updating of probabilities in between the voting rounds. The model is thus not based on Bayesian updating. This assumption may be justified with an appeal to the computational constraints. The fact that the probability that any alternative j beats any other alternative k is the same irrespective of the voting round (except the last) saves computing time and memory tremendously. However, one could also argue that voters actually cannot learn other voters' utilities from their behaviour. Representatives in parliaments could do so only if they knew about differences and similarities in the content of the various bills. The present model is based on the assumption that they do not have such information about differences and similarities in content.

A *simulated election g* consists of a set of utilities created by a random number generator, beliefs based on these utilities and orderings, voters' perturbed signals, and voting outcomes under the different behavioural assumptions. All the variables are defined for a given simulated election g, but I will omit an index denoting the election in order to avoid unnecessary clutter.

3.1 Signals for the Last Round of Voting

In the last round, voters do not have an incentive to vote strategically. Given that the beliefs for the last round are the same as in Lehtinen (2007b), only a very brief account is given here. Let $\succ_i$ denote voter i's preference relation. Let $n(j \succ k)$ denote the number of voters who prefer alternative j to alternative k in simulated game g. Voters are assumed to obtain a randomly perturbed signal on $n(j \succ k)$. Using a standardised variable $Q(j \succ k) = \frac{n(j \succ k) - Np}{\sqrt{Np^2}}$ (where p denotes the probability that the Bernoulli trial $j \succ_i k = 1$) allows constructing signals for which reasonable values of the perturbances are independent of the number of voters. If the number of voters is at least about 30, the variable $Q(j \succ k)$ asymptotically approximates the standard normal distribution. Since the impartial culture implies that $p = \frac{1}{2}$, a *signal* of voter i concerning the preferences for j and k can be written as

$$S_i(j,k) = \frac{2n(j \succ k) - N}{\sqrt{N}} + \varepsilon \cdot r_i\,(j,k)\,, \tag{3}$$

[5] Note that the model does not require that voters are able to make interpersonal comparisons in order to formulate their signals. The difference in the sums of utilities is used as a rough proxy for estimating other voters' behaviour. Lehtinen (2008) also provides a signal model for plurality and approval voting which is based on sums of utilities. I refer to this paper for further justifying arguments for this assumption.

where $r_i\,(j,k)$ is a realization of an i.i.d. standard normal random variable, and ε is a scaling factor that reflects the *reliability* of the signals. The smaller ε is, the more reliable voters' signals are. In this paper, voters are assumed to know the reliability of their signals. Lehtinen (2006) shows that voters' beliefs can be derived from such signals. They are given by Eq. (4).

$$p^i_{jk} = 1 - \Phi\left(\frac{-S_i(j,k)}{\varepsilon\sqrt{1+\varepsilon^2}}\right), \tag{4}$$

where Φ denotes the standard normal probability density function.

3.2 Signals for Voting Rounds from the Second to the Penultimate

Let $\Delta_i(j,k) = U_i\,(j) - U_i\,(k)$, and $\Delta(j,k) = \sum_{i=1}^{N} \Delta_i(j,k) = U\,(j) - U\,(k)$. A signal consists of the difference in the sum of utility $\Delta(j,k) = U\,(j) - U\,(k)$ and a random term $\varepsilon r_i(j,k)$. A signal for a contest before the last round is given by:

$$S_i(j,k) = U\,(j) - U\,(k) + \varepsilon r_i(j,k), \tag{5}$$

The standardized variable $Q\,(j,k) \sim N(0,1)$ is now given by:

$$Q\,(j,k) = \frac{\Delta\,(j,k) - N \cdot E\,[\Delta\,(j,k)]}{\sigma_\Delta\sqrt{N}} = \frac{\Delta\,(j,k)}{\sigma_\Delta\sqrt{N}} = \frac{U\,(j) - U\,(k)}{\sigma_\Delta\sqrt{N}}, \tag{6}$$

where σ_Δ is the standard deviation of the variable $\Delta_i\,(j,k)$ and E is an expectation operator.[6] Calculating the probability that candidate j beats candidate k $(p_i(jBk))$, given a signal $S_i(j,k)$, requires knowing the variance of Δ_i. "Appendix" at the end of the paper shows that the standard deviation of Δ_i is $\sigma_\Delta = \sqrt{\frac{1}{6}}$, and the utility-based signal is thus given by

$$S^u_i(j,k) = \frac{U\,(j) - U\,(k)}{\sqrt{\frac{N}{6}}} + R_i\,(j,k)\,. \tag{7}$$

The corresponding probabilities are derived by applying Eq. (4). In order to study the effects of including intensity information in the signals, composite signals were used. Let $\lambda \in [0,1]$ denote the relative share of utility information in the signals. A *composite signal* consists of a combination of preference and utility information, and a random term:

$$S^\lambda_i(j,k) = \lambda\frac{U\,(j) - U\,(k)}{\sqrt{\frac{N}{6}}} + (1-\lambda)\frac{2n(j \succ k) - N}{\sqrt{N}} + R_i\,(j,k)\,. \tag{8}$$

When $\lambda = 0$ the probabilities are based only on preference ordering information.

[6] It is obvious that $E\,[\Delta\,(j,k)] = 0$.

4 Simulation and Setups

A setup is a set of assumptions used in a set of G = 1000 simulated games. The number of voters was 201. Expected utility setups differ with respect to the reliability of voters' signals (ε) and the degree of correlation between voter types and preference intensities (C). In *uniform* setups voters' utilities are drawn from a uniform distribution on [0,1] The simulations were thus based on the impartial anonymous culture assumption: each voter type is equally likely (see Regenwetter et al. 2006). In setups with intensity correlation the preference orderings remain the same, but the utility of alternative 1 is increased and the utilities of all other alternatives decreased.

In order to generate such setups without affecting the interpersonal comparisons or the preference orderings, the individual utilities were derived as follows. $U_1, U_2, \ldots, U_n$ were first generated from the uniform distribution on [0,1] for each voter. U_1 and U_n were then used for defining the voter's utility scale as the $[U_1,U_n]$ interval. The utility for alternative 1 was then increased, and the utilities of all other alternatives decreased. For the sake of clarity, however, let us temporarily denote the alternatives by letters x, y, z, and w in the example presented below, and increase the utility of x at the expense of the other alternatives. Suppose, for example, that voter i had the following utilities: $U_x = .55$, $U_y = .80$, $U_z = .05$, and $U_w = .52$ (i.e., the voter's ranking is yxwz). These utilities define 'scales' that express the difference in utility between the alternatives. If K_{jk} denotes the scale between the j'th best and the k'th best alternative, we have $K_{12} = .25$, $K_{23} = .03$, and $K_{34} = .47$. K_{12} now expresses how much the utility of x can be increased without changing the preference ordering. Let the starred variables denote their values after the conversion. The utility of x is now increased by setting $U_x^* = U_x + (1 - C)K_{12}$. Thus, if for example, C = .5, we get $U_x^* = .55 + (1 - .5).25 = .675$. If x is already the most preferred alternative, we use the scale between it and the second-best alternative. The scales are then redrawn such that K_{12}^*=.125, $K_{23}^* = .155$, and $K_{34}^* = .47$. Then the utility of alternative y is decreased by setting $U_y^* = U_y - (1-C)K_{12}^* = .8 - (1 - .5).125 = .7375$ and the utility of alternative w is decreased by setting $U_w^* = U_w - (1-C)K_{23}^* = .52 - (1 - .5) * .47 = .285$. U_z is finally decreased using the scale K_{34}^* into $U_3^* = .05 - (1 - .5) * .47 = -.185$. A similar conversion of utilities is conducted for all voters. Table 4 provides a tabular representation of the various steps: the conversions begin at the top and end at the bottom of this table. As a result of such conversions, the utilities no longer remain in the [0,1] interval. Furthermore, the average utility shrinks to well below 0.5 because the utility of only one alternative is increased, but the utility of several others is diminished. This means that the results on average utilities in simulations as reported in Lehtinen (2007b) would not have been comparable to those that would have been derivable from the present model. This is why this paper presents results on utilitarian efficiency rather than average utility. The utility conversions presented here provide a simple way of increasing the popularity of one alternative, but it is easy to imagine other ways in which voters intensities could be unevenly distributed. For example, one could increase or decrease the utility of two alternatives rather than one, or increase the utility of x more than that of y, and so on. Studying these further possibilities is surely warranted but given the length limitations, must be left to future work.

Table 4 Modifying utilities

Initial utilities		Re-ordered	
U_x	.55	U_y	.80
U_y	.80	U_x	.55
U_z	.05	U_w	.52
U_w	.52	U_z	.05
Scales	From	Result	
K_{12} (y vs. x)	.80–.55	.25	
K_{23} (x vs. w)	.55–.52	.03	
K_{34} (w vs. z)	.52–.05	.47	
New U_1	Equation	Example	
U_x^*	$U_x^* = U_x + (1 - C)K_{12}$	$.55 + (1 - .50) * .25 = .675$	
New scales	From	Result	
K_{12}^*	.80–.675	.125	
K_{23}^*	.675–.52	.155	
K_{34}^*	.52–.05	.47	
New utilities	Equation	Example	Final utilities
U_x^*	.675	.675	.675
U_y^*	$U_y^* = U_2 - (1 - C)K_{12}^*$	$.80 - (1 - .5) * .125$	.7375
U_z^*	$U_z^* = U_2 - (1 - C)K_{34}^*$	$.05 - (1 - .5) * .47$	−.185
U_w^*	$U_w^* = U_2 - (1 - C)K_{23}^*$	$.52 - (1 - .5) * .155$	.285

5 Path-Dependence

To the best of my knowledge, path-independence (Plott 1973) in voting has only been studied in complete information settings.[7] This is why the existence of a Condorcet winner is often considered sufficient for path-independent social choices. Hammond (1977) shows that a social welfare functional satisfies a condition which is related to path-independence (metastatic consistency) if it satisfies Arrow's Independence of Irrelevant Alternatives. The result means that there is a close relation between ordinal choice and path-independence. The existence of a Condorcet winner does not guarantee path-independence under incomplete information, however. It is thus natural to investigate how important the existence of a Condorcet winner is for path-independence under incomplete information.

5.1 The Degree of Path-Dependence

I will now present a criterion for the *extent* to which social choices depend on the order of voting: the *degree of path-dependence*. When the number of alternatives is

[7] See the special issue on path-dependence in *Political Analysis* 2012, 20(2) for incomplete information accounts of path-dependence in non-voting contexts.

n, there are $\frac{n!}{2}$ *different* binary agendas and thus $\frac{n!}{2}$ different voting orders.[8] Each simulated election g has a fixed preference profile, but the outcomes may be different under different agendas. Let a_k^g denote the number of agendas in which alternative k is the outcome in the simulated election g. Let f_k^g denote the relative frequency of agendas in which alternative k is the outcome in a given simulated election g, i.e. the ratio between the number of agendas in which k is the outcome and the number of behaviourally different agendas:

$$f_k^g = \frac{a_k^g}{\left(\frac{n!}{2}\right)}. \tag{9}$$

The *degree of path-dependence* in a simulated election g is one minus the sum of squared relative frequencies for each of the alternatives

$$1 - \sum_{k=1}^{n} \left(f_k^g\right)^2.$$

The degree of path-dependence obtains its theoretical maximum when each alternative wins in an equal number of agendas: $1 - \frac{1}{n} = \frac{n-1}{n}$, and the theoretical minimum when one alternative wins under all agendas, i.e., when voting is path-independent. The theoretical minimum is always zero but the theoretical maximum depends on the number of alternatives. We will be more interested in the *average degree of path-dependence*:

$$DPD = 1 - \frac{1}{G}\sum_{g=1}^{G}\sum_{k=1}^{n} \left(f_k^g\right)^2. \tag{10}$$

Given that this functional form has been already applied in order to study homogeneity in various different fields, I take its use to be self-evidently justified.[9]

6 Simulation Results

6.1 Utilitarian Efficiencies

Utilitarian efficiency is defined as the percentage of simulated elections in which the utilitarian winner is selected. Figures 2 and 3 display utilitarian efficiencies with four

[8] The total number of agendas n! is divided by 2 so as remove behaviourally indistinguishable agendas. For example, agenda (123) is behaviourally identical to (213) because alternatives 1 and 2 are put to a vote against each other in the first round. Only the labelling of the alternatives is different. Each pair of alternatives thus always has two behaviourally indistinguishable agendas.

[9] Corrado Gini was one of the first to apply it for studying the inequality of the distribution of income. Carnap (1952, pp. 65–68) discussed it under the name ‘degree of order’. Carnap also used the terms ‘homogeneity’ and the ‘uniformity of the world’. Patil and Taillie (1982) show various applications (e.g., biodiversity, industrial concentration) for the functional form used here and related forms. Political scientists are perhaps best acquainted with the so called ‘effective number of parties’ (Laakso and Taagepera 1979).

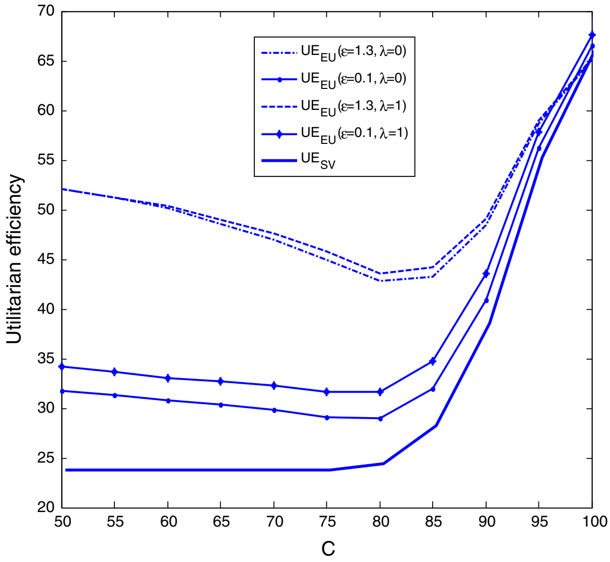

Fig. 2 Utilitarian efficiencies with n = 4

and six alternatives, respectively. These two figures also show how the quality of voters' information (ε) and the share of intensity information in the signals (λ) affect utilitarian efficiencies. The results were derived with $\varepsilon = .1, .5, .9, 1.3$, $\lambda = 1, .8, .2, 0$, and C = 0.5, …, 1.[10] They will be displayed only for the extreme values in order to maximise their informativeness. It seems clear that strategic voting continues to increase utilitarian efficiencies also when there are more than three alternatives. Furthermore, under EU behaviour the utilitarian efficiencies only have a slight tendency to decrease as the number of alternatives increases. Note that the utilitarian efficiencies were here calculated as averages over all different agendas. In other words, these results also mean that Lehtinen's (2007b) result that strategic voting increases utilitarian efficiency under amendment agendas does not depend on using a particular (123) agenda. Given that utilitarian efficiencies are consistently higher when the signals are unreliable ($\varepsilon = 1.3$) than when they are reliable ($\varepsilon = .1$), it seems that more exact information is usually harmful. The primary explanation for this is that with $\varepsilon = .1$ voters' have almost complete information, and under these circumstances, their preference intensities have very little effect on their behaviour. Finally, as expected, utilitarian efficiencies are higher when voters signals contain intensity information with $\varepsilon = 1.3$. When voters' signals are highly reliable, however, this result is reversed. The reason for this is that their signals would be correct in a world in which preference intensities affect voters' behaviour. Here, however, they mostly depend on preference orderings.

Let F_1 denote the percentage of voters who vote strategically for alternative 1 in a simulation setup, and A_1 the percentage of voters who would sincerely vote for 1 but

[10] The figures report C values in the range of 50–100 because parameter C was implemented with another parameter b in the computer code: b = 100 ∗ C.

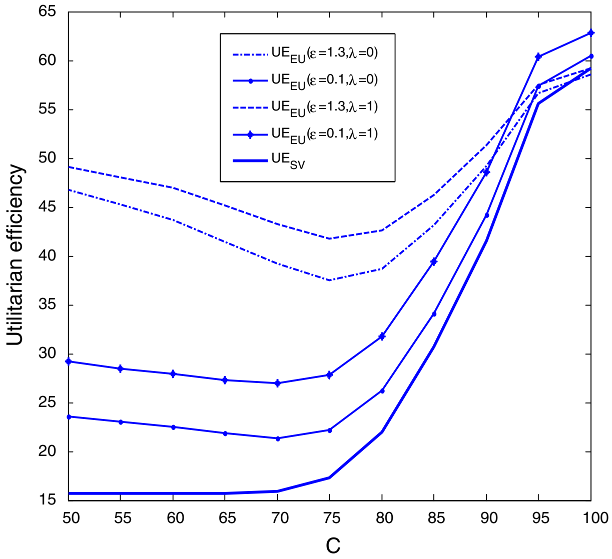

Fig. 3 Utilitarian efficiencies with n = 6

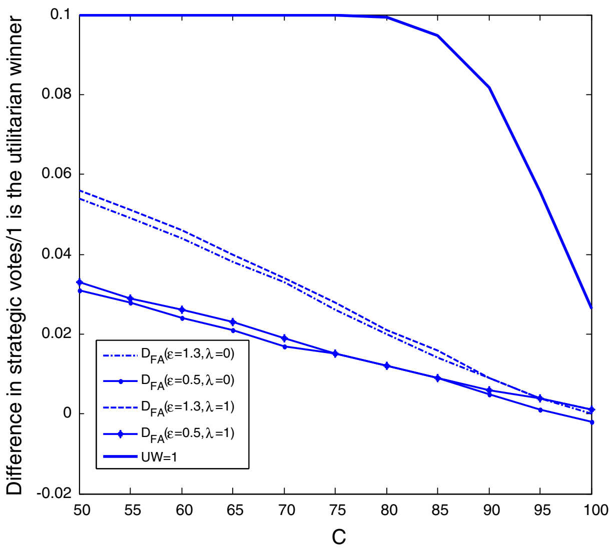

Fig. 4 Strategic votes for and against alternative 1, n = 4

vote strategically. Let $D_{FA} = F_1 - A_1$ denote the difference between the two. The trough at about C = 85 can be explained by inspecting Fig. 4.

It shows the percentage of simulated elections in which alternative 1 is the utilitarian winner (UW = 1), and D_{FA}. Recall that C reflects how much preference intensities for alternative 1 have been increased at the expense of other alternatives, keeping the preference orderings fixed. The smaller C is, the more likely it is that alternative 1 is

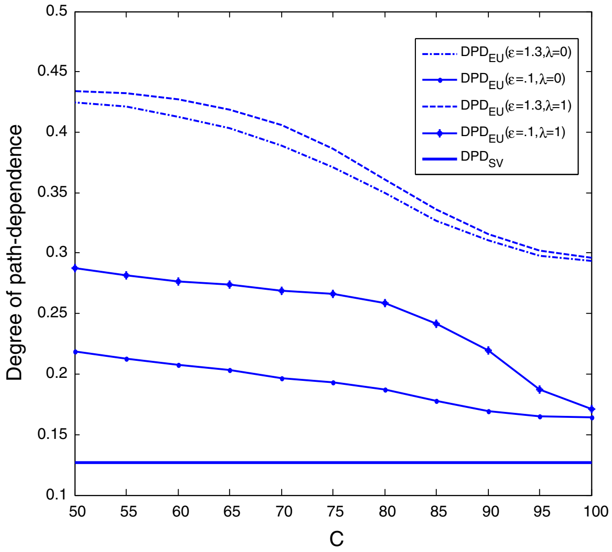

Fig. 5 Degree of path-dependence with n = 4

the utilitarian winner. With C = 90 Alternative 1 is the utilitarian winner in 81.8 % of the simulated elections. With C = 85 this raises to 94.8 and with C = 80–99.3 %. Alternative 1 is thus practically always the utilitarian winner with C = 85. This alternative does obtain more and more strategic votes as C gets smaller. However, with C = 85 such strategic votes are not sufficient to make it the winner as often as it is the utilitarian winner. When C becomes smaller than 80, alternative 1 is always the utilitarian winner, and more and more clearly so. This, in turn, increases the number of strategic votes for it. This explains why the curves rise as one goes from C = 85 towards C = 50.

6.2 Degree of Path-Dependence

Figures 5 and 6 show degrees of path-dependence with 4, and 6 alternatives. These figures show that the degree of path-dependence is higher under EU than under SV behaviour. The main reason for this result is that there is often a Condorcet winner even when the number alternatives is relatively large. With 6 alternatives, for example, there is a Condorcet winner among the alternatives in 69.9 % of the simulated elections. Under those elections, DPD_{SV} is always zero but DPD_{EU} is usually far from zero. However, when there is no Condorcet winner, sincere voting yields a very high degree of path-dependence, but strategic voting decreases the degree of path-dependence because the utilitarian winner obtains many more strategic votes than other alternatives. This can be seen from Figs. 7 and 8. They display degrees of path-dependence under those elections in which there is no Condorcet winner with $\varepsilon = .1$, and $\varepsilon = 1.3$, respectively. These results were derived with $\lambda = 1$.

What happens with more than 6 alternatives? Given that the probability of a preference cycle increases with the number alternatives, is it reasonable to suggest that with

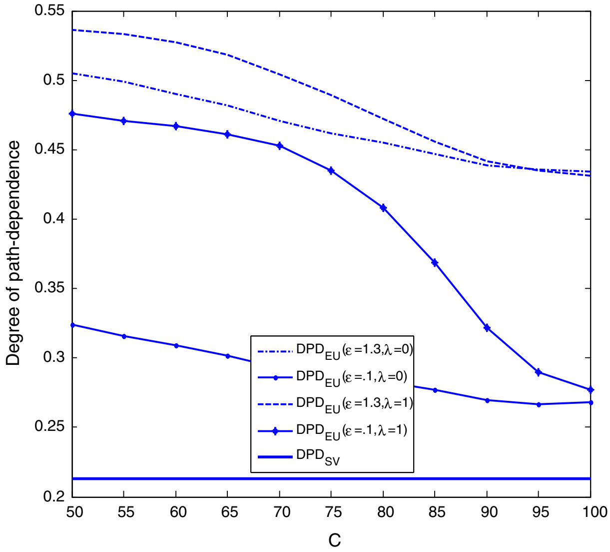

Fig. 6 Degree of path-dependence with n = 6

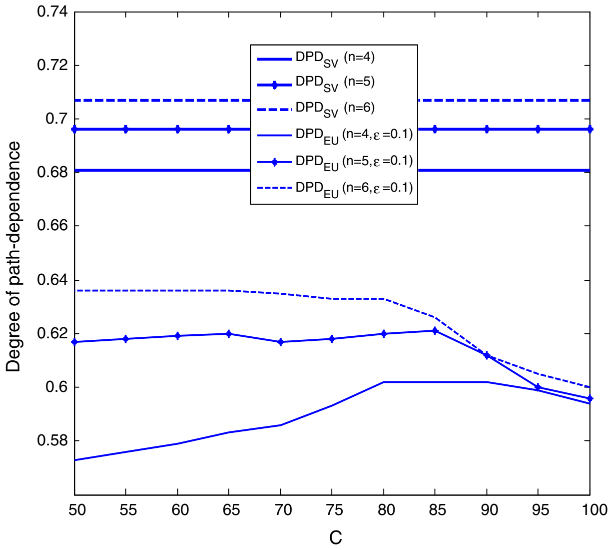

Fig. 7 DPD when there is no Condorcet winner, almost complete information

10 or 15 alternatives, strategic voting should decrease the degree of path-dependence? I was able to run a simulation with 10 alternatives and 50 repeats. The results in Fig. 9 are thus highly tentative. Comparing them figure to Fig. 5 shows, as expected, that the degree of path-dependence increases under SV behaviour as the number of alternatives increases. The degree of path-dependence also increases under EU behaviour but not as much as under SV behaviour. Although strategic voting decreases the degree of

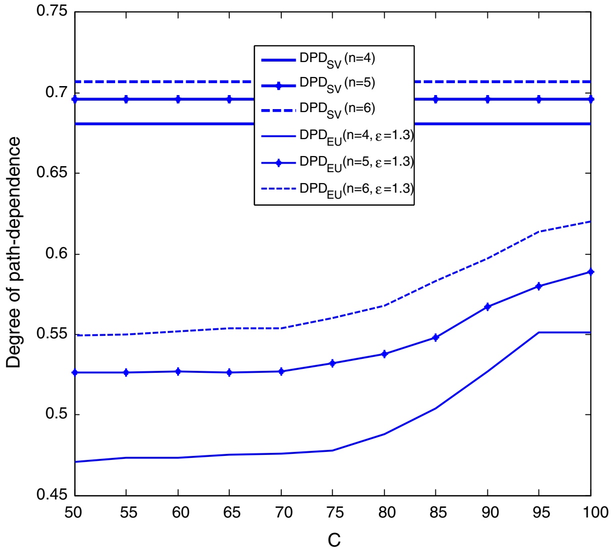

Fig. 8 DPD when there is no Condorcet winner, unreliable information

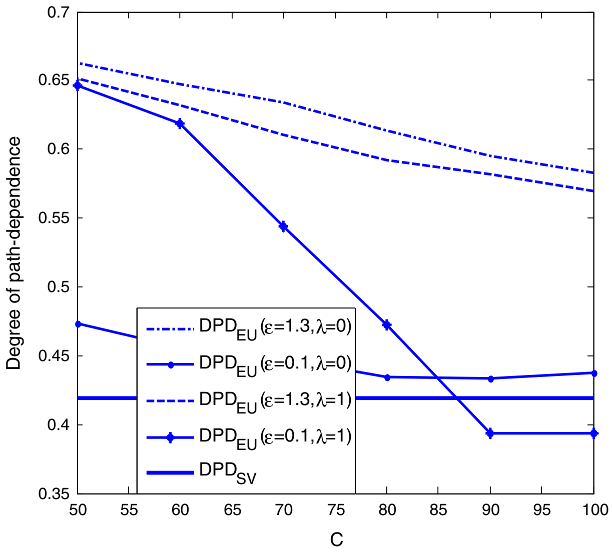

Fig. 9 Degree of path-dependence with n = 10

path-dependence when C = 90–100 with $\lambda = 1$ and $\varepsilon = 0.1$, voters are too close to having complete information for these results to be relevant. Given such results, it is possible that with a very large number of alternatives, strategic voting might decrease the degree of path-dependence also with reasonable values of ε. However, the number of alternatives required for such a result seems to be so large as to be of no practical relevance.

7 Conclusions

Strategic voting increases the degree of path-dependence mainly because it may generate path-dependent choices even when there is a Condorcet winner. The simulation setups were constructed in such a way that the Condorcet winner and the utilitarian winner are often different. Given that strategic voting increases the number of votes for the latter, it increases utilitarian efficiency. However, under amendment agendas, the Condorcet winner always wins under at least those agendas under which it is introduced on the last round of voting. This creates a situation in which a utilitarian winner wins under a large number of agendas, but never all of them unless it is the same alternative as the Condorcet winner. This, in turn, leads to relatively high degrees of path-dependence. When there is no Condorcet winner strategic voting decreases the degree of path-dependence because it concentrates votes on the utilitarian winner.

8 Appendix: The Standard Deviation of $\varDelta_i$

The sum of utilities for candidate j can be viewed as the sum of N random variables U_i, one for each voter: $U_1 + U_2 +, \ldots, U_i, \ldots, +U_N = \sum_{i=1}^{N} U_i = U(j)$. Let $\varDelta_i(j,k) = U_i(j) - U_i(k)$, the variance of $\varDelta_i$ is

$$Var(\varDelta_i) = E\left[U_i(j) - U_i(k) - E\left[U_i(j) - U_i(k)\right]\right]^2.$$

Each U_i is a uniformly distributed random variable on (0,1) with expected value $\frac{1}{2}$, and variance $\int_0^1 \left(u_i - \frac{1}{2}\right)^2 dU_i = \frac{1}{12}$. It is obvious that $E\left[U_i(j) - U_i(k)\right] = 0$. The variance of $\varDelta_i$ is thus given by

$$Var(\varDelta_i) = E\left[U_i(j)^2\right] - 2E\left[U_i(j)\,U_i(k)\right] + E\left[U_i(k)^2\right].$$

Since by definition

$$E(U_i(j)^2) = Var(U_i(j)) + [E(U_i(j))]^2 = \frac{1}{12} + \left(\frac{1}{2}\right)^2 = \frac{1}{3},$$

$U_i(j)$ and $U_i(k)$ are independent random variables so that $E\left[U_i(j)\,U_i(k)\right] = E\left[U_i(j)\right]E[U_i(k)] = \frac{1}{2}\frac{1}{2} = \frac{1}{4}$. The variance of $\varDelta_i$ is thus $\frac{1}{3} - 2\left(\frac{1}{4}\right) + \frac{1}{3} = \frac{1}{6}$. Since $\varDelta$ is the sum of N independent random variables $\varDelta = \Sigma_{i=1}^{N} \varDelta_i$, the standard deviation of $\varDelta$, $\sigma_\varDelta$, is$\sqrt{\frac{N}{6}}$.

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