Appendix to Lehtinen, Aki: *Inferential Rules for Confirmatory Robustness*

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A full treatment of the tacking problem for the Derivational Confirmation Rule (DCR) was omitted due to space constraints. This appendix provides a more detailed account of how tools from genuine confirmation resolve the issue, and how these solutions extend to the inferential rules and to indirect confirmation.

It is straightforward to construct an apparent counterexample to the rationality of DCR. Consider again model

$$M_1 = (CA_1A_2A_3) \vdash R. \quad (1)$$

Now add any arbitrary statement A—for instance, 'the moon is made of green cheese'. By the monotonicity of entailment, we obtain

$$M_1' = (CA_1A_2A_3A) \vdash R. \quad (2)$$

Applying DCR would then yield $p(R|A,B_1)>p(R|A,B_0)$, since this conjunction of elements had not previously been used to derive R, and A was among them. The problem is that DCR seems to have no resources to avoid this conclusion even though A is known to be irrelevant for R. This is the problem of tacking (also known as the conjunction problem) applied to DCR.[1] Yet this result is clearly irrational: A is known to be irrelevant to R and plays no genuine role in the derivation.

The tacking problem affects not only DCR but the broader account of confirmatory robustness. In the next section, the inferential rules are applied to indirect confirmation, which also fails if tacking cannot be resolved (see esp. Okasha 1997). A piece of evidence E confirms a component X indirectly when X does not entail E, yet E still supports X. This section explains how the tacking objection can be addressed, thereby justifying DCR, DDR, and the account of indirect confirmation alike.

A simple response to the tacking challenge is to apply DDR to (1) and set $p(A\vdash_C R|B)=0$. In other words, DDR serves as the antidote to tacking, provided that the background information for DCR includes the inferential results from DDR.

This reply, however, will not satisfy everyone, since DDR itself requires justification. Such justification cannot rely on the entailment relation, as doing so would also commit the account to the monotonicity of entailment. The central idea behind genuine confirmation (Gemes 1993; Schurz 1991) is that elements irrelevant to an entailment are not confirmed. Formally, this is achieved by replacing entailment with the content-element relation, which is non-monotonic

[1] A reviewer raised this objection.

and thus excludes demonstrably irrelevant conjuncts. The following discussion examines how accounts of genuine confirmation handle tacking in both hypothetico-deductive and probabilistic contexts, beginning with the former.

In Laudan and Leplin's (1991) account, E indirectly confirms hypothesis H when E confirms H_1 and a general theory T entails both H and H_1. This can be represented schematically as:

$$\begin{array}{ccc} T & \vdash & H_1 \\ \top & & \top \\ H & & E \end{array}$$

Here, E indirectly confirms H because both H_1 and H are consequences of the broader theory T, allowing the confirmation of H_1 by E to flow upward to T and then downward to H. This intuition can be clarified using Hempel's conditions for confirmation. The Converse Consequence Condition (CCC) states that if E confirms H and H´ entails H, then E also confirms H´. The Special Consequence Condition (SCC) holds that if E confirms H, it confirms everything entailed by H. In this example, by CCC, E confirms T because H_1 entails E and T entails H_1; and by SCC, this confirmation extends to H, since T entails H.

However, these conditions are well known to be too permissive, as they allow for the tacking paradox (see, e.g., Sober 1999 for examples). To illustrate, consider tacking A to T:

$$\begin{array}{cccc} (T & A) & \vdash & H_1 \\ \top & \top & & \top \\ H & A & & E \end{array} \tag{3}$$

Because entailment is monotonic, if E confirms H_1, it also confirms the conjunction H_1&A. By the Converse Consequence Condition (CCC), E therefore confirms both. Then, given that H_1&A entails A, according to SCC, E also confirms A - an obviously unwarranted result.

When A is entailed by T&A yet irrelevant to entailing H_1, A is said to *cut the content* of T&A (Gemes 1999). The problem, then, is that if E indirectly confirms H, it would also indirectly confirm 'the moon is made of green cheese'. This paradoxical outcome is unavoidable as long as SCC and CCC are formulated in terms of the entailment relation.

Fortunately, accounts of *genuine confirmation* resolve the tacking paradox. They capture the intuition that if E can be derived from a conjunction M that excludes A, then E cannot confirm A even if it confirms M. Formally, E is a *content element* of a set of propositions M iff E is a relevant logical consequence of M—that is, M⊢E, and no predicate in E can be replaced in any of its occurrences by another predicate (of the same degree) salva validitate of M⊢E. To block tacking, the standard entailment relation (⊢) is replaced with the content-element relation ($\vdash_C$), and irrelevant conjuncts are excluded from confirmation. If an assumption (such as A) can be removed from M while E remains derivable from the pruned conjunction, then A is not confirmed by E. In Schippers and Schurz (2017, 2020), a content part of M is defined as a non-empty conjunction of its content elements.

These approaches reject CCC, holding that only those parts of a conjunction necessary for deriving E are confirmed. SCC must likewise be rejected, though its underlying intuition can be preserved through a reformulation that permits only genuine confirmation. The Special Consequence Condition for Genuine Confirmation (SCCG) can be stated as follows: if E confirms a conjunction of assumptions M, and C is a genuinely confirmed element of M (i.e., p(C|E)>p(C)) because it is necessary for deriving E, then E also confirms the other content elements of C (cf. Gemes 1999; Bartelborth 2020).

To apply this framework to robustness, consider the following reasoning. If C is part of a conjunction M´ that jointly entails C_M, then genuine confirmation of C from another conjunction M (with p(E|M)>p(E)) indirectly confirms C_M iff C is necessary within M´ to derive C_M. This first condition may hold, for instance, if

$$M' = (CA_j) \vdash C_M,$$

and it is known that C, but not A_j, is necessary for deriving C_M. Suppose further that C, but not A_i, is necessary for deriving E from M. This yields the following structure:

$$\begin{array}{lcccc} M = & (C & A_i) & \vdash & E \\ & \top_c & & & \\ & C_M & & & \end{array}$$

Here C_M is placed below C, but not below A_i, to indicate that C alone is needed for deriving C_M. According to SCCG, if E is a content element of C in M, and E confirms C, then E also confirms C_M, since C_M is a content element of C. By definition of content elements, although C entails $C_M \vee A$ for any A whenever C entails C_M, E does not confirm A as A is not a content element of C. Thus, E confirms C_M indirectly because E confirms C genuinely and C_M is a content element of C.

It is crucial to use Schippers and Schurz's (2017, 2020) definition of content elements, rather than content parts. For example, E may confirm the conjunction C&A_i (a content part but not a content element of E), such that p(E|C&A_i)>p(E), and also confirm the element C: p(E|C)>p(E), but not A_i: p(E|A_i)≤p(E). Since A_i is a content element of C&A_i, defining SCCG in terms of content parts would wrongly imply that, because E confirms C&A_i, it also confirms A_i. Under SCCG, however, A_i is not confirmed. More generally, because conditional probabilities are non-monotonic, cases may arise where p(E|C)<p(E) and p(E|A_i)<p(E) but p(E|C&A_i)>p(E) holds. In such cases, E confirms neither the content elements of C nor of A_i, and SCCG would fail. Hence, SCCG requires that only genuinely confirmed individual assumptions transmit confirmation to their content elements.

When conjunctions of assumptions are confirmed by evidence E, they indirectly confirm only those content parts whose derivation depends on genuinely confirmed elements. For instance, if E confirms both the conjunction

$C\&A_i$ ($p(E|C\&A_i)>p(E)$) and the content element C ($p(E|C)>p(E)$), but not A_i ($p(E|A_i)<p(E)$), then E confirms neither A_i nor anything entailed by A_i. Let a proposition C_M be *genuinely indirectly confirmed* when it does not entail E, is genuinely confirmed by the content element C, and C is genuinely confirmed by E.

Thus far, the discussion of genuine confirmation has been framed in hypothetico-deductive terms, but probabilistic accounts make similar distinctions. Schurz (2014) specifies the conditions under which confirmation of a conjunction M transfers to its content elements, depending on whether a given element is required to increase the likelihood of the evidence. If E raises M's probability so that $p(M|E) > p(M)$, this probability increase spreads to an E-transcending content element C of M only if C is necessary within M to make E highly probable, i.e., there exists no conjunction M* of content elements of M that makes E at least equally probable ($p(E|M^*) \geq p(E|M)$) but does not entail C. This is a necessary condition for being able to conclude that $p(C|E)>p(C)$. When a content element C is confirmed in this manner, it is said to be *genuinely confirmed.*

Although Schurz's condition is formulated for confirmation relations, it can be adapted to justify DDR. Since content elements identify which components are necessary for deriving a consequence, the distinction between empirical and theoretical consequences is irrelevant from the standpoint of relevance logic. Recall the earlier equations:

$$M_1 = (CA_1A_2A_3) \vdash R \tag{4}$$

$$M_1' = (CA_1A_2A_3A) \vdash R. \tag{5}$$

Applying Schurz' condition, $p(M_1'|R) > p(M_1')$, but this probability increase spreads to the R-transcending content element A only if A is necessary within M_1' to make R highly probable. Yet there *exists* a conjunction of content elements of M_1' that does not entail A and makes R at least as probable - namely M_1, so that $p(R|M_1) \geq p(R|M_1')$. Consequently, Schurz' condition is not satisfied by A. Hence, Schurz's condition is not satisfied by A, which justifies DDR.

Although tacking would undermine DCR without genuine confirmation, no separate prohibition on irrelevant conjuncts is needed. Modellers' background information already incorporates the inferences warranted by DDR: whenever tacking arises, the original model (M_1) provides an immediate counterexample showing that the tacked statement fails Schurz's condition. Thus, DDR automatically rules out any spurious probability increase under DCR.

We can now clarify the interpretation of $p(X\vdash_C R|B)$. It expresses a belief about whether X is a content element necessary within some model for deriving R—or, equivalently, whether X could be genuinely confirmed by empirical evidence for R.

Schurz's condition blocks content cutting by requiring that confirmation extends only to those E-transcending elements that are necessary for deriving E. These conditions function as constraints on rational belief: it is not enough to propose a probability distribution that violates them—one must also show that such a distribution would be rational.

Thus, the tacking problem poses no threat to the rationality of DCR or indirect confirmation.

References

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