— Problem Set 1 —
Y1: Macroeconometrics
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—Exercise 1. (Gymnastics of recursive substitution)—

Consider the following scalar stochastic process

$$x_t = \rho x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{iid}(0, \sigma^2), \; t = 1, 2, \ldots, \infty.$$

Assume further the initial condition $x_0 = a \in \mathbb{R}$.

(a) Express x_t as the sum of current and past ϵ_ts and x_0.

(b) Is x_t stable or non-explosive if $|\rho| < 1$? Explain why?

(c) What happens if $|\rho| > 1$? What happens if $\rho = 1$?

(d) Are your answers to (b) and (c) different if we don't assume an initial condition, i.e.: $t = -\infty, \ldots, -1, 0, 1, 2, \ldots, \infty$

(e) Suppose that $x_0 = 0$, $\varepsilon_1 = 1$ and $\varepsilon_j = 0$ $j > 2$. What is x_1, x_2, x_t? (Impulse responses)

(f) Compute one period forecast of x_t, ie $\mathrm{E}_t\, x_{t+1}$? Compute h-period forecast of x_t, ie $\mathrm{E}_t\, x_{t+h}$? The conditioning information set is $\mathcal{F}_t = \{\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$.

—Exercise 2. (Intermediate steps of the example)—

Calculate the intermediate steps, ie matrices A_0^{-1}, $\tilde{A}$ and E, of the example 'NK model in BK form'.

—Exercise 3. (Indeterminacy in NK model)—

Consider the New-Keynesian model (equations (13)–(15) in the hand-out/slides). Some assumptions related to the model parameters are listed in early parth of the slides. Assume that instead of (15), the monetary policy is given by

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t + s_t,$$

where s_t is *a monetary policy shock*. Let's assume it is (strong) white noise[1].

[1] See wikipedia for the definition.

(a) Write the process in the matrix form as equation (16) in the example in the handout.

(b) Assume the following parameter values $\sigma = 1$, $\beta = 0.99$ $\lambda = 0.1$, shock variances 0.01 (if needed), $\phi_\pi = 1.5$, $\phi_y = 0.2$ and $\rho = 0$. Use your matrix computing program (R, Octave, Matlab, Gauss, whatever), and calculate *numerically* the matrices A_0, A_0^{-1}, $\tilde{A}$ and E.

(c) What does the BK conditions say about this problem? Calculate the eigenvalues of $\tilde{A}$, ie the BK conditions.

(d) Try (again numerically) different values of ϕ_π. What are the values that do not satisfy BK condition?