— Problem Set 2 —
MA0910: Macroeconomic Theory — Part II, Section 2
Monetary Policy in Dynamic Macromodels
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Exercise 1

This exercise is based on Galí's exercise 3.4. Consider the Calvo model of staggered price setting with the following modification: In the periods between price reoptimizations firms mechanically adjust their prices according to some indexation rule. Formally, a firm that reoptimizes its price in period t (an event that occurs with probability $1-\theta$) sets a price $P_t^\star$ in that period. In subsequent preiods (ie, until it reoptimizes prices again), its price is adjusted according to the following rule:

$$P_{t+k|t} = P_{t+k-1|t}\Pi_{t+k-1}^{\omega}\Pi^{1-\omega} \quad \text{for } k = 1, 2, 3, \ldots$$

and

$$P_{t|t} = P_t^\star,$$

where $P_{t+k|t}$ denotes the price effective in period $t+k$ for a firm that last repotimized its price in period t, $\Pi_t \equiv P_t/P_{t-1}$ is the gross inflation rate, and $\omega \in [0,1]$ is an exogenous parameter that measures the degree of indexation[1]. This means that those firms that may not reoptimize their price level may update the price level with a rule where the previous period price level is updated with a geometric mean of previous period gross inflation and steady-state inflation[2]. Note, that — contrary to the book — the inflation is not zero anymore!

Suppose that all firms have access to the same *constant return to scale* technology, $\alpha = 0$, and face a demand schedule with a constant price elasticity ϵ.

[1] This is called partial indexing in the New-Keynesian litterature.

[2] The steady-state inflation is the same as inflation target of the central bank.

The objective function for a firm reoptimizing its price in period t (ie, choosing $P_t^\star$) is given by

$$\max_{P_t^\star} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[P_{t+k|t} Y_{t+k|t} - \Psi_{t+l}(Y_{t+k|t}) \right] \right\}$$

subject to a sequence of demand constraints, and the rule of indexation described above. $Y_{t+k|t}$ denotes the output in period $t+k$ of a firm that last reoptimized its price in period t, $\Psi(\cdot)$ is the cost function, and θ is the probability of not being able to reoptimize the price in any given period, and

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$$

is the usual stochastic discount factor for nominal payoffs.

a) (Weight is 10 %) Using the definition of the price level index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}},$$

derive a log-linear expression for the evolution of inflation π_t as a function of the average price adjustment term $\hat{\underline{p}}_t^\star$ ($\equiv \log(P_t^\star/P_t) - \log(P^\star/P)$).[3]

Note! Due to the non-zero steady-state inflation[4], Galí's way of using p_{t-1} is not a wise strategy here.

b) (Weight is 10 %) Derive the first-order condition for the firm's problem that determines the optimal price $P_t^\star$.

c) (Weight is 20 %) Stationarize the first order condition: you may do that by dividing both sides by P_t continue working with relative prices.[5] Then, log-linearize the first-order condition around the corresponding steady state and derive an expression for $\hat{\underline{p}}_t^\star (\equiv \log(P_t^\star/P_t) - \log(P^\star/P))$.

d) (Weight is 10 %) Combine the results of (a) and (c) to obtain a log-linearized inflation equation, that express inflation as a function of lagged inflation, expected inflation and real marginal costs.

[3] I use underscore notation for stationarized variables — like relative prices.

[4] This is an assumption we make. It is less restricted than the zero-inflation assumption.

[5] Note that relative prices are stationary even in the case of non-zero steady-state inflation.

e) (Weight is 20 %) Code this to the `Dynare`.[6] Use the parameter values that are mentioned in the book (section 3.4.1) and the following $\theta = 2/3$, $\sigma = \varphi = 1$, $\phi_\pi = 1.5$, $\phi_y = 0.5/4$ and $\omega = 0.6$. The other part of the system of equations follow the basic new Keynesian model. Monetary policy follows the Taylor rule with inflation target (and, hence, steady state inflation) $\pi = 0.02/4$. The Taylor rule is as follows

$$i_t = \rho + \phi_\pi(\pi_t - \pi) + \phi_y \tilde{y}_t + v_t,$$

where $v_t = \rho_v v_{t-1} + \varepsilon_t^v$, $\varepsilon_t^v \sim iid(0, \sigma_v^2)$, and $\rho_v = 0.5$ and $\sigma_v = 0.03$. The technology shock follows AR(1) process too: the persistence parameter is $\rho_a = 0.95$ and the standard deviation of the shock is $\sigma_a = 0.05$.

f) (Weight is 20 %) Compute the impulse responses (3.1) and (3.2) in the book (and in the slides). How they differ from the original impulse responses? Explain why!

Note that the dynare uses the shock variance as the size of the shock in impulse responses. This explains the magnitude of the responses.

g) (Weight is 10 %) Consider the following central bank loss function

$$\mathcal{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\mathrm{Var}(\tilde{y}_t) + \frac{\epsilon}{\lambda}\mathrm{Var}(\pi_t)\right],$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$. Compare economies with different indexing schemes $\omega = \{0.3, 0.9\}$ (US and Euro area) by computing the above losses.

Hint! The computation of loss $\mathcal{L}$ is very easy since dynare repors the relevant variances.

[6] Do not even dream coding in original nonlinear form (because the decision rule for $P_t^\star$ is not in recursive form). The idea is to use the dynamic IS curve; mapping between real marginal costs and natural level of output; and relationship between natural level of output and technology shock.