# — Problem Set 1 —
# MA0910: Macroeconomic Theory — Part II, Section 2
# Monetary Policy in Dynamic Macromodels
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## —Exercise 1—

We start by some gymnastics related to infinite sums. Introductions/appendices in Sargent (1987a,b); Hamilton (1994) provide algebra for difference equations and lag (backshift) operator.

(a) Denote $\pi_t = p_t - p_{t-1}$. Show the following

$$
\begin{aligned}
&(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k}-p_{t-1}) \\
&\qquad = (1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k(\sum_{i=0}^{k}\pi_{t+i}) = \sum_{k=0}^{\infty}(\beta\theta)^k\pi_{t+k}. \quad (1)
\end{aligned}
$$

Hint! Everything becomes straightforward if you write down the first few terms of the sum.

(b) If you managed to do (a), then showing the following will be simple.

$$
\begin{aligned}
&(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k}-p_t) \\
&\qquad = (1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k(\sum_{i=1}^{k}\pi_{t+i}) = \sum_{k=0}^{\infty}(\beta\theta)^k\pi_{t+k} - \pi_t. \quad (2)
\end{aligned}
$$

(c) Show the following (Galí's book's equation (25)):

$$
\begin{aligned}
p_t &= \frac{1}{1+\eta}\,\mathrm{E}_t\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k m_{t+k} + u'_t \\
&= m_t + \mathrm{E}_t\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \Delta m_{t+k} + u'_t \quad (3)
\end{aligned}
$$

## —Exercise 2—

Galí 2.1.

## —Exercise 3—

The aim is to study the model with money-in-the-utility function in a non-separable form (section "Motivation of money: MIUF" in the slides). The model is the basic classical model. Note that the firm's problem is the same as in earlier part of the chapter. The biggest difference arises from the fact that the utility function is non-separable wrt real money and consumption.

The key point is to log-linearize the expression

$$X_t = \left[ (1-\mathcal{V})C_t^{1-\nu} + \mathcal{V}\left(\frac{M_t}{P_t}\right)^{1-\nu} \right]^{\frac{1}{1-\nu}}$$

around zero inflation and constant output (ie output is not growing). To get the similar results as in Galí's book, you need to use the steady-state version of the money demand equation

$$\frac{M_t}{P_t} = C_t(1-Q_t)^{-1/\nu}\left(\frac{\mathcal{V}}{1-\mathcal{V}}\right)^{1/\nu}.$$

Note also that in the zero inflation steady-state: $Q = \beta$.

(a) Log-linearize $X_t$.

(b) Code the model into dynare and use the following calibration: $\mathcal{V} = 0.5$; $\delta = 1$; $\nu = 0.8$; $\beta = 0.998$; $\rho = -\log(\beta)$; $\rho_v = 0.5$; $\rho_a = 0.9$; $\phi_\pi = 1.5$; $\eta = 1.3$; $\phi = 1$; $\alpha = 1/3$; assume further that shock variances are 1.

The key equations are (32)–(34), the firm's equations, the shock processes (page 30) and the market equilibrium.

(c) Replicate the impulse response graph that was presented in the slides.

(d) Study the case when $\nu = \delta$. What happens? Explain why?

(e) Re-specify the technical process $a_t$ such that $\mathrm{E}_t\, a_{t+1} > a_t$, where the conditioning information contains $\{a_t, a_{t-1}, \dots\}$. This means that $a_t$ should grow on average. Compute the impulse response of real interest rate $r_t \equiv i_t - \mathrm{E}_t\, \pi_{t+1}$ to a shock in $a_t$ process. Explain the impulse response.

# References


Hamilton, James D. (1994) *Time Series Analysis* (Princeton, New Jersey: Princeton University Press)

Sargent, Thomas J. (1987a) *Dynamic Macroeconomic Theory* (Harvard University Press)

_ (1987b) *Macroeconomic Theory,* second ed. (Orlando, Florida: Academic Press)