

MONETARY POLICY, INFLATION AND THE BUSINESS CYCLE

MA0910: MACROECONOMIC THEORY — PART II, SECTION 2

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OUTLINE

We follow chapters 1–5, 7 in Galí's excellent book:

- ① Introduction
- ② Monetary Policy in Classical Model
- ③ The Basic New Keynesian Model
- ④ Monetary Policy Design in the Basic New Keynesian Model
- ⑤ Time-Consistency in Monetary Policy: Discretion vs. Commitment
- ⑥ Monetary Policy in Open Economy

Note: many of the slides are reproduced from Galí's book's slide collection

(<http://www.crei.cat/people/gali/monograph.html>)

- 1 INTRODUCTION
- 2 MONETARY POLICY IN CLASSICAL MODEL
- 3 THE BASIC NEW KEYNESIAN MODEL
- 4 MONETARY POLICY DESIGN IN THE BASIC NEW KEYNESIAN MODEL
- 5 DISCRETION VS COMMITMENT: MONETARY POLICY AND TIME-CONSISTENCY
- 6 OPEN ECONOMY AND MONETARY POLICY
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OUTLINE

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 - New Keynesian Model
- 2 MONETARY POLICY IN CLASSICAL MODEL
- 3 THE BASIC NEW KEYNESIAN MODEL
- 4 MONETARY POLICY DESIGN IN THE BASIC NEW KEYNESIAN MODEL
- 5 DISCRETION VS COMMITMENT: MONETARY POLICY AND TIME-CONSISTENCY
- 6 OPEN ECONOMY AND MONETARY POLICY

MOTIVATION

- Monetary policy has been a central area of macroeconomic research
- The macroeconomics of monetary policy studies the interaction between monetary policy, inflation and business cycles (fluctuations in economic activity)
- The modern models of monetary policy build on — as most of the recent macroeconomic research — on real business cycle models by Kydland and Prescott (1982) and Prescott (1986).

REAL BUSINESS CYCLE REVOLUTION

METHODOLOGICAL REVOLUTION

Intertemporally optimizing agents. Budget and technology constraints.

CONCEPTUAL REVOLUTION

- In a frictionless markets under perfect competition business cycles are efficient: no need for stabilization; stabilization may be counter-productive.
- Economic fluctuations are caused by technology shocks: they are the main source of fluctuation.
- Monetary factors (price level) has a limited (or no) role: money (price level) has no effect on the real economy, real wages, relative prices, consumption, investments, employment,

NEW KEYNESIAN MODEL

- Methodologically similar to RBC models.
- Builds on the following features:
 - monopolistic competition
 - nominal rigidities
 - short-run non-neutrality of money: real interest rate affect money supply.
- Leads to differences w.r.t RBC models: economy's response to shocks is generally inefficient.
- Removing the effects of non-neutrality is potentially welfare improving → role for monetary policy.

EMPIRICAL EVIDENCE

SUPPORTIVE TO PRICE RIGIDITY

4–6 quarters by Bils and Klenow (2004), 8–11 quarters by Nakamura and Steinsson (2008), 8-11 by Dhyne et al. (2006).

Wages are also rigid: 1 year, downward rigidity by Dickens et al. (2007)

NON-NEUTRALITY HARD TO MEASURE

The expected inflation does not move as nominal interest rates, when latter is changed, the real interest rate changes and, as a result, equilibrium output and employment.

This is hard to show empirically. Christiano et al. (2005) provides example.

OUTLINE

1 INTRODUCTION

2 MONETARY POLICY IN CLASSICAL MODEL

- Households
- Sidestep: log-linearization
- Firms
- Equilibrium
- Monetary Policy Rules
- Motivation of money
- Motivation of money: MIUF
- Optimal policy

3 THE BASIC NEW KEYNESIAN MODEL

HOUSEHOLD PROBLEM

Households

- Decide how much they consume, C_t , in each period and
- how much they work, N_t .
- They know the *current and historical* values of nominal wages W_t ,
- and their savings in bonds B_t , with bond price Q_t , and
- their lump-sum taxes, T_t .
- Price level is P_t .

They cannot accumulate infinite debt (transversality condition):

$$\lim_{T \rightarrow \infty} E_t B_T \geq 0$$

Households maximize the expected present value of utility

$$\max_{\{C_t, N_t\}} E_0 \sum_{t=0}^{\infty} U(C_t, N_t)$$

subject to the following (flow) budget constraint

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t. \quad (3.1)$$

FIRST ORDER CONDITIONS

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

Marginal rate of substitution between consumption and leisure (equaling real wages).

$$Q_t = \beta E_t \left(\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right)$$

Intertemporal marginal rate of substitution in nominal terms.

PARAMETRIC VERSION

Assume the following functional form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\sigma \geq 0$ and $\varphi \geq -1$.

Then

$$U_{N,t} = -N_t^\varphi \quad U_{C,t} = C_t^{-\sigma}$$

and the first order conditions as follows

$$C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \tag{3.2}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{3.3}$$

SIDESTEP TO LOG-LINEARIZATIONS

TAYLOR APPROXIMATIONS ARE ACCURATE ONLY IN THE NEIGHBORHOOD OF A POINT

As a consequence (loosely speaking) an economic model has to be stationary. This means that very often we need to stationarize the model first, ie to express it in terms of stationary variables.

Suppose we have

$$f(X_t, Y_t) = g(Z_t), \quad (3.4)$$

with strictly positive X, Y, Z (ie the linearization point). The steady state counterpart is $f(X, Y) = g(Z)$.

This simple summarization is, for example, in the slides by Jürg Adamek.
(http://www.vwl.unibe.ch/studies/3076_e/linearisation_slides.pdf)

Let's denote $x_t = \log(X_t)$ (for any variable). Start from replacing $X_t = \exp(\log(X_t))$ in (3.4),

$$f\left(e^{\log(X_t)}, e^{\log(Y_t)}\right) = g\left(e^{\log(Z_t)}\right),$$

Taking first-order Taylor approximations from both sides:

$$\begin{aligned} f(X, Y) + f'_1(X, Y)X(x_t - x) + f'_2(X, Y)Y(y_t - y) \\ = g(Z) + g'(Z)Z(z_t - z) \end{aligned} \quad (3.5)$$

Often we denote $\hat{x}_t \equiv x_t - x$.

LOG-LINEARIZATION OF (3.3) AROUND NON-ZERO INFLATION AND GROWTH

In a growing economy $C_{t+1} > C_t$ and in a non-zero-inflation economy $P_{t+1} > P_t$. Hence, neither C nor P is a point. The growth rates are, however, stationary. Let's denote $\dot{X}_t \equiv X_t/X_{t-1}$. Then (3.3) is

$$Q_t = \beta E_t (\dot{C}_{t+1}^{-\sigma} \dot{P}_{t+1}^{-1}).$$

In the steady state: $Q = \beta \dot{C}^{-\sigma} / \dot{P}$. Denote $\rho \equiv -\log \beta$, then $\rho = -q - (\sigma \Delta c + \pi)$, where $\pi_t \equiv \dot{P}_t$ and $\Delta x_t \equiv \dot{x}_t$. Note also that $i_t = -q_t$.

Apply mechanically (3.5)

$$Q + Q(q_t - q) = \underbrace{\beta \dot{C}^{-\sigma} / \dot{P}}_{=Q} - \sigma \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{C}} \dot{C} (\Delta E_t c_{t+1} - \Delta c) + (-1) \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{P}} \dot{P} (E_t \pi_{t+1} - \pi).$$

and divide by Q and get rid of constants to obtain

$$q_t - q = -\sigma (E_t \Delta c_{t+1} - \Delta c) - (E_t \pi_{t+1} - \pi)$$

and combine with log of steady-state to obtain

$$i_t - E_t \pi_{t+1} - \rho = \sigma E_t \Delta c_{t+1} \quad (3.6)$$

$$y_t = c_t \quad (11)$$

Labor market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha)$$

Asset market clearing:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate production relationship:

$$y_t = a_t + (1 - \alpha) n_t$$

FIRMS

Firms use only labour N to produce output Y :

$$Y_t = A_t N_t^{1-\alpha}. \quad (3.9)$$

Log-linearized as $y_t = a_t + (1 - \alpha)n_t$. A_t is exogenously given stationary process (a shock). Firms maximize profits

$$P_t Y_t - W_t N_t$$

subject to production function (3.9) and obtain the following FOC:

$$(1 - \alpha)A_t N_t^{-\alpha} = \frac{W_t}{P_t}. \quad (3.10)$$

which gives labour demand schedule and tells us how much labour the firm is willing to hire for given real wages and technological process A_t .

$$n_t = \psi_{na} a_t + \vartheta_n$$

$$y_t = \psi_{ya} a_t + \vartheta_y$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho + \sigma E_t\{\Delta y_{t+1}\} = \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\}$$

$$\omega_t \equiv w_t - p_t = y_t - n_t + \log(1 - \alpha) = \psi_{\omega a} a_t + \log(1 - \alpha)$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$; $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$

$$\vartheta_y \equiv (1 - \alpha)\vartheta_n \quad ; \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$$

⇒ real variables determined *independently of monetary policy* (neutrality)

⇒ *optimal policy*: undetermined.

⇒ specification of monetary policy needed to determine nominal variables

DISCUSSION ON THE PARAMETERS

TECHNOLOGY SHOCK HAS ALWAYS POSITIVE IMPACT ON OUTPUT

$$\phi_{ya} = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha}$$

is always positive given reasonable values of $\varphi > -1$.

TECHNOLOGY SHOCK MAY REDUCE OR INCREASE EMPLOYMENT

$$\phi_{na} = \frac{1 - \sigma}{\sigma(1 - \alpha) + \varphi + \alpha} = \begin{cases} \leq 0 & \text{if } \sigma > 1 \\ = 0 & \text{if } \sigma = 1 \\ \geq 0 & \text{if } \sigma < 1. \end{cases}$$

A CLOSER LOOK AT EMPLOYMENT RESPONSE

Let's rewrite (3.2) as follows

$$N_t^\varphi = \frac{W_t}{P_t} C_t^{-\sigma}$$

The elasticity of substitution is unity (or $1/\varphi$), and wealth elasticity is $-\sigma$ (or $-\sigma/\varphi$).

- The substitution effect dominates the negative wealth effect if $\sigma < 1$ and
- *vice versa* if $\sigma > 1$.
- They cancel each other when $\sigma = 1$ (logarithmic utility).

TECHNOLOGY PROCESS DEFINES PROPERTIES OF REAL INTEREST RATE

r_t will go down if $E_t a_{t+1} < a_t$ and go up if $E_t a_{t+1} > a_t$ (like in the growing economy).

SUMMARY OF EQUILIBRIUM

In equilibrium, output, consumption, employment, real wages and real rate of return are function of productivity shock only — not of anything else! Hence monetary factors play no role in real economy, ie monetary policy is neutral w.r.t. real variables.

FIXED INTEREST RATE RULE

Consider standard Fisher equation that we have derived above

$$i_t = E_t \pi_{t+1} + r_t = E_t p_{t+1} - p_t + r_t. \quad (3.12)$$

Its solution should be of the form

$$p_t = -E_t \sum_{i=0}^{\infty} (i_{t+i} - r_{t+i}).$$

It is easy to see that this does not converge in general.

SIMPLE INFLATION BASED INTEREST RATE RULE

where $v_t \equiv \eta^{-1}(u_t + y_t)$ is independent of policy.

Consider an interest rate rule

$$i_t = \rho + \phi_\pi \pi_t$$

and combine it with the Fisher equation to obtain

$$\rho + \phi_\pi \pi_t = E_t \pi_{t+1} + r_t.$$

Its solution is of the form

$$\pi_t = E_t \sum_{i=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^i (r_{t+i} - \rho),$$

which is convergent if $0 \leq 1/\phi_\pi < 1$, ie

TAYLOR PRINCIPLE

$$\phi_\pi > 1$$

MONEY GROWTH RULE

Substitute (3.12) into the money demand equation to obtain

$$m_t - p_t = y_t - \eta(E_t \pi_{t+1} + r_t).$$

Solve price level forward

$$\begin{aligned} p_t &= \frac{1}{1+\eta} E_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta} (m_{t+i} + \eta r_{t+i} - y_{t+i}) \\ &= m_t + E_t \sum_{i=1}^{\infty} \frac{\eta}{1+\eta} \Delta m_{t+i} + E_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta} (\eta r_{t+i} - y_{t+i}) \end{aligned}$$

and the implied nominal interest rate

SUMMARY

- Real variables are independent of monetary policy.
- Monetary policy has an important impact on nominal variables.
- No monetary policy rule is better than any other.
- The non-existence of the interaction between nominal and real variables is in contrast to empirical evidence.

VARIOUS APPROACHES TO MOTIVATE MONEY

$$P_t C_t + Q_t B_t + \frac{U_{c,t}^{n_t}}{U_{c,t}} M_t \leq 1 B_t \exp\{M_t\} + W_t N_t - T_t$$

Being a good utility, evaluated at $(C_t, \frac{M_t}{P_t}, N_t)$

In the above classical models, money had a role of unit of account:

Two cases: $P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t \leq A_{t+1} + W_t N_t - T_t$

cashless economy, cashless limit.

Interest rate non-sustainable in real business cycles (e.g. $U_{cw} > 0$)
 Money provides liquidity services. They can be modelled, for example, as

- Real balances generate utility: Money-in-the-utility-function (MIUF)
- The transaction cost approach
 - Explicit microfounded matching models starting from double coincidence of wants
 - Cash-in-advance (CIA) constraint

We study MIUF and leave CIA as an exercise. First item can be found from micro courses.

Utility specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

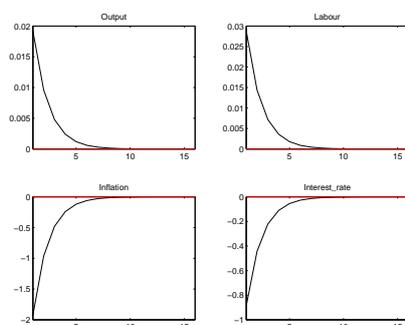
$$X(C_t, M_t/P_t) \equiv \left[(1-\vartheta) C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t}\right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \quad \text{for } \nu \neq 1$$

$$\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t}\right)^{\vartheta} \quad \text{for } \nu = 1$$

Policy Rule: $\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$ Calibration: $\nu = 2.56$; $\sigma = 2 \implies U_{cm} > 0$

Effects of Exogenous Monetary Policy Shock (Fig 2.3 and 2.4)

RESPONSES TO A POSITIVE MONEY SUPPLY SHOCK



OPTIMAL MONETARY POLICY IN A CLASSICAL SETTING

Assume a hypothetical social planner that maximize the utility of representative household, that contains real money.

Social planner faces a static problem, since only an individual household (not the society as a whole) can smooth its consumption over time. The planner's problem is to maximize

$$\max U\left(C_t, \frac{M_t}{C_t}, N_t\right)$$

subject to resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

for some $\phi > 1$. Combined with the definition of the real rate:

The optimality conditions are given by

$$E_t \{ i_{t+1} \} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t .
Implied equilibrium inflation:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) A_t N_t^{-\alpha} \quad (3.13)$$

$$\pi_t = -r_{t-1}$$

$$U_{m,t} = 0 \quad (3.14)$$

(3.13) First corresponds the labour market equilibrium that is independent of monetary policy (except in the non-separable MIUF case)

(3.14) Second condition equates marginal utility of real balances to the "social" marginal cost of producing them (zero!).

From household's problem we know

$$\frac{U_{m,t}}{U_{c,t}} = 1 - e^{-i_t}.$$

RHS can be zero only if $i_t = 0$. This is called Friedman rule. (In steady-state) this results $\pi = -\rho (\equiv -\log(\beta)) < 0$, ie in the steady state, the price level declines at the rate of time preference.

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- 1 INTRODUCTION
- 2 MONETARY POLICY IN CLASSICAL MODEL
- 3 THE BASIC NEW KEYNESIAN MODEL
 - Preliminaries
 - Introduction
 - Households
 - Firms
 - Optimal price setting
 - Aggregate prices
 - Equilibrium
 - Monetary policy in a new Keynesian model

MONETARY POLICY DESIGN IN THE BASIC NEW KEYNESIAN MODEL

CES AGGREGATOR

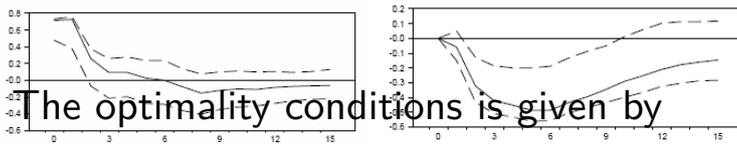
- 6 OPEN ECONOMY AND MONETARY POLICY
- In modern macro models with imperfect competition, the Dixit-Stiglitz, or Constant-Elasticity-of-Substitution aggregator plays an important role. Consider a static optimization problem of a firm that buy infinite number of intermediate products $C(i)$, puts them together using technology

$$C = \left[\int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}, \quad (4.1)$$

where ϵ is the elasticity of substitution (and also the price elasticity of the demand function). Its optimization problem is

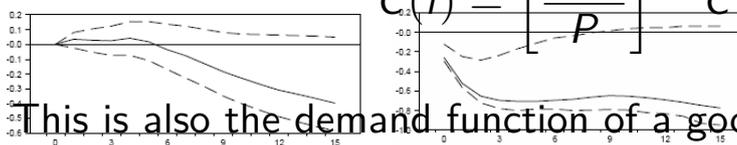
$$\max_{C(i)} C \cdot P - \int_0^1 C(i)P(i)di$$

subject to the production technology (4.1).



The optimality conditions is given by

$$C(i) = \left[\frac{P(i)}{P} \right]^{-\epsilon} C \quad \forall i \in [0, 1].$$



This is also the demand function of a good $C(i)$. (You must work out the details by yourself.) Plug this to the profits and use zero profit constraint to get the aggregate price level (=price index=marginal costs):

Source: Christiano, Eichenbaum and Evans (1999)

$$P = \left[\int_0^1 P(i)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{4.2}$$

INTRODUCTION

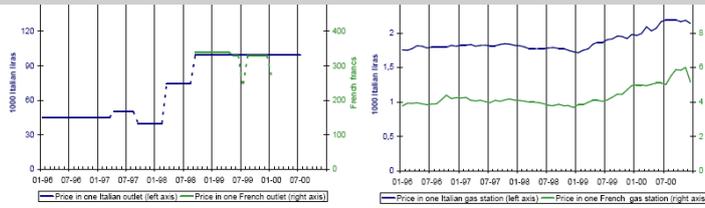
The basic new Keynesian model consists of two key ingredients:

IMPERFECT COMPETITION

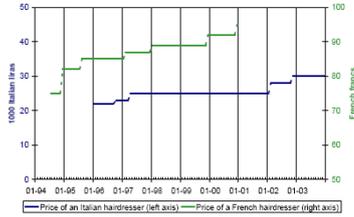
We assume that there is a continuum of firms and each produce a differentiated intermediate good for which it sets the price.

PRICE RIGIDITIES

We assume (*a la* Calvo (1983)) that, in each period, only a fraction of firms can change their price.



Haircut (men)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhvne et al. WP 05

HOUSEHOLDS

Household problem is the same as in the case of classical model except that the aggregate consumption consists of continuum of goods:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}$$

Household must allocate its consumption to different goods according to their relative price

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t, \quad (4.3)$$

where the aggregate price index is as in (4.2).

The optimality conditions are, of course, the same and assuming same functional form of utility function, and loglinearizing, we obtain

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (4.4)$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1} - \rho) \quad (4.5)$$

$$m_t - p_t = y_t - \eta i_t. \quad (4.6)$$

FIRMS

Assume continuum of identical firms indexed by $i \in [0, 1]$ that use the following common production technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}.$$

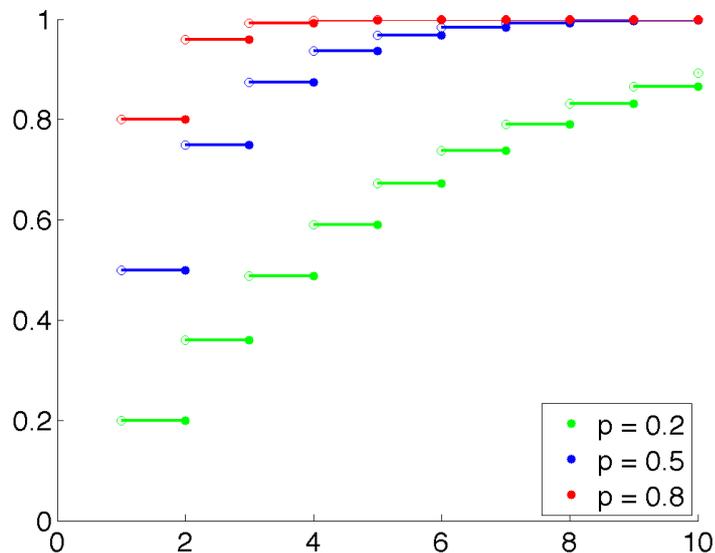
All firms face an identical demand curve (4.3) and take aggregate price index P_t and aggregate consumption index C_t as given.

CALVO FAIRY

- A firm may change price of its product only when Calvo Fairy visits.
- The probability of a visit is $1 - \theta$.
- It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $1 - \theta$ share of firms may change their price and rest, θ , keep their price unchanged.
- Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process).
- The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution.
- The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1 - \theta}.$$

CUMULATIVE DISTRIBUTION FUNCTION OF GEOMETRIC DISTRIBUTION



Source: http://en.wikipedia.org/wiki/Geometric_distribution

OPTIMAL PRICE SETTING

Let P_t^* denote the price level of the firm that receives price change signal. This is the price level of the firm that Calvo Fairy visits.

When making its pricing decision, the firm takes into account that it can change its price with the probability $1 - \theta$, i.e. the chosen price remains the same with probability θ .

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \{ P_t^* Y_{t+k}(P_t^*) - \Psi_{t+k} [Y_{t+k}(P_t^*)] \},$$

where $\Psi_{t+k} [Y_{t+k}(P_t^*)]$ is firm's total costs that depends on the demand function $Y_{t+k}(P_t^*)$ with a relative price of P_t^*/P_{t+k} .

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} - \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

and

Perfect Foresight, Zero Inflation Steady State: $Y_{t+k}(P_t^*) = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC \equiv \frac{\mathcal{M}}{P_{t+k}}$$

The first one is the demand function that the firm faces and is due to the households' consumption index.

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right).$$

The second equation is the nominal *stochastic discount factor* (pricing kernel), that household use to price any financial asset. Note, that in our standard household's optimisation problem, the household price one-period bond using the very same pricing kernel.

The first-order condition

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^*) \left(P_t^* - \underbrace{\frac{\epsilon}{1-\epsilon}}_{\equiv \mathcal{M}} \psi_{t,t+k} \right) = 0,$$

where

$$\psi_{t,t+k} = \Psi'_{t+k}(Y_{t+k}(P_t^*))$$

is nominal marginal costs and \mathcal{M} is the desired or frictionless markup. In the case of no frictions ($1 - \theta = 1$):

$$P_t^* = \mathcal{M} \psi_{t,t}.$$

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1} \}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc_t$.

Equivalently,

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + p_{t+k} \}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$.

Flexible prices ($\theta = 0$):

$$p_t^* = \mu + mc_t + p_t$$

$\implies mc_t = -\mu$ (symmetric equilibrium)

AGGREGATE PRICE LEVEL

Note that all those firms that may set their price level (ie where Calvo Fairy visits) will choose identical price P_t^* . Let $S(t) \subset [0, 1]$ represent the set of firms that do not reoptimize their price in period t . Note that “the distribution of prices among firms not adjusting in period t corresponds to the distribution effective prices in period $t - 1$, though with total mass reduced to θ ”. Remember the aggregate price index P_t in eq (4.2). The aggregate price level has to follow this functional form

$$P_t = \left[\int_{S(t)} P_t(i)^{1-\epsilon} di + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

$$= [\theta P_{t-1}^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}. \quad (4.7)$$

LOG-LINEARIZATION OF AGGREGATE PRICE LEVEL

Stationarize equation (4.7), ie express it in inflation rates and in relative prices: Divide (4.7) by P_{t-1} and raise it power $1 - \epsilon$:

$$\Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon},$$

where $\Pi_t = P_t/P_{t-1}$.

Log-linearizing it around $\Pi = 1$ and $P_t^*/P_t = 1$ (zero steady state inflation, unit relative prices in steady-state):

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}).$$

EQUILIBRIUM

Combining (1) and (2): $p_{t+k} - \frac{1}{1-\alpha} (a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$,

$$\begin{aligned}
 \pi_t &= \beta E_t \{ \pi_{t+1} \} + \lambda \widehat{mc}_t \\
 mc_{t+k|t} &= mc_{t+k} + \frac{1}{1-\alpha} (y_{t+k|t} - y_{t+k}) \\
 &= mc_{t+k} - \frac{\alpha \epsilon}{(1-\theta)(1-\alpha-\beta\theta)} (p_t^* - p_{t+k}) \quad (3)
 \end{aligned}$$

where

Goods market

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda \widehat{mc}_t = C_t(i) \quad \forall i \in [0,1] \text{ and } \forall t.$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{1-\alpha-\alpha\epsilon}$$

Due to aggregator (4.1)

$$Y_t = C_t. \quad \forall t$$

Labour markets

$$N_t = \int_0^1 N_t(i) di.$$

Goods market clearing $Y_t = \int_0^1 N_t(i) di$

for all $i \in [0, 1]$ and all t . $= \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \bar{c}_t(i) di$

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\alpha} di \right)^{\frac{1}{1-\alpha}} \left(\frac{\bar{c}_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\alpha}{1-\alpha}} di$
 $Y_t = C_t$

Taking logs for all t . Combined with the consumer's Euler equation:

$$(1 - \alpha) n_t = y_t - a_t + d_t$$

$$y_t = E_t\{y_{t+1}\} - \frac{\sigma_c}{\sigma_c + 1} (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{5}$$

where $d_t \equiv (1 - \alpha) \log \int_0^1 (P_t(i)/P_t)^{-\frac{\alpha}{1-\alpha}} di$ (second order).

Up to a first order approximation:

$$y_t = a_t + (1 - \alpha) n_t$$

$$mc_t = (\omega_t - \alpha) - \rho + E_t \pi_{t+1} + \log y_t \quad (9)$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t + \varphi n_t) - (y_t - n_t) - \log(1 - \alpha)$
 $= \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha)$ (6)

Dynamic IS equation

Under flexible prices

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t - \frac{1}{\rho} (i_t - E_t \pi_{t+1} - r_t^n) \quad (10)$$

where r_t^n is the natural rate of interest, given by

$$r_t^n \equiv \rho + \sigma E_t \Delta y_{t+1}^n a_t$$

where $\delta_y \equiv \frac{(\mu - \log(1 - \alpha))(1 - \alpha)}{\sigma + \varphi + \alpha(1 - \sigma)} > 0$ and $\psi_{ya} \equiv \frac{E_t \{\Delta y_{t+1}^n\}}{\sigma + \varphi + \alpha(1 - \sigma)}$.

Missing block: description of (monetary) policy (determination of i_t) (8)
 $\Rightarrow mc_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n)$

where $y_t - y_t^n \equiv \tilde{y}_t$ is the output gap

MONETARY POLICY

Determination of nominal interest rate, i_t , gives the path for actual real rate. It is a description how monetary policy is conducted.

MONETARY POLICY IS NON-NEUTRAL

When prices are sticky nominal interest rate path determines output gap.

DYNAMICS

Equilibrium dynamics. combining (9), (10), and (11)

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T (\hat{r}_t^n - v_t) \quad (12)$$

where

To study the dynamics of a linear rational expectation model, we write it as follows

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_x}$

$$\mathbf{A}_0 \mathbf{x}_t = \mathbf{A}_1 E_t \mathbf{x}_{t+1} + \mathbf{B} z_t.$$

and study the properties of the transition matrix $\mathbf{A}_T \equiv \mathbf{A}_0^{-1} \mathbf{A}_1$ of the following form

$$\mathbf{x}_t = \underbrace{\mathbf{A}_0^{-1} \mathbf{A}_1}_{\equiv \mathbf{A}_T} E_t \mathbf{x}_{t+1} + \underbrace{\mathbf{A}_0^{-1} \mathbf{B}}_{\equiv \mathbf{B}_T} z_t.$$

\mathbf{A}_0 should be of full rank (some solution algorithms allow reduced rank cases too).

Let $v_t = v$ (nominal shocks).

Let v_t follow an AR(1) process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Calibration:

$$\rho_v = 0.5, \phi_\pi = 1.5, \phi_y = 0.5/4, \beta = 0.99, \sigma = \varphi = 1, \theta = 2/3, \eta = 4.$$

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution

ANALYTICAL SOLUTION: UNDETERMINED COEFFICIENTS

- ① Guess that the solution takes the form $\tilde{y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$ (why?)
- ② Substitute (11) into (10),
- ③ Let's assume that $\hat{r}_t^n \equiv r_t^n - \rho = 0$ (because it is not affected by monetary policy shocks,
- ④ Impose these to (9) and (10) (note that $E_t v_{t+1} = \rho_v v_t$) and
- ⑤ solve the unknown ψ_{yv} and $\psi_{\pi v}$ to obtain

$$\begin{aligned}\tilde{y}_t &= -(1 - \beta\rho_v)\Lambda_v v_t \\ \pi_t &= -\kappa\Lambda_v v_t,\end{aligned}$$

$$\text{where } \Lambda_v = \{(1 - \beta\rho_v)[\sigma(1 - \rho_v)\theta\phi_y] + \kappa(\phi_\pi - \rho_v)\}^{-1}$$

ANALYTICAL SOLUTION: UNDETERMINED COEFFICIENTS...

- ⑥ Note, however, that $v_t = \rho_v v_{t-1} + \varepsilon_t^v = \frac{1}{1 - \rho_v L} \varepsilon_t^v$, where L is lag-operator (or backshift operator), ie $Lx_t = x_{t-1}$.
- ⑦ Then, the solution ('VAR' representation) is

$$\begin{aligned}\tilde{y}_t &= \rho_v \tilde{y}_{t-1} - (1 - \beta\rho_v)\Lambda_v \varepsilon_t^v \\ \pi_t &= \rho_v \pi_{t-1} - \kappa\Lambda_v \varepsilon_t^v.\end{aligned}$$

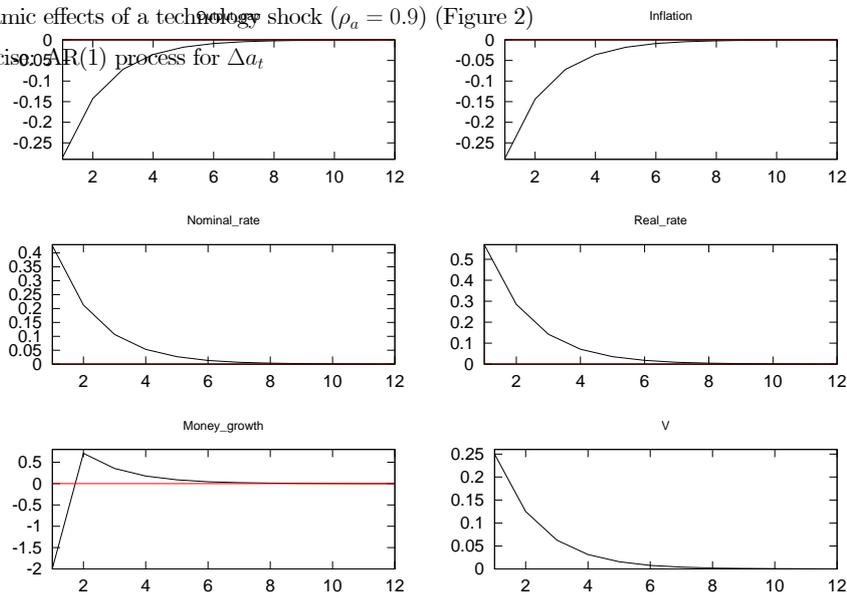
Note, that the persistence (lagged endogenous variable) is *inherited* from the shock process v_t .

RESPONSES TO A MONETARY POLICY SHOCK (INTEREST RATE RULE)

$$r_t = \bar{r} + \psi(\pi_t - \pi_t^e) + \rho_a \omega_t$$

Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise 1: AR(1) process for Δa_t



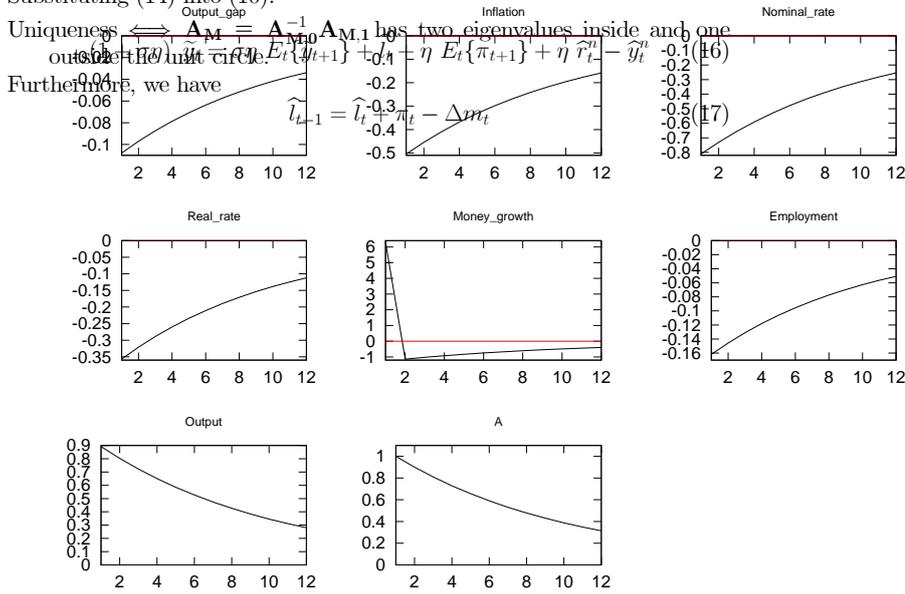
RESPONSES TO A TECHNOLOGY SHOCK (INTEREST RATE RULE)

where $\hat{l}_t \equiv m_t - \rho$ denotes \log real money balances. $\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}$

Substituting (14) into (10):

Uniqueness $\iff \Delta M = \Delta M^{-1}$. ΔM has two eigenvalues inside and one outside the unit circle.

Furthermore, we have



Money growth process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \quad (19)$$

where $\rho_m \in [0, 1)$

Figure 3 (based on $\rho_m = 0.5$)

Effects of a Technology Shock

Set $\Delta m_t = 0$ (no monetary shocks).

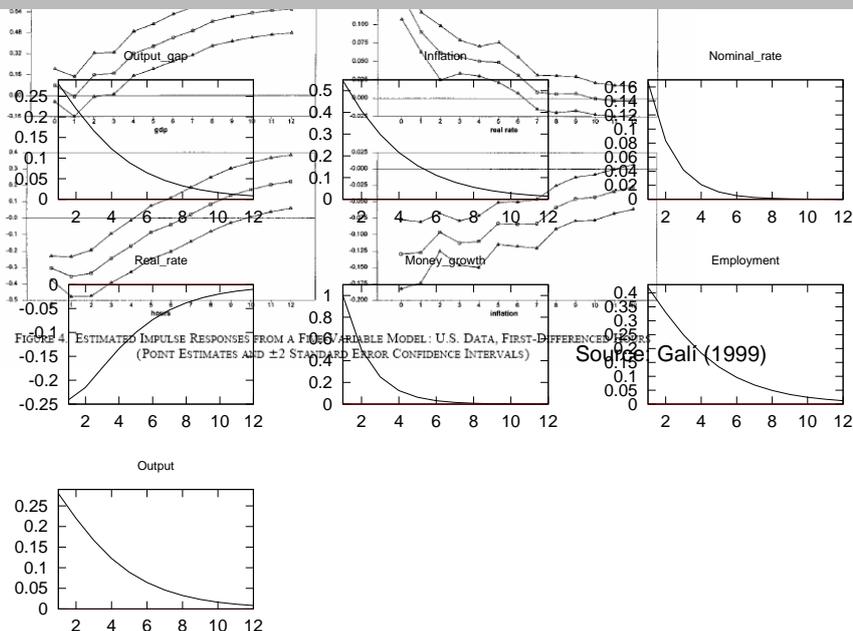
Technology process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

Figure 4 (based on $\rho_a = 0.9$).

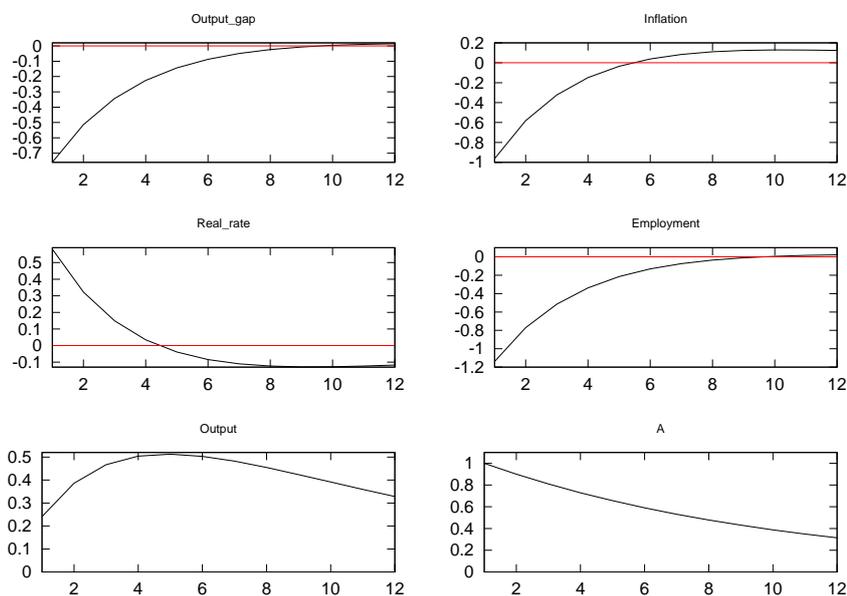
Empirical Evidence

RESPONSES TO A MONETARY POLICY SHOCK (MONEY GROWTH RATE RULE)



Note that liquidity effect is not present (due to the calibration)!

RESPONSES TO A TECHNOLOGY SHOCK (MONEY GROWTH RATE RULE)



**Monetary Policy Design
in the Basic New Keynesian Model**

by

Jordi Galí

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1. Distortions unrelated to nominal rigidities:

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$, subject to θ_i
 • *Monopolistic competition:* $P_t = \mathcal{M} \frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1} > 1$

$$C_t(i) = A_t N_t(i)^{1-\alpha}, \text{ all } i \in [0, 1]$$

$$\implies \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{N_t(i) \mathcal{M} i} < MPN_t$$

Optimality conditions: subsidy τ . Under flexible prices, $P_t = \mathcal{M} \frac{(1-\tau)W_t}{MPN_t}$.

$$\implies \frac{C_t(i) U_{n,t}}{N_t(i) U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{N_t(i) \mathcal{M} (1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) \frac{U_{n,t}}{U_{c,t}} = MPN_t$ or, equivalently, $\tau = \frac{1}{\epsilon}$.

Monopolistic competition (frictionless economy with valued money): assumed to be negligible

Assumptions:

- **Optimal employment subsidy** from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t M}{W_t/MPN_t}$ (as flexible price equilibrium allocation is efficient)
- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1] \implies -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{M}{\mathcal{M}_t} \neq MPN_t$

\implies Optimal employment subsidy is attained by a policy that stabilizes the price level at a level consistent with firms' desired markup, *given existing prices*:

- **Relative price distortions**, resulting from staggered price setting: no firm has an incentive to adjust its price, i.e. $P_t^i = P_{t-1}^i$ and $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods and firms.
- As a result the aggregate price level is fully stabilized and no relative price distortions emerge. Accordingly, markups should be identical across firms/goods at all times.

Equilibrium dynamics: $\tilde{y}_t = 0$

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_O \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

for all t .

Implementation: Some Candidate Interest Rate Rules

Not a policy block. The solution $\tilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of \mathbf{A}_O is strictly greater than one.

$$\tilde{y}_t = \frac{\sigma}{\sigma + \kappa} \pi_t \quad \text{and} \quad \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t$$

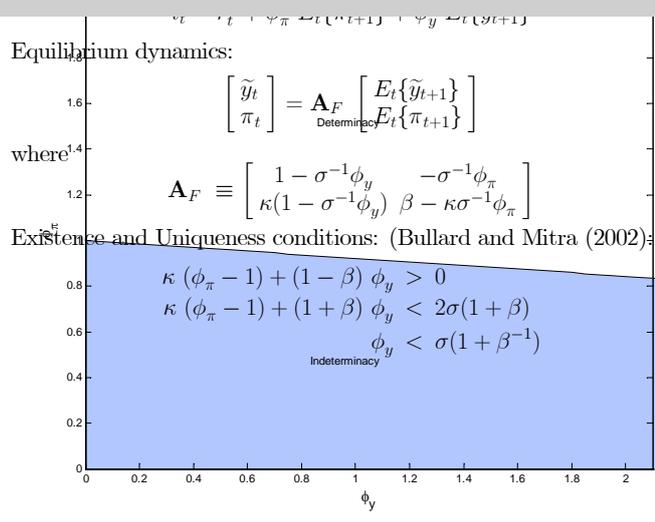
$$\kappa (\phi_\pi - r^* \beta) + (1 - \beta) \phi_y \tilde{y}_t > 0$$

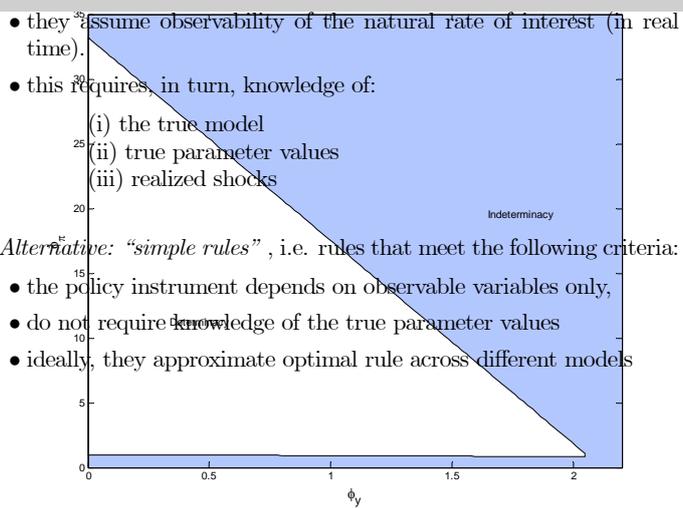
Equilibrium conditions interpretation (Woodford (2000)):

$$d \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t \{ \tilde{y}_{t+1} \} \\ E_t \{ \pi_{t+1} \} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{\begin{pmatrix} \phi_\pi + \phi_y (1 - \beta) \\ \sigma + \phi_y + \kappa \phi_\pi \end{pmatrix}}{\begin{bmatrix} \sigma & 1 \\ \sigma \kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}}$$





Welfare-based evaluation: $v_t = \rho + \phi_\pi \pi_t + \phi_y y_t$

Equivalently:

$$\mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - \rho U_{t+1}}{U_c C} \right) \pi_t + \frac{\phi_y}{2\lambda} \tilde{y}_0 \sum_{t=0}^{\infty} \beta^t (\kappa \tilde{y}_t^2 + \epsilon \pi_t^2)$$

where $v_t \equiv \phi_y \tilde{y}_t^n$

Equilibrium dynamics:

$$\Rightarrow \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \tilde{y}_{t+1} \\ \pi_{t+1} \end{bmatrix} + \mathbf{B}_T (\hat{r}_t^n - \phi_y \tilde{y}_t^n)$$

$\mathbb{L} = \frac{1}{2\lambda} \begin{bmatrix} E_t \{ \pi_{t+1} \} \\ \kappa \text{var}(\tilde{y}_t) + \epsilon \text{var}(\pi_t) \end{bmatrix}$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

See Appendix for Derivation.

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$. Note that $\hat{r}_t^n - \phi_y \tilde{y}_t^n = -\psi_{ya}^n [\sigma(1 - \rho_a) + \phi_y] a_t$

Exercise: $\Delta a_t \sim AR(1) + \text{modified Taylor rule } i_t = \rho + \phi_\pi \pi_t + \phi_y \Delta y_t$

money market clearing condition

$$\mathbf{A}_{M,0} \begin{bmatrix} y_t \\ \pi_t \\ l_t^+ \end{bmatrix} = \mathbf{A}_{M,1} \begin{bmatrix} E_t\{y_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{y}_t^n - b\hat{y}_t^r - \zeta_t \end{bmatrix} + \mathbf{B}_M \begin{bmatrix} r_t^m \\ \hat{y}_t^m \\ \Delta\zeta_t \end{bmatrix}$$

where $l_t \equiv m_t - p_t$ and ζ_t is a money demand shock following the

process

$$\mathbf{A}_{M,0} \equiv \begin{bmatrix} 1 + \sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^\zeta \\ \varepsilon_t^m \\ \varepsilon_t^r \end{bmatrix}; \quad \mathbf{A}_{M,1} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \mathbf{B}_M \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Define $l_t^+ \equiv l_t - \zeta_t$. \implies

Simulations and Evaluation of the Simplified Rules

$$\hat{l}_{t-1}^+ = \hat{l}_t^+ + \pi_t - \Delta\zeta_t$$

	<i>Taylor Rule</i>				<i>Constant Money Growth</i>	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	(0, 0)	(0.0063, 0.6)
$\sigma(\tilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
<i>welfare loss</i>	0.30	0.08	0.002	1.92	0.08	0.38

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