# Monetary Policy, Inflation and the Business Cycle

## MA0910: Macroeconomic Theory — Part II, Section 2

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7 Feb 2010 recompiling old slides

# OUTLINE

We follow chapters 1–5, 7 in Galí's excellent book:

1. Introduction
2. Monetary Policy in Classical Model
3. The Basic New Keynesian Model
4. Monetary Policy Design in the Basic New Keynesian Model
5. Time-Consistency in Monetary Policy: Discretion vs. Commitment
6. Monetary Policy in Open Economy

Note: many of the slides are reproduced from Galí's book's slide collection

(http://www.crei.cat/people/gali/monograph.html)

# Outline

1. Introduction
   - New Keynesian Model

2. Monetary Policy in Classical Model

3. The Basic New Keynesian Model

4. Monetary Policy Design in the Basic New Keynesian Model

5. Discretion vs Commitment: Monetary Policy and Time-Consistency

6. Open Economy and Monetary Poli

# Motivation

- Monetary policy has been a central area of macroeconomic research
- The macroeconomics of monetary policy studies the interaction between monetary policy, inflation and business cycles (fluctuations in economic activity)
- The modern models of monetary policy build on — as most of the recent macroeconomic research — on real business cycle models by Kydland and Prescott (1982) and Prescott (1986).

# Real Business Cycle Revolution

## Methodological revolution

Intertemporally optimizing agents. Budget and technology constraints.

## Conceptual revolution

- In a frictionless markets under perfect competition business cycles are efficient: no need for stabilization; stabilization may be counter-productive.
- Economic fluctuations are caused by technology shocks: they are the main source of fluctuation.
- Monetary factors (price level) has a limited (or no) role: money (price level) has no effect on the real economy, real wages, relative prices, consumption, investments, employment, . . . .

# NEW KEYNESIAN MODEL

- Methodologically similar to RBC models.
- Builds on the following features:
  - monopolistic competition
  - nominal rigidities
  
  $\rightarrow$ short-run non-neutrality of money: real interest rate affect money supply.
- Leads to differences w.r.t RBC models: economy's response to shocks is generally inefficient.
- Removing the effects of non-neutrality is potentially welfare improving $\rightarrow$ role for monetary policy.

# Empirical evidence

## Supportive to price rigidity

4–6 quarters by Bils and Klenow (2004), 8–11 quarters by Nakamura and Steinsson (2008), 8-11 by Dhyne et al. (2006).
Wages are also rigid: 1 year, downward rigidity by Dickens et al. (2007)

## Non-neutrality hard to measure

The expected inflation does not move as nominal interest rates, when latter is changed, the real interest rate changes and, as a result, equilibrium output and employment.
This is hard to show empirically. Christiano et al. (2005) provides example.

# OUTLINE

# Household problem

Households

- Decide how much they consume, $C_t$, in each period and
- how much they work, $N_t$.
- They know the *current and historical* values of nominal wages $W_t$,
- and their savings in bonds $B_t$, with bond price $Q_t$, and
- their lump-sum taxes, $T_t$.
- Price level is $P_t$.

They cannot accumulate infinite debt (transversality condition):

$$\lim_{T\to\infty} E_t B_T \geq 0$$

Households maximize the expected present value of utility

$$\max_{\{C_t, N_t\}} \mathrm{E}_0 \sum_{t=0}^{\infty} U(C_t, N_t)$$

subject to the following (flow) budget constraint

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t. \tag{3.1}$$

# First order conditions

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

Marginal rate of substitution between consumption and leisure (equaling real wages).

$$Q_t = \beta\,\mathsf{E}_t\left(\frac{U_{C,t+1}}{U_{C,t}}\frac{P_t}{P_{t+1}}\right)$$

Intertemporal marginal rate of substitution in nominal terms.

# Parametric version

Assume the following functional form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\sigma \geq 0$ and $\varphi \geq -1$.
Then

$$U_{N,t} = -N_t^{\varphi} \quad U_{C,t} = C_t^{-\sigma}$$

and the first order conditions as follows

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{3.2}$$

$$Q_t = \beta \, \mathrm{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{3.3}$$

# Sidestep to log-linearizations

## Taylor approximations are accurate only in the neighborhood of a point

As a consequence (loosely speaking) an economic model has to be stationary. This means that very often we need to stationarize the model first, ie to express it in terms of stationary variables.

Suppose we have

$$f(X_t, Y_t) = g(Z_t), \tag{3.4}$$

with strictly positive $X, Y, Z$ (ie the linearization point). The steady state counterpart is $f(X, Y) = g(Z)$.

This simple summarization is, for example, in the slides by Jürg Adamek).
(http://www.vwl.unibe.ch/studies/3076_e/linearisation_slides.pdf)

Let's denote $x_t = \log(X_t)$ (for any variable). Start from replacing $X_t = \exp(\log(X_t))$ in (3.4),

$$f\left(e^{\log(X_t)}, e^{\log(Y_t)}\right) = g\left(e^{\log(Z_t)}\right),$$

Taking first-order Taylor approximations from both sides:

$$\begin{aligned} f(X,Y) + f_1'(X,Y)X(x_t - x) + f_2'(X,Y)Y(y_t - y) \\ = g(Z) + g'(Z)Z(z_t - z) \quad (3.5) \end{aligned}$$

Often we denote $\hat{x}_t \equiv x_t - x$.

# Log-linearization of (3.3) around non-zero inflation and growth

In a growing economy $C_{t+1} > C_t$ and in a non-zero-inflation economy $P_{t+1} > P_t$. Hence, neither $C$ nor $P$ is a point. The growth rates are, however, stationary. Let's denote $\dot{X}_t \equiv X_t / X_{t-1}$. Then (3.3) is

$$Q_t = \beta \, \mathrm{E}_t \left( \dot{C}_{t+1}^{-\sigma} \dot{P}_{t+1}^{-1} \right).$$

In the steady state: $Q = \beta \dot{C}^{-\sigma} / \dot{P}$. Denote $\rho \equiv -\log \beta$, then $\rho = -q - (\sigma \Delta c + \pi)$, where $\pi_t \equiv \dot{P}_t$ and $\Delta x_t \equiv \dot{x}_t$. Note also that $i_t = -q_t$.

Apply mechanically (3.5)

$$Q + Q(q_t - q) = \underbrace{\beta \dot{C}^{-\sigma}/\dot{P}}_{=Q} - \sigma \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{C}} \dot{C}(\Delta \operatorname{E}_t c_{t+1} - \Delta c) + (-1) \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{P}} \dot{P}(\operatorname{E}_t \pi_{t+1} - \pi).$$

and divide by $Q$ and get rid of constants to obtain

$$q_t - q = -\sigma(\operatorname{E}_t \Delta c_{t+1} - \Delta c) - (\operatorname{E}_t \pi_{t+1} - \pi)$$

and combine with log of steady-state to obtain

$$i_t - \operatorname{E}_t \pi_{t+1} - \rho = \sigma \operatorname{E}_t \Delta c_{t+1} \tag{3.6}$$

*Log-linear versions*

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t \tag{7}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{8}$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$.(interpretation)

Perfect foresight steady state (with zero growth):

$$i = \pi + \rho$$

hence implying a real rate

$$r \equiv i - \pi = \rho$$

*Ad-hoc money demand*

$$m_t - p_t = y_t - \eta \ i_t$$

# Firms

Firms use only labour $N$ to produce output $Y$:

$$Y_t = A_t N_t^{1-\alpha}. \tag{3.9}$$

Log-linearized as $y_t = a_t + (1-\alpha)n_t$. $A_t$ is exogenously given stationary process (a shock). Firms maximize profits

$$P_t Y_t - W_t N_t$$

subject to production function (3.9) and obtain the following FOC:

$$(1-\alpha)A_t N_t^{-\alpha} = \frac{W_t}{P_t}. \tag{3.10}$$

which gives labour demand schedule and tells us how much labour the firm is willing to hire for given real wages and technological process $A_t$.

## Equilibrium

*Goods market clearing*

$$y_t = c_t \tag{11}$$

*Labor market clearing*

$$\sigma \; c_t + \varphi \; n_t = a_t - \alpha \; n_t + \log(1 - \alpha)$$

*Asset market clearing:*

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho)$$

*Aggregate production relationship:*

$$y_t = a_t + (1 - \alpha) \; n_t$$

Implied equilibrium values for real variables

$$n_t = \psi_{na}\ a_t + \vartheta_n$$

$$y_t = \psi_{ya}\ a_t + \vartheta_y$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho + \sigma\ E_t\{\Delta y_{t+1}\} = \rho + \sigma\psi_{ya}\ E_t\{\Delta a_{t+1}\}$$

$$\omega_t \equiv w_t - p_t = y_t - n_t + \log(1-\alpha) = \psi_{\omega a}\ a_t + \log(1-\alpha)$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$ ; $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$ ; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$

$$\vartheta_y \equiv (1-\alpha)\vartheta_n \quad ; \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$$

$\Longrightarrow$ real variables determined *independently of monetary policy* (neutrality)

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ specification of monetary policy needed to determine nominal variables

# Discussion on the parameters

## Technology shock has always positive impact on output

$$\phi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

is always positive given reasonable values of $\varphi > -1$.

## Technology shock may reduce or increase employment

$$\phi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} = \begin{cases} = 0 & \text{if } \sigma = 1 \\ \geq 0 & \text{if } \sigma < 1. \end{cases}$$

### Technology process defines properties of real interest rate

$r_t$ will go down if $\mathrm{E}_t\, a_{t+1} < a_t$ and go up if $\mathrm{E}_t\, a_{t+1} < a_t$ (like in the growing economy).

### Summary of equilibrium

In equilibrium, output, consumption, employment, real wages and real rate of return are function of productivity shock only — not of anything else! Hence monetary factors play no role in real economy, ie monetary policy is neutral w.r.t. real variables.

# Fixed interest rate rule

Consider standard Fisher equation that we have derived above

$$i_t = \mathrm{E}_t\, \pi_{t+1} + r_t = \mathrm{E}_t\, p_{t+1} - p_t + r_t. \tag{3.12}$$

Its solution should be of the form

$$p_t = -\mathrm{E}_t \sum_{i=0}^{\infty} (i_{t+i} - r_{t+i}).$$

It is easy to see that this does not converge in general.

# SIMPLE INFLATION BASED INTEREST RATE RULE

Consider an interest rate rule

$$i_t = \rho + \phi_\pi \pi_t$$

and combine it with the Fisher equation to obtain

$$\rho + \phi_\pi \pi_t = \mathsf{E}_t\, \pi_{t+1} + r_t .$$

Its solution is of the form

$$\pi_t = \mathsf{E}_t \sum_{i=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^i (r_{t+i} - \rho),$$

which is convergent if $0 \leq 1/\phi_\pi < 1$, ie

## TAYLOR PRINCIPLE

$\phi_\pi > 1$

# MONEY GROWTH RULE

Substitute (3.12) into the money demand equation to obtain

$$m_t - p_t = y_t - \eta(\mathrm{E}_t\, \pi_{t+1} + r_t).$$

Solve price level forward

$$\begin{aligned} p_t &= \frac{1}{1+\eta}\, \mathrm{E}_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta}(m_{t+i} + \eta\, r_{t+i} - y_{t+i}) \\ &= m_t + \mathrm{E}_t \sum_{i=1}^{\infty} \frac{\eta}{1+\eta} \Delta m_{t+i} + \mathrm{E}_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta}(\eta\, r_{t+i} - y_{t+i}) \end{aligned}$$

and the implied nominal interest rate

$$
\begin{aligned}
i_t &= \eta^{-1}\left[y_t - (m_t - p_t)\right] \\
&= \eta^{-1}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + v_t
\end{aligned}
$$

where $v_t \equiv \eta^{-1}(u_t + y_t)$.is independent of policy.

# Summary

- Real variables are independent of monetary policy.
- Monetary policy has an important impact on nominal variables.
- No monetary policy rule is better that any other.
- The non-existence of the interaction between nominal and real variables is in contrast to empirical evidence.

# Various approaches to motivate money

In the above classical models, money had a role of unit of account: *cashless economy, cashless limit.*

Money provides liquidity services. They can be modelled, for example, as

- Real balances generate utility: Money-in-the-utility-function (MIUF)
- The transaction cost approach
    - Explicit microfounded matching models starting from double coincidence of wants
    - Cash-in-advance (CIA) constraint

We study MIUF and leave CIA as an exercise. First item can be found from micro courses.

## A Model with Money in the Utility Function

*Preferences*

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

*Budget constraint*

$$P_t \, C_t + Q_t \, B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \, N_t - T_t$$

Letting $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$ :

$$P_t \, C_t + Q_t \mathcal{A}_{t+1} + (1 - Q_t) \, M_t \leq \mathcal{A}_t + W_t \, N_t - T_t$$

Interpretation: $\qquad (1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$

$\Longrightarrow$ opportunity cost of holding money

*Optimality Conditions*

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

where marginal utilities evaluated at $\left( C_t, \frac{M_t}{P_t}, N_t \right)$

Two cases:

- utility separable in real balances $\implies$ neutrality
- utility non-separable in real balances (e.g. $U_{cm} > 0$) $\implies$ non-neutrality

*How Important is the implied non-neutrality? (Walsh, ch. 2)*

Utility specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

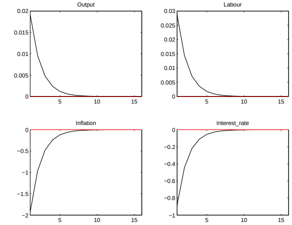
$$X(C_t, M_t/P_t) \equiv \left[(1-\vartheta)\, C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-v}} \quad for\ \nu \neq 1$$

$$\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t}\right)^{\vartheta} \quad for\ \nu = 1$$

Policy Rule: $\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m$
Calibration: $\nu = 2.56$ ; $\sigma = 2$ $\Longrightarrow$ $U_{cm} > 0$

Effects of Exogenous Monetary Policy Shock (Fig 2.3 and 2.4)

# Responses to a positive money supply shock



# Optimal monetary policy in a classical setting

Assume a hypothetical social planner that maximize the utility of representative household, that contains real money.

Social planner faces a static problem, since only an individual household (not the society as a whole) can smooth its consumption over time. The planner's problem is to maximize

$$\max U(C_t, \frac{M_t}{C_t}, N_t)$$

subject to resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

The optimality conditions are given by

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha} \quad (3.13)$$

$$U_{m,t} = 0 \quad (3.14)$$

(3.13) First corresponds the labour market equilibrium that is independent of monetary policy (except in the non-separable MIUF case)

(3.14) Second condition equates marginal utility of real balances to the "social"marginal cost of producing them (zero!).

From household's problem we know

$$\frac{U_{m,t}}{U_{c,t}} = 1 - e^{-i_t}.$$

RHS can be zero only if $i_t = 0$. This is called Friedman rule. (In steady-state) this results $\pi = -\rho(\equiv -\log(\beta)) < 0$, ie in the steady state, the price level declines at the rate of time preference.

*Implementation*

$$i_t = \phi \left( r_{t-1} + \pi_t \right)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi \; i_t$$

whose only stationary solution is $i_t = 0$ for all $t$.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$

# OUTLINE

# CES aggregator

In modern macro models with imperfect competition, the Dixit-Stiglitz, or Constant-Elasticity-of-Substitution aggregator plays an important role. Consider a static optimization problem of a firm that buy infinite number of intermediate products $C(i)$, puts them together using technology

$$C = \left[ \int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}, \tag{4.1}$$

where $\epsilon$ is the elasticity of substitution (and also the price elasticity of the demand function). Its optimization problem is

$$\max_{C(i)} C \cdot P - \int_0^1 C(i) P(i) di$$

subject to the production technology (4.1).

The optimality conditions is given by

$$C(i) = \left[\frac{P(i)}{P}\right]^{-\epsilon} C \quad \forall i \in [0,1].$$

This is also the demand function of a good $C(i)$. (You must work out the details by yourself.) Plug this to the profits and use zero profit constraint to get the aggregate price level (=price index=marginal costs):

$$P = \left[\int_0^1 P(i)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{4.2}$$

# INTRODUCTION

The basic new Keynesian model consists of two key ingredients:

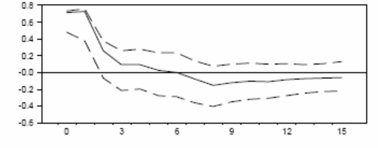
## IMPERFECT COMPETITION

We assume that there is a continuum of firms and each produce a differentiated intermediate good for which it sets the price.

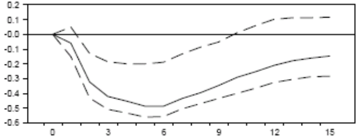
## PRICE RIGIDITIES

We assume (*a la* Calvo (1983)) that, in each period, only a fraction of firms can change their price.

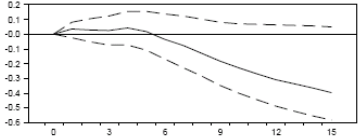
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



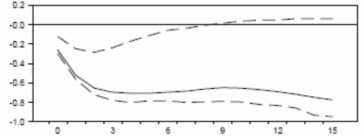

Federal funds rate




GDP

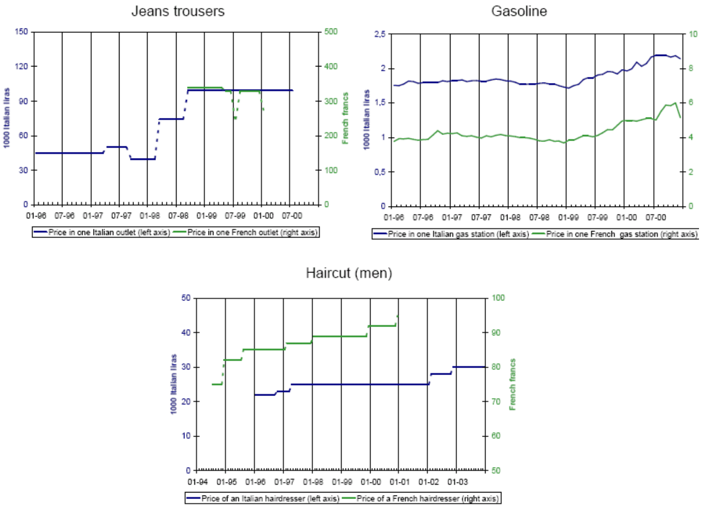



GDP deflator




M2

Source: Christiano, Eichenbaum and Evans (1999)




Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

# Households

Household problem is the same as in the case of classical model except that the aggregate consumption consists of continuum of goods:

$$C_t = \left[ \int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}$$

Household must allocate its consumption to different goods according to their relative price

$$C_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t, \tag{4.3}$$

where the aggregate price index is as in (4.2).

The optimality condition are, of course, the same and assuming same functional form of utility function, and loglinearizing, we obtain

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{4.4}$$

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho) \tag{4.5}$$

$$m_t - p_t = y_t - \eta i_t. \tag{4.6}$$

# FIRMS

Assume continuum of identical firms indexed by $i \in [0, 1]$ that use the following common production technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}.$$

All firms face an identical demand curve (4.3) and take aggregate price index $P_t$ and aggregate consumption index $C_t$ as given.

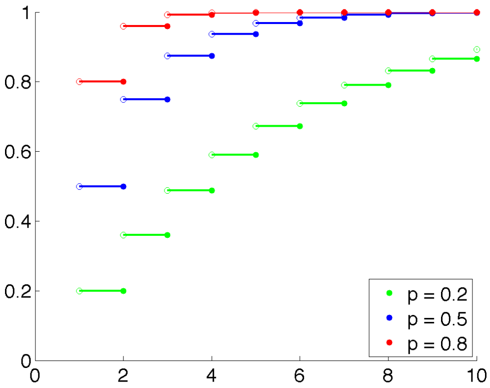
# Calvo Fairy

A firm may change price of its product only when Calvo Fairy visits. The probability of a visit is $1-\theta$. It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $1-\theta$ share of firms may change their price and rest, $\theta$, keep their price unchanged.

Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process). The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution. The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1-\theta}.$$

# Cumulative distribution function of geometric distribution




Source: `http://en.wikipedia.org/wiki/Geometric_distribution`

# Optimal price setting

Let $P_t^\star$ denote the price level of the firm that receives price change signal. When making its pricing decision, the firm takes into account that it can change is price with the probability $1-\theta$, ie the chosen price remains the same with probability $\theta$.

$$\max_{P_t^\star} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left\{ P_t^\star Y_{t+k}(P_t^\star) - \Psi_{t+k}\left[Y_{t+k}(P_t^\star)\right]\right\},$$

where

$$Y_{t+k}(P_t^\star) = \left(\frac{P_t^\star}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right).$$

The first one is the demand function that the firm faces and is due to the households' consumption index. The second equation is the nominal

The first-order condition

$$\mathsf{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left( P_t^\star - \underbrace{\frac{\epsilon}{1-\epsilon}}_{\equiv \mathcal{M}} \psi_{t,t+k} \right) = 0,$$

where

$$\psi_{t,t+k} = \Psi'_{t+k}\left(Y_{t+k}(P_t^\star)\right)$$

is nominal marginal costs and $\mathcal{M}$ is the desired or frictionless markup. In the case of no frictions $(1-\theta=1)$:

$$P_t^\star = \mathcal{M}\psi_{t,t}.$$

Equivalently,

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ Y_{t+k|t} \ \left( \frac{P_t^*}{P_{t-1}} - \mathcal{M} \ MC_{t+k|t} \ \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

*Perfect Foresight, Zero Inflation Steady State:*

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\mathcal{M}}$$

Log-linearization around zero inflation steady state:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$.

Equivalently,

$$p_t^* = \mu + (1 - \beta\theta) \ \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{mc_{t+k|t} + p_{t+k}\}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon - 1}$.

Flexible prices ($\theta = 0$):

$$p_t^* = \mu + mc_t + p_t$$

$\Longrightarrow \qquad mc_t = -\mu$ (symmetric equilibrium)

# AGGREGATE PRICE LEVEL

Note that all those firms that may set their price level (ie where Calvo Fairy visits) will choose identical price $P_t^\star$. Let $S(t) \subset [0, 1]$ represent the set of firms that do not reoptimize their price in period $t$. Note that "the distribution of prices among firms not adjusting in period $t$ corresponds to the distribution effective prices in period $t - 1$, though with total mass reduced to $\theta$". Remember the aggregate price index $P_t$ in eq (4.2). The aggregate price level has to follow this functional form

$$P_t = \left[ \int_{S(t)} P_t(i)^{1-\epsilon} di + (1-\theta)(P_t^\star)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

$$= \left[ \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^\star)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} . \quad (4.7)$$

# Log-linearization of aggregate price level

Stationarize equation (4.7), ie express it in inflation rates and in relative prices: Divide (4.7) by $P_{t-1}$ and rise it power $1-\epsilon$:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^\star}{P_{t-1}}\right)^{1-\epsilon},$$

where $\Pi_t = P_t/P_{t-1}$.
Log-linearizing it around $\Pi = 1$ and $P_t^\star/P_t = 1$ (zero steady state inflation, unit relative prices in steady-state):

$$\pi_t = (1-\theta)(p_t^\star - p_{t-1}).$$

# Equilibrium

Goods market

$$Y_t(i) = C_t(i) \quad \forall i \in [0.1] \text{ and } \forall t.$$

Due to aggregator (4.1)

$$Y_t = C_t. \quad \forall t$$

Labour markets

$$N_t = \int_0^1 N_t(i) di.$$

*Particular Case:* $\alpha = 0$ (constant returns)

$$\Longrightarrow \; MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta\theta \; E_t\{p_{t+1}^*\} + (1-\beta\theta) \; \widehat{mc}_t + (1-\beta\theta) p_t \tag{2}$$

Combining (1) and (2):

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

*Generalization to* $\alpha \in (0,1)$ (decreasing returns)

Define

$$\begin{aligned} mc_t &\equiv (w_t - p_t) - mpn_t \\ &\equiv (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$,

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \qquad (3) \end{aligned}$$

Implied inflation dynamics

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \lambda\, \widehat{mc}_t \qquad (4)$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\,\frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

## Equilibrium

*Goods markets clearing*

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all $t$.

Letting $Y_t \equiv \left( \int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$,

$$Y_t = C_t$$

for all $t$. Combined with the consumer's Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right) \tag{5}$$

*Labor market clearing*

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)\,di \\
&= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\
&= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di
\end{aligned}
$$

Taking logs,

$$(1-\alpha)\,n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log\int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\,di$ (second order).

Up to a first order approximation:

$$y_t = a_t + (1-\alpha)\,n_t$$

*Marginal Cost and Output*

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma\ y_t + \varphi\ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ y_t - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \end{aligned} \tag{6}$$

Under *flexible prices*

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ y_t^n - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \tag{7}$$

$$\implies y_t^n = -\delta_y + \psi_{ya}\ a_t$$

where $\delta_y \equiv \frac{(\mu - \log(1-\alpha))(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$.

$$\implies \widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ (y_t - y_t^n) \tag{8}$$

where $y_t - y_t^n \equiv \widetilde{y}_t$ is the *output gap*

*New Keynesian Phillips Curve*

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ \widetilde{y}_t \tag{9}$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

*Dynamic IS equation*

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) \tag{10}$$

where $r_t^n$ is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma \ E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma\psi_{ya} \ E_t\{\Delta a_{t+1}\} \end{aligned}$$

*Missing block:* description of monetary policy (determination of $i_t$).

# Monetary policy

Determination of nominal interest rate, $i_t$, gives the path for actual real rate. It is a description how monetary policy is conducted.

## Monetary policy is non-neutral

When prices are sticky nominal interest rate path determines output gap.

# Dynamics

To study the dynamics of a linear rational expectation model, we write is as follows

$$A_0 x_t = A_1 \mathrm{E}_t \, x_{t+1} + B z_t .$$

and study the properties of the transition matrix $A_T \equiv A_0^{-1} A_1$ of the following form

$$x_t = \underbrace{A_0^{-1} A_1}_{\equiv A_T} \mathrm{E}_t \, x_{t+1} + \underbrace{A_0^{-1} B}_{\equiv B_T} z_t .$$

$A_0$ should be of full rank (some solution algorithms allow reduced rank cases too).

## Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t \tag{11}$$

where $v_t$ is exogenous (possibly stochastic) with zero mean.

*Equilibrium Dynamics:* combining (9), (10), and (11)

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - v_t) \tag{12}$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

*Uniqueness* $\iff \mathbf{A}_T$ has both eigenvalues within the unit circle

Given $\phi_\pi \geq 0$ and $\phi_y \geq 0$, (Bullard and Mitra (2002)):

$$\kappa \left(\phi_\pi - 1\right) + \left(1 - \beta\right) \phi_y > 0$$

is necessary and sufficient.

*Effects of a Monetary Policy Shock*

Set $\widehat{r}_t^n = 0$ (no real shocks).

Let $v_t$ follow an AR(1) process

$$v_t = \rho_v \, v_{t-1} + \varepsilon_t^v$$

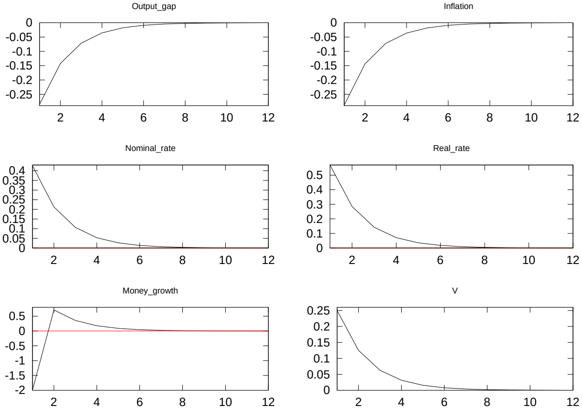
Calibration:

$$\rho_v = 0.5,\ \phi_\pi = 1.5,\ \phi_y = 0.5/4,\ \beta = 0.99,\ \sigma = \varphi = 1,\ \theta = 2/3,\ \eta = 4.$$

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution

# Responses to a monetary policy shock (interest rate rule)



*Effects of a Technology Shock*

Set $v_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a.$$

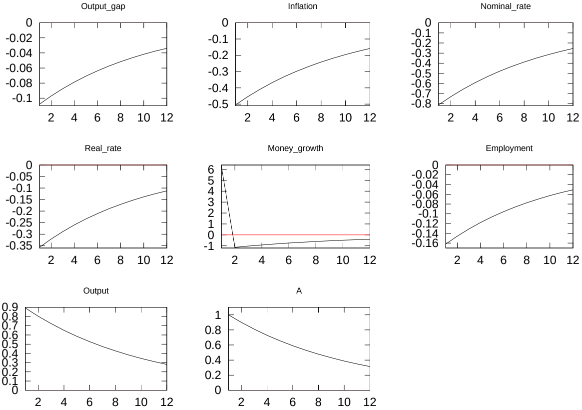
Implied natural rate:

$$\widehat{r}_t^n = -\sigma \psi_{ya}(1 - \rho_a) \; a_t$$

Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise: AR(1) process for $\Delta a_t$

# RESPONSES TO A TECHNOLOGY SHOCK (INTEREST RATE RULE)



**Equilibrium under an Exogenous Money Growth Process**

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{13}$$

Money market clearing

$$\widehat{l}_t = \widehat{y}_t - \eta \, \widehat{i}_t \tag{14}$$
$$= \widetilde{y}_t + \widehat{y}_t^n - \eta \, \widehat{i}_t \tag{15}$$

where $l_t \equiv m_t - p_t$ denotes (log) real money balances.

Substituting (14) into (10):

$$(1 + \sigma\eta) \; \widetilde{y}_t = \sigma\eta \; E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta \; E_t\{\pi_{t+1}\} + \eta \; \widehat{r}_t^n - \widehat{y}_t^n \tag{16}$$

Furthermore, we have

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t \tag{17}$$

*Equilibrium dynamics*

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_{t-1} \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta m_t \end{bmatrix} \tag{18}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness $\iff$ $\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}} \mathbf{A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

*Effects of a Monetary Policy Shock*

Set $\widehat{r}_t^n = y_t^n = 0$ (no real shocks).

Money growth process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{19}$$

where $\rho_m \in [0, 1)$

Figure 3 (based on $\rho_m = 0.5$)

*Effects of a Technology Shock*

Set $\Delta m_t = 0$ (no monetary shocks).

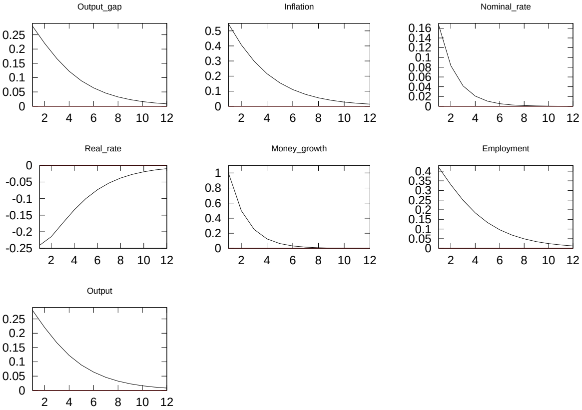
Technology process:

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a .$$

Figure 4 (based on $\rho_a = 0.9$).

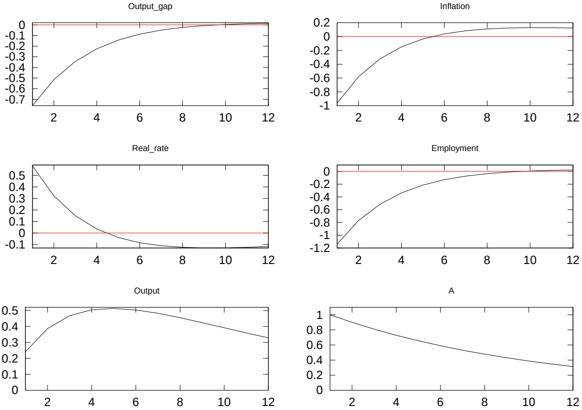
Empirical Evidence

# Responses to a monetary policy shock (money growth rate rule)




Note that liquidity effect is not present (due to the calibration)!

# RESPONSES TO A TECHNOLOGY SHOCK (MONEY GROWTH RULE)



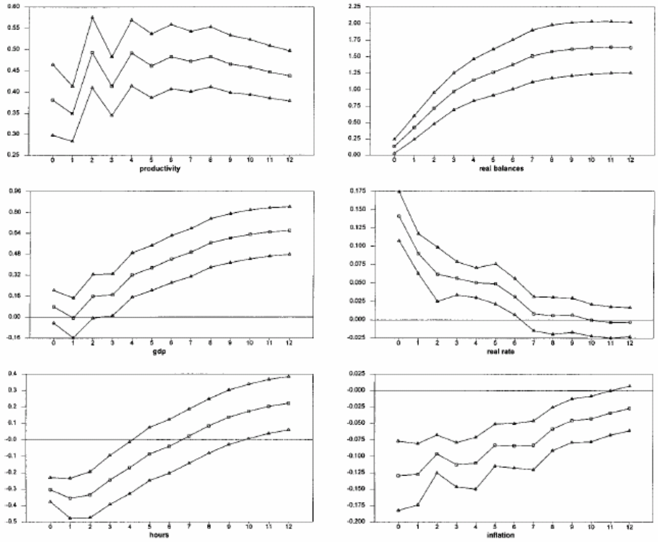



Figure 4. Estimated Impulse Responses from a Five-Variable Model: U.S. Data, First-Differenced Hours (Point Estimates and ±2 Standard Error Confidence Intervals)

Source: Galí (1999)

# Outline

# Lectures on Monetary Policy, Inflation and the Business cycle

## Monetary Policy Design in the Basic New Keynesian Model

*by*

*Jordi Galí*

## The Efficient Allocation

$$\max U\left(C_t, N_t\right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t\ N_t(i)^{1-\alpha},\ \ all\ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i)\ di$$

*Optimality conditions*:

$$C_t(i) = C_t,\ \ all\ i \in [0,1]$$

$$N_t(i) = N_t,\ \ all\ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha)\ A_t N_t^{-\alpha}$.

## Sources of Suboptimality of Equilibrium

### 1. Distortions unrelated to nominal rigidities:

- *Monopolistic competition*: $P_t = \mathcal{M} \, \frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

*Solution:* employment subsidy $\tau$. Under flexible prices, $P_t = \mathcal{M} \, \frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

- *Transactions friction* (economy with valued money): assumed to be negligible

**2. Distortions associated with the presence of nominal rigidities:**

- *Markup variations* resulting from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$ (assuming optimal subsidy)

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \, \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Optimality requires that the average markup be stabilized at its frictionless level.

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods. Accordingly, markups should be identical across firms/goods at all times.

## Optimal Monetary Policy in the Basic NK Model

*Assumptions:*

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$

$\Longrightarrow$ the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$As a result the aggregate price level is fully stabilized and no relative price distortions emerge.
- equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

## Equilibrium under the Optimal Policy

$$\widetilde{y}_t = 0$$
$$\pi_t = 0$$
$$i_t = r_t^n$$

for all $t$.

## Implementation: Some Candidate Interest Rate Rules

*Non-Policy Block:*

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$
$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \kappa\, \widetilde{y}_t$$

*An Exogenous Interest Rate Rule*

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

*Shortcoming*: the solution $\widetilde{y}_t = \pi_t = 0$ for all $t$ is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one.

$\rightarrow$ indeterminacy. (real and nominal).

*An Interest Rate Rule with Feedback from Target Variables*

$$i_t = r_t^n + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

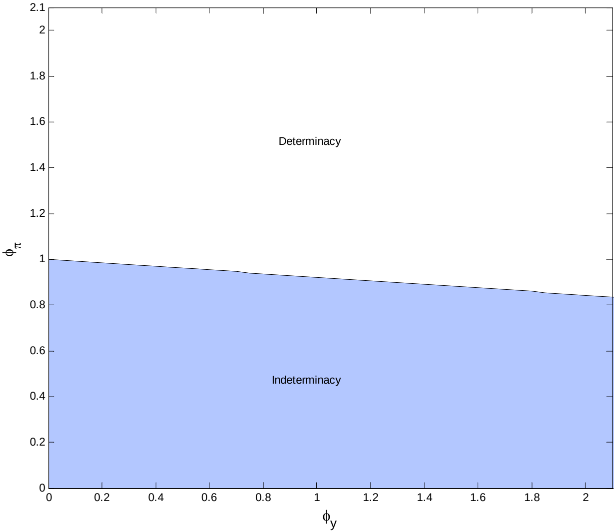
Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa\ (\phi_\pi - 1) + (1 - \beta)\ \phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned} di &= \phi_\pi\ d\pi + \phi_y\ d\widetilde{y} \\ &= \left( \phi_\pi + \frac{\phi_y\ (1 - \beta)}{\kappa} \right)\ d\pi \end{aligned}$$

Figure 4.1


Determinacy
Indeterminacy
$\phi_\pi$
$\phi_y$
2.1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

*A Forward-Looking Interest Rate Rule*

$$i_t = r_t^n + \phi_\pi \, E_t\{\pi_{t+1}\} + \phi_y \, E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

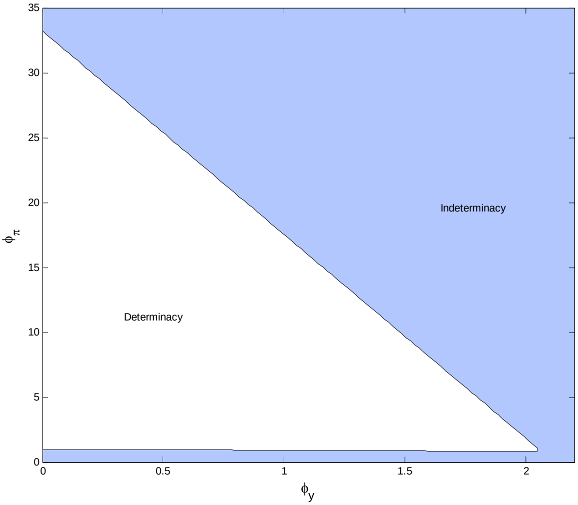
where

$$\mathbf{A}_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}$$

Existence and Uniqueness conditions: (Bullard and Mitra (2002):

$$\begin{aligned} \kappa\,(\phi_\pi - 1) + (1 - \beta)\,\phi_y &> 0 \\ \kappa\,(\phi_\pi - 1) + (1 + \beta)\,\phi_y &< 2\sigma(1 + \beta) \\ \phi_y &< \sigma(1 + \beta^{-1}) \end{aligned}$$

Figure 4.2



*Shortcomings of Optimal Rules*

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

*Alternative: "simple rules"* , i.e. rules that meet the following criteria:

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

## Simple Monetary Policy Rules

*Welfare-based evaluation:*

$$\mathbb{W} \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left( \kappa \, \widetilde{y}_t^2 + \epsilon \, \pi_t^2 \right)$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2\lambda} \left[ \kappa \; var(\widetilde{y}_t) + \epsilon \; var(\pi_t) \right]$$

See Appendix for Derivation.

*A Taylor Rule*

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \, \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - \phi_y \, \widehat{y}_t^n)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$. Note that $\widehat{r}_t^n - \phi_y \, \widehat{y}_t^n = -\psi_{ya}^n \, [\sigma(1-\rho_a) + \phi_y] \, a_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \Delta y_t$

*Money Growth Peg*

$$\Delta m_t = 0$$

money market clearing condition

$$\widehat{l_t} = \widetilde{y}_t + \widehat{y}_t^n - \eta\,\widehat{i}_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ and $\xi_t$ is a money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta\,\Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

Define $l_t^+ \equiv l_t - \zeta_t$. $\Longrightarrow$

$$\widehat{i}_t = \frac{1}{\eta}\left(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+\right)$$

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l^+_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l^+_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}^n_t \\ \widehat{y}^n_t \\ \Delta\zeta_t \end{bmatrix}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

*Simulations and Evaluation of Simple Rules*

**Table 4.1: Evaluation of Simple Monetary Policy Rules**

| | *Taylor Rule* | | | | *Constant Money Growth* | |
|---|---|---|---|---|---|---|
| $\phi_\pi$ | 1.5 | 1.5 | 5 | 1.5 | - | - |
| $\phi_y$ | 0.125 | 0 | 0 | 1 | - | - |
| $(\sigma_\zeta, \rho_\zeta)$ | - | - | - | - | $(0, 0)$ | $(0.0063, 0.6)$ |
| $\sigma(\widetilde{y})$ | 0.55 | 0.28 | 0.04 | 1.40 | 1.02 | 1.62 |
| $\sigma(\pi)$ | 2.60 | 1.33 | 0.21 | 6.55 | 1.25 | 2.77 |
| $welfare\ loss$ | 0.30 | 0.08 | 0.002 | 1.92 | 0.08 | 0.38 |

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