MONETARY POLICY, INFLATION AND THE BUSINESS CYCLE

MA0910: MACROECONOMIC THEORY — PART II, SECTION 2

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OUTLINE

We follow chapters 1–5, 7 in Galí's excellent book:

1. Introduction
2. Monetary Policy in Classical Model
3. The Basic New Keynesian Model
4. Monetary Policy Design in the Basic New Keynesian Model
5. Time-Consistency in Monetary Policy: Discretion vs. Commitment
6. Monetary Policy in Open Economy

Note: many of the slides are reproduced from Galí's book's slide collection

(http://www.crei.cat/people/gali/monograph.html)

Outline

Motivation

- Monetary policy has been a central area of macroeconomic research
- The macroeconomics of monetary policy studies the interaction between monetary policy, inflation and business cycles (fluctuations in economic activity)
- The modern models of monetary policy build on — as most of the recent macroeconomic research — on real business cycle models by Kydland and Prescott (1982) and Prescott (1986).

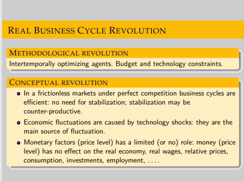
Real Business Cycle Revolution

Methodological revolution

Intertemporally optimizing agents. Budget and technology constraints.

Conceptual revolution

- In a frictionless markets under perfect competition business cycles are efficient: no need for stabilization; stabilization may be counter-productive.
- Economic fluctuations are caused by technology shocks: they are the main source of fluctuation.
- Monetary factors (price level) has a limited (or no) role: money (price level) has no effect on the real economy, real wages, relative prices, consumption, investments, employment, ...



Now all macro research follows this methodological approach.

New Keynesian Model

- Methodologically similar to RBC models.
- Builds on the following features:
  - monopolistic competition
  - nominal rigidities
  - $\longrightarrow$ short-run non-neutrality of money: real interest rate affect money supply.
- Leads to differences w.r.t RBC models: economy's response to shocks is generally inefficient.
- Removing the effects of non-neutrality is potentially welfare improving $\longrightarrow$ role for monetary policy.

Empirical evidence

Supportive to price rigidity

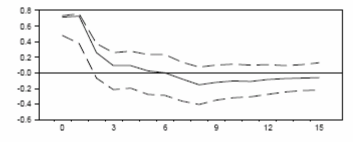
4–6 quarters by Bils and Klenow (2004), 8–11 quarters by Nakamura and Steinsson (2008), 8-11 by Dhyne et al. (2006).
Wages are also rigid: 1 year, downward rigidity by Dickens et al. (2007)

Non-neutrality hard to measure

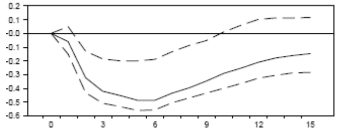
The expected inflation does not move as nominal interest rates, when latter is changed, the real interest rate changes and, as a result, equilibrium output and employment.
This is hard to show empirically. Christiano et al. (2005) provides example.

Response to monetary policy shock

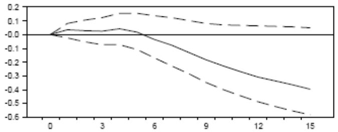
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



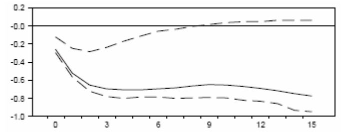
Federal funds rate



GDP



GDP deflator



M2

Outline

Household problem

Households

- Decide how much they consume, C_t, in each period and
- how much they work, N_t.
- They know the *current and historical* values of nominal wages W_t,
- and their savings in bonds B_t, with bond price Q_t, and
- their lump-sum taxes, T_t.
- Price level is P_t.

They cannot accumulate infinite debt (transversality condition):

$$\lim_{T \to \infty} E_t B_T \geq 0$$

Households maximize the expected present value of utility

$$\max_{\{C_t, N_t\}} \mathrm{E}_0 \sum_{t=0}^{\infty} U(C_t, N_t)$$

subject to the following (flow) budget constraint

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t. \tag{3.1}$$

First order conditions

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

Marginal rate of substitution between consumption and leisure (equaling real wages).

$$Q_t = \beta\, \mathrm{E}_t \left(\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right)$$

Intertemporal marginal rate of substitution in nominal terms.

Parametric version

Assume the following functional form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\sigma \geq 0$ and $\varphi \geq -1$.
Then

$$U_{N,t} = -N_t^{\varphi} \quad U_{C,t} = C_t^{-\sigma}$$

and the first order conditions as follows

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{3.2}$$

eq:foc1

$$Q_t = \beta \, \mathrm{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}. \tag{3.3}$$

eq:foc2

Sidestep to log-linearizations

Taylor approximations are accurate only in the neighborhood of a point

As a consequence (loosely speaking) an economic model has to be stationary. This means that very often we need to stationarize the model first, ie to express it in terms of stationary variables.

Suppose we have

$$f(X_t, Y_t) = g(Z_t), \tag{3.4}$$

eq:loglin:1

with strictly positive X, Y, Z (ie the linearization point). The steady state counterpart is $f(X, Y) = g(Z)$.

This simple summarization is, for example, in the slides by Jürg Adamek).
(http://www.vwl.unibe.ch/studies/3076_e/linearisation_slides.pdf)

Let's denote $x_t = \log(X_t)$ (for any variable). Start from replacing $X_t = \exp(\log(X_t))$ in (3.4),

$$f\left(e^{\log(X_t)}, e^{\log(Y_t)}\right) = g\left(e^{\log(Z_t)}\right),$$

Taking first-order Taylor approximations from both sides:

$$f(X,Y) + f_1'(X,Y)X(x_t - x) + f_2'(X,Y)Y(y_t - y) \\ = g(Z) + g'(Z)Z(z_t - z) \quad \text{eq:loglin:2} \quad (3.5)$$

Often we denote $\hat{x}_t \equiv x_t - x$.

Log-linearization of (3.3) around non-zero inflation and growth

In a growing economy $C_{t+1} > C_t$ and in a non-zero-inflation economy $P_{t+1} > P_t$. Hence, neither C nor P is a point. The growth rates are, however, stationary. Let's denote $\dot{X}_t \equiv X_t / X_{t-1}$. Then (3.3) is

$$Q_t = \beta \, \mathrm{E}_t \left(\dot{C}_{t+1}^{-\sigma} \dot{P}_{t+1}^{-1} \right).$$

In the steady state: $Q = \beta \dot{C}^{-\sigma} / \dot{P}$. Denote $\rho \equiv -\log \beta$, then $\rho = -q - (\sigma \Delta c + \pi)$, where $\pi_t \equiv \dot{P}_t$ and $\Delta x_t \equiv \dot{x}_t$. Note also that $i_t = -q_t$.

Apply mechanically (3.5)

$$Q + Q(q_t - q) = \underbrace{\beta \dot{C}^{-\sigma}/\dot{P}}_{=Q} - \sigma \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{C}} \dot{C}(\mathrm{E}_t \, \Delta c_{t+1} - \Delta c)$$
$$+ (-1) \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{P}} \dot{P}(\mathrm{E}_t \, \pi_{t+1} - \pi).$$

and divide by Q and get rid of constants to obtain

$$q_t - q = -\sigma(\mathrm{E}_t \, \Delta c_{t+1} - \Delta c) - (\mathrm{E}_t \, \pi_{t+1} - \pi)$$

and combine with log of steady-state to obtain

$$i_t - \mathrm{E}_t \, \pi_{t+1} - \rho = \sigma \, \mathrm{E}_t \, \Delta c_{t+1} \quad \text{eq:loglin:loglin3} \tag{3.6}$$

Log-linear versions

$$w_t - p_t = \sigma\ c_t + \varphi\ n_t \tag{7}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) \tag{8}$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$.(interpretation)

Perfect foresight steady state (with zero growth):

$$i = \pi + \rho$$

hence implying a real rate

$$r \equiv i - \pi = \rho$$

Ad-hoc money demand

$$m_t - p_t = y_t - \eta\ i_t$$

FIRMS

Firms use only labour N to produce output Y:

$$Y_t = A_t N_t^{1-\alpha}. \quad \text{eq:f} \tag{3.9}$$

Log-linearized as $y_t = a_t + (1-\alpha)n_t$. A_t is exogenously given stationary process (a shock). Firms maximize profits

$$P_t Y_t - W_t N_t$$

subject to production function (3.9) and obtain the following FOC:

$$(1-\alpha)A_t N_t^{-\alpha} = \frac{W_t}{P_t}. \tag{3.10}$$

which gives labour demand schedule and tells us how much labour the firm is willing to hire for given real wages and technological process A_t.

Equilibrium

Goods market clearing

$$y_t = c_t \tag{11}$$

Labor market clearing

$$\sigma \; c_t + \varphi \; n_t = a_t - \alpha \; n_t + \log(1 - \alpha)$$

Asset market clearing:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right)$$

Aggregate production relationship:

$$y_t = a_t + (1 - \alpha) \; n_t$$

Implied equilibrium values for real variables

$$n_t = \psi_{na}\ a_t + \vartheta_n$$

$$y_t = \psi_{ya}\ a_t + \vartheta_y$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho + \sigma\ E_t\{\Delta y_{t+1}\} = \rho + \sigma\psi_{ya}\ E_t\{\Delta a_{t+1}\}$$

$$\omega_t \equiv w_t - p_t = y_t - n_t + \log(1-\alpha) = \psi_{\omega a}\ a_t + \log(1-\alpha)$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$; $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$

$$\vartheta_y \equiv (1-\alpha)\vartheta_n \quad ; \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$$

$\Longrightarrow$ real variables determined *independently of monetary policy* (neutrality)

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ specification of monetary policy needed to determine nominal variables

Discussion on the parameters

Technology shock has always positive impact on output

$$\phi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

is always positive given reasonable values of $\varphi > -1$.

Technology shock may reduce or increase employment

$$\phi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} = \begin{cases} <0 & \text{if } \sigma > 1 \\ =0 & \text{if } \sigma = 1 \\ >0 & \text{if } \sigma < 1. \end{cases}$$

A closer look at employment response

Let's rewrite (3.2) as follows

$$N_t^{\varphi} = \frac{W_t}{P_t} C_t^{-\sigma}$$

The elasticity of substitution is unity (or $1/\varphi$), and wealth elasticity is $-\sigma$ (or $-\sigma/\varphi$).

- The substitution effect dominates the negative wealth effect if $\sigma < 1$ and
- *vice versa* if $\sigma > 1$.
- They cancel each other when $\sigma = 1$ (logarithmic utility).

Technology process defines properties of real interest rate

r_t will go down if $\mathrm{E}_t\, a_{t+1} < a_t$ and go up if $\mathrm{E}_t\, a_{t+1} > a_t$ (like in the growing economy).

Summary of equilibrium

In equilibrium, output, consumption, employment, real wages and real rate of return are function of productivity shock only — not of anything else! Hence monetary factors play no role in real economy, ie monetary policy is neutral w.r.t. real variables.

Fixed interest rate rule

Consider standard Fisher equation that we have derived above

$$i_t = \mathrm{E}_t\,\pi_{t+1} + r_t = \mathrm{E}_t\,p_{t+1} - p_t + r_t. \qquad \text{eq:fisher} \quad (3.12)$$

Its solution should be of the form

$$p_t = -\,\mathrm{E}_t \sum_{i=0}^{\infty} (i_{t+i} - r_{t+i}).$$

It is easy to see that this does not converge in general.

Simple inflation based interest rate rule

Consider an interest rate rule

$$i_t = \rho + \phi_\pi \pi_t$$

and combine it with the Fisher equation to obtain

$$\rho + \phi_\pi \pi_t = \mathrm{E}_t\, \pi_{t+1} + r_t.$$

Its solution is of the form

$$\pi_t = \mathrm{E}_t \sum_{i=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^i (r_{t+i} - \rho),$$

which is convergent if $0 \leq 1/\phi_\pi < 1$, ie

Taylor principle

$\phi_\pi > 1$

Money growth rule

Substitute (3.12) into the money demand equation to obtain

$$m_t - p_t = y_t - \eta(\mathrm{E}_t \, \pi_{t+1} + r_t).$$

Solve price level forward

$$\begin{aligned} p_t &= \frac{1}{1+\eta} \mathrm{E}_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta} (m_{t+i} + \eta r_{t+i} - y_{t+i}) \\ &= m_t + \mathrm{E}_t \sum_{i=1}^{\infty} \frac{\eta}{1+\eta} \Delta m_{t+i} + \mathrm{E}_t \sum_{i=0}^{\infty} \frac{\eta}{1+\eta} (\eta r_{t+i} - y_{t+i}) \end{aligned}$$

and the implied nominal interest rate

$$\begin{aligned} i_t &= \eta^{-1}\left[y_t - (m_t - p_t)\right] \\ &= \eta^{-1}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + v_t \end{aligned}$$

where $v_t \equiv \eta^{-1}(u_t + y_t)$.is independent of policy.

Summary

- Real variables are independent of monetary policy.
- Monetary policy has an important impact on nominal variables.
- No monetary policy rule is better that any other.
- The non-existence of the interaction between nominal and real variables is in contrast to empirical evidence.

Various approaches to motivate money

In the above classical models, money had a role of unit of account: *cashless economy, cashless limit.*

Money provides liquidity services. They can be modelled, for example, as

- Real balances generate utility: Money-in-the-utility-function (MIUF)
- The transaction cost approach
  - Explicit microfounded matching models starting from double coincidence of wants
  - Cash-in-advance (CIA) constraint

We study MIUF and leave CIA as an exercise. First item can be found from micro courses.

A Model with Money in the Utility Function

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

Budget constraint

$$P_t \, C_t + Q_t \, B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \, N_t - T_t$$

Letting $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$:

$$P_t \, C_t + Q_t \mathcal{A}_{t+1} + (1 - Q_t) \, M_t \leq \mathcal{A}_t + W_t \, N_t - T_t$$

Interpretation: $\quad (1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$

$$\Longrightarrow \text{opportunity cost of holding money}$$

Optimality Conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

where marginal utilities evaluated at $\left(C_t, \frac{M_t}{P_t}, N_t\right)$

Two cases:

- utility separable in real balances $\implies$ neutrality
- utility non-separable in real balances (e.g. $U_{cm} > 0$) $\implies$ non-neutrality

How Important is the implied non-neutrality? (Walsh, ch. 2)

Utility specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$X(C_t, M_t/P_t) \equiv \left[(1-\vartheta)\, C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-v}} \quad for\ \nu \neq 1$$

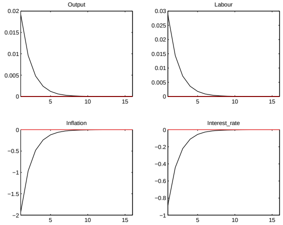
$$\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t}\right)^{\vartheta} \quad for\ \nu = 1$$

Policy Rule: $\Delta m_t = \rho_m\, \Delta m_{t-1} + \varepsilon_t^m$

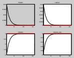
Calibration: $\nu = 2.56 \quad ; \quad \sigma = 2 \quad \Longrightarrow \quad U_{cm} > 0$

Effects of Exogenous Monetary Policy Shock (Fig 2.3 and 2.4)

RESPONSES TO A POSITIVE MONEY SUPPLY SHOCK



Responses to a positive money supply shock



Optimal monetary policy in a classical setting

Assume a hypothetical social planner that maximize the utility of representative household, that contains real money.
Social planner faces a static problem, since only an individual household (not the society as a whole) can smooth its consumption over time. The planner's problem is to maximize

$$\max U(C_t, \frac{M_t}{C_t}, N_t)$$

subject to resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

The optimality conditions are given by

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha} \quad \text{eq:2:plan1} \quad (3.13)$$

$$U_{m,t} = 0 \quad \text{eq:2:plan2} \quad (3.14)$$

(3.13) First corresponds the labour market equilibrium that is independent of monetary policy (except in the non-separable MIUF case)

(3.14) Second condition equates marginal utility of real balances to the "social"marginal cost of producing them (zero!).

From household's problem we know

$$\frac{U_{m,t}}{U_{c,t}} = 1 - e^{-i_t}.$$

RHS can be zero only if $i_t = 0$. This is called Friedman rule. (In steady-state) this results $\pi = -\rho(\equiv -\log(\beta)) < 0$, ie in the steady state, the price level declines at the rate of time preference.

Implementation

$$i_t = \phi\,(r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi\; i_t$$

whose only stationary solution is $i_t = 0$ for all t.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$

Outline

CES aggregator

In modern macro models with imperfect competition, the Dixit-Stiglitz, or Constant-Elasticity-of-Substitution aggregator plays an important role. Consider a static optimization problem of a firm that buy infinite number of intermediate products $C(i)$, puts them together using technology

$$C = \left[\int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}, \tag{4.1}$$

eq:CES 1

where ϵ is the elasticity of substitution (and also the price elasticity of the demand function). Its optimization problem is

$$\max_{C(i)} C \cdot P - \int_0^1 C(i)P(i)di$$

subject to the production technology (4.1).

The optimality conditions are given by

$$C(i) = \left[\frac{P(i)}{P}\right]^{-\epsilon} C \quad \forall i \in [0,1].$$

This is also the demand function of a good $C(i)$. (You must work out the details by yourself.) Plug this to the profits and use zero profit constraint to get the aggregate price level (=price index=marginal costs):

$$P = \left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}. \quad \text{eq:P} \quad (4.2)$$

Introduction

The basic new Keynesian model consists of two key ingredients:

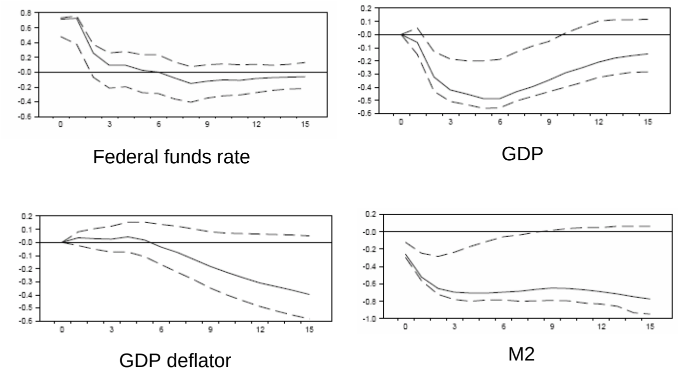
Imperfect competition

We assume that there is a continuum of firms and each produce a differentiated intermediate good for which it sets the price.

Price rigidities

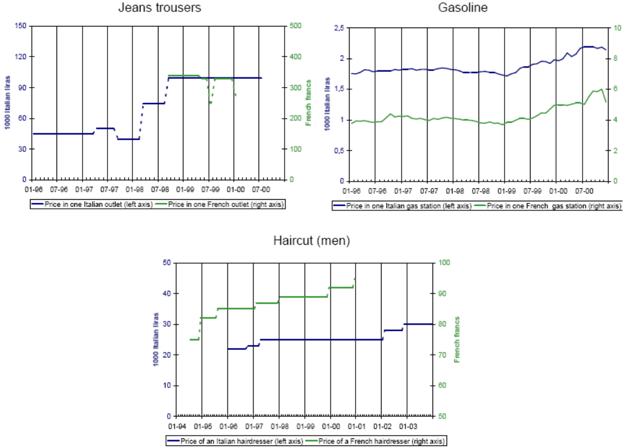
We assume (*a la* Calvo (1983)) that, in each period, only a fraction of firms can change their price.

Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

Households

Household problem is the same as in the case of classical model except that the aggregate consumption consists of continuum of goods:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{1-\epsilon}}$$

Household must allocate its consumption to different goods according to their relative price

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t, \qquad \text{eq:C(i)} \quad (4.3)$$

where the aggregate price index is as in (4.2).

The optimality condition are as before and assuming same functional form of utility function, and loglinearizing, we obtain

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{4.4}$$

eq:ld

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho) \tag{4.5}$$

eq:c

$$m_t - p_t = y_t - \eta i_t. \tag{4.6}$$

eq:m

Firms

Assume continuum of identical firms indexed by $i \in [0,1]$ that use the following common production technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}.$$

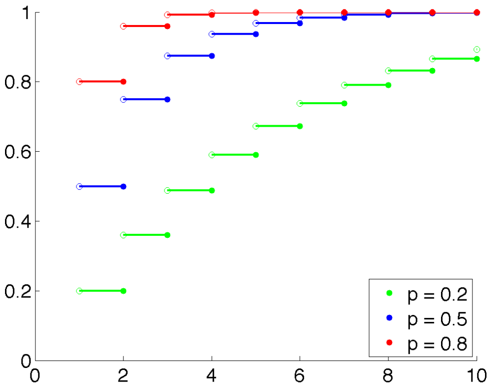
All firms face an identical demand curve (4.3) and take aggregate price index P_t and aggregate consumption index C_t as given.

Calvo Fairy

- A firm may change price of its product only when Calvo Fairy visits.
- The probability of a visit is $1-\theta$.
- It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $1-\theta$ share of firms may change their price and rest, θ, keep their price unchanged.
- Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process).
- The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution.
- The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1-\theta}.$$

Cumulative distribution function of geometric distribution



Source: http://en.wikipedia.org/wiki/Geometric_distribution

Optimal price setting

Let $P_t^\star$ denote the price level of the firm that receives price change signal. This is the price level of the firm that Calvo Fairy visits.

When making its pricing decision, the firm takes into account that it can change is price with the probability $1-\theta$, ie the chosen price remains the same with probability θ.

$$\max_{P_t^\star} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left\{ P_t^\star Y_{t+k}(P_t^\star) - \Psi_{t+k}\left[Y_{t+k}(P_t^\star) \right] \right\},$$

where $\Psi_{t+k}\left[Y_{t+k}(P_t^\star)\right]$ is firm's total costs that depends on the demand function $Y_{t+k}(P_t^\star)$ with a relative price of $P_t^\star / P_{t+k}$.

and

$$Y_{t+k}(P_t^\star) = \left(\frac{P_t^\star}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

The first one is the demand function that the firm faces and is due to the households' consumption index.

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right).$$

The second equation is the nominal *stochastic discount factor* (pricing kernel), that household use to price any financial asset. Note, that in our standard household's optimisation problem, the household price one-period bond using the very same pricing kernel.

and

$$Y_{t+k}(P_t^*) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

The first one is the demand function that the firm faces and is due to the households' consumption index.

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right).$$

The second equation is the nominal *stochastic discount factor* (pricing kernel), that household use to price any financial asset. Note, that in our standard household's optimisation problem, the household price one-period bond using the very same pricing kernel.

Tell here more about the $Q_{t,t+k}$.

The first-order condition

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left(P_t^\star - \underbrace{\frac{\epsilon}{\epsilon - 1}}_{\equiv \mathcal{M}} \psi_{t,t+k} \right) = 0,$$

where

$$\psi_{t,t+k} = \Psi'_{t+k}\left(Y_{t+k}(P_t^\star)\right)$$

is nominal marginal costs and $\mathcal{M}$ is the desired or frictionless markup. In the case of no frictions $(1 - \theta = 1)$:

$$P_t^\star = \mathcal{M} \psi_{t,t}.$$

Equivalently,

$$\sum_{k=0}^{\infty} \theta^k \, E_t \left\{ Q_{t,t+k} \, Y_{t+k|t} \, \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} \; MC_{t+k|t} \, \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

Perfect Foresight, Zero Inflation Steady State:

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\mathcal{M}}$$

Log-linearization around zero inflation steady state:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \; E_t\{\widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1}\}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$.

Equivalently,

$$p_t^* = \mu + (1-\beta\theta)\;\sum_{k=0}^{\infty}(\beta\theta)^k \; E_t\{mc_{t+k|t} + p_{t+k}\}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$.

Flexible prices ($\theta = 0$):

$$p_t^* = \mu + mc_t + p_t$$

$$\Longrightarrow \qquad mc_t = -\mu \text{ (symmetric equilibrium)}$$

Aggregate price level

Note that all those firms that may set their price level (ie where Calvo Fairy visits) will choose identical price $P_t^\star$. Let $S(t) \subset [0,1]$ represent the set of firms that do not reoptimize their price in period t. Note that "the distribution of prices among firms not adjusting in period t corresponds to the distribution effective prices in period $t-1$, though with total mass reduced to θ". Remember the aggregate price index P_t in eq (4.2). The aggregate price level has to follow this functional form

$$P_t = \left[\int_{S(t)} P_t(i)^{1-\epsilon} di + (1-\theta)(P_t^\star)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

$$= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^\star)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} . \quad \text{eq:aggP} \quad (4.7)$$

Log-linearization of aggregate price level

Stationarize equation (4.7), ie express it in inflation rates and in relative prices: Divide (4.7) by P_{t-1} and rise it power $1-\epsilon$:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^\star}{P_{t-1}}\right)^{1-\epsilon},$$

where $\Pi_t = P_t/P_{t-1}$.
Log-linearizing it around $\Pi = 1$ and $P_t^\star/P_t = 1$ (zero steady state inflation, unit relative prices in steady-state):

$$\pi_t = (1-\theta)(p_t^\star - p_{t-1}).$$

EQUILIBRIUM

Goods market

$$Y_t(i) = C_t(i) \quad \forall i \in [0,1] \text{ and } \forall t.$$

Due to aggregator (4.1)

$$Y_t = C_t. \quad \forall t$$

Labour markets

$$N_t = \int_0^1 N_t(i) di.$$

Particular Case: $\alpha = 0$ (constant returns)

$$\Longrightarrow \; MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta\theta \; E_t\{p_{t+1}^*\} + (1-\beta\theta) \; \widehat{mc}_t + (1-\beta\theta)p_t \tag{2}$$

Combining (1) and (2):

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \lambda \; \widehat{mc}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

Generalization to $\alpha \in (0,1)$ (decreasing returns)

Define

$$\begin{aligned} mc_t &\equiv (w_t - p_t) - mpn_t \\ &\equiv (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$,

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \qquad (3) \end{aligned}$$

Implied inflation dynamics

$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \lambda\, \widehat{mc}_t \qquad (4)$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

Equilibrium

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t.

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}}\, di\right)^{\frac{\epsilon}{\epsilon-1}}$,

$$Y_t = C_t$$

for all t. Combined with the consumer's Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) \tag{5}$$

Labor market clearing

$$\begin{aligned} N_t &= \int_0^1 N_t(i)\, di \\ &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \end{aligned}$$

Taking logs,

$$(1-\alpha)\, n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\, di$ (second order).

Up to a first order approximation:

$$y_t = a_t + (1-\alpha)\, n_t$$

Marginal Cost and Output

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma\ y_t + \varphi\ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ y_t - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \end{aligned} \quad (6)$$

Under *flexible prices*

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ y_t^n - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \quad (7)$$

$$\Longrightarrow\ y_t^n = -\delta_y + \psi_{ya}\ a_t$$

where $\delta_y \equiv \frac{(\mu-\log(1-\alpha))(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$.

$$\Longrightarrow\ \widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\ (y_t - y_t^n) \quad (8)$$

where $y_t - y_t^n \equiv \widetilde{y}_t$ is the *output gap*

New Keynesian Phillips Curve

$$\pi_t = \beta \, E_t\{\pi_{t+1}\} + \kappa \, \widetilde{y}_t \tag{9}$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) \tag{10}$$

where r_t^n is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma \, E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \psi_{ya} \, E_t\{\Delta a_{t+1}\} \end{aligned}$$

Missing block: description of monetary policy (determination of i_t).

Monetary policy

Determination of nominal interest rate, i_t, gives the path for actual real rate. It is a description how monetary policy is conducted.

Monetary policy is non-neutral

When prices are sticky nominal interest rate path determines output gap.

Dynamics

To study the dynamics of a linear rational expectation model, we write is as follows

$$A_0 x_t = A_1 \operatorname{E}_t x_{t+1} + B z_t.$$

and study the properties of the transition matrix $A_T \equiv A_0^{-1} A_1$ of the following form

$$x_t = \underbrace{A_0^{-1} A_1}_{\equiv A_T} \operatorname{E}_t x_{t+1} + \underbrace{A_0^{-1} B}_{\equiv B_T} z_t.$$

A_0 should be of full rank (some solution algorithms allow reduced rank cases too).

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A_0 should be of full rank (some solution algorithms allow reduced rank cases too).

Muista laskea taululla välivaiheet seuraavasta kalvosta

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t \tag{11}$$

where v_t is exogenous (possibly stochastic) with zero mean.

Equilibrium Dynamics: combining (9), (10), and (11)

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - v_t) \tag{12}$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

Uniqueness $\iff \mathbf{A}_T$ has both eigenvalues within the unit circle

Given $\phi_\pi \geq 0$ and $\phi_y \geq 0$, (Bullard and Mitra (2002)):

$$\kappa\left(\phi_\pi - 1\right) + \left(1 - \beta\right)\phi_y > 0$$

is necessary and sufficient.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = 0$ (no real shocks).

Let v_t follow an AR(1) process

$$v_t = \rho_v \, v_{t-1} + \varepsilon_t^v$$

Calibration:

$$\rho_v = 0.5,\ \phi_\pi = 1.5,\ \phi_y = 0.5/4,\ \beta = 0.99,\ \sigma = \varphi = 1,\ \theta = 2/3,\ \eta = 4.$$

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution

Analytical solution: undetermined coefficients

1. Guess that the solution takes the form $\tilde{y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$ (why?)
2. Substitute (11) into (10),
3. Let's assume that $\hat{r}_t^n \equiv r_t^n - \rho = 0$ (because it is not affected by monetary policy shocks,
4. Impose these to (9) and (10) (note that $\mathrm{E}_t\, v_{t+1} = \rho_v v_t$) and
5. solve the unknown ψ_{yv} and $\psi_{\pi v}$ to obtain

$$\tilde{y}_t = -(1-\beta\rho_v)\Lambda_v v_t$$
$$\pi_t = -\kappa\Lambda_v v_t,$$

where $\Lambda_v = \{(1-\beta\rho_v)[\sigma(1-\rho_v)0\phi_y] + \kappa(\phi_\pi - \rho_v)\}^{-1}$

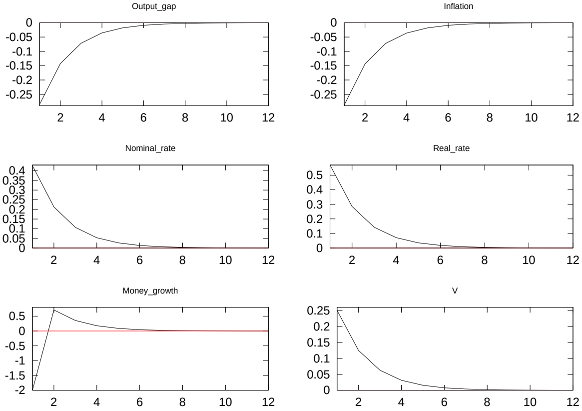
Analytical solution: undetermined coefficients...

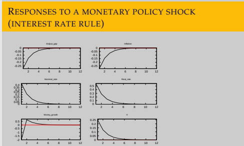
6. Note, however, that $v_t = \rho_v v_{t-1} + \varepsilon_t^v = \frac{1}{1-\rho_v L}\varepsilon_t^v$, where L is lag-operator (or backshift operator), ie $Lx_t = x_{t-1}$.
7. Then, the solution ('VAR' representation) is

$$\tilde{y}_t = \rho_v \tilde{y}_{t-1} - (1 - \beta\rho_v)\Lambda_v \varepsilon_t^v$$
$$\pi_t = \rho_v \pi_{t-1} - \kappa\Lambda_v \varepsilon_t^v.$$

Note, that the persistence (lagged endogenous variable) is *inherited* from the shock process v_t.

Responses to a monetary policy shock (interest rate rule)





- Real rate increases.
- Nominal interest rate increases but if the persistence of monetary policy shock is sufficiently high, the nominal interest rate will decline (this is due t the policy rule).
- Exogenous increase in interest rate leads to persistent decline in output gap and inflation.
- output response matches that of the output gap.
- If i_t rises after mopo shock, then money demand will contract (liquidity effect).

Effects of a Technology Shock

Set $v_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a.$$

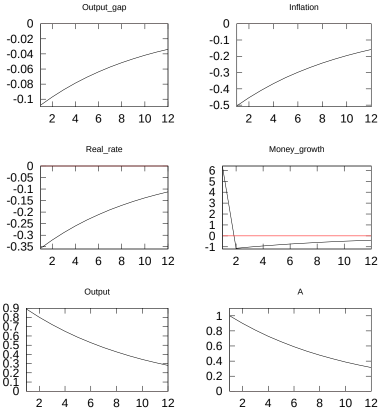
Implied natural rate:

$$\widehat{r}_t^n = -\sigma \psi_{ya}(1 - \rho_a) \, a_t$$

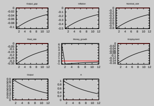
Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise: AR(1) process for Δa_t

RESPONSES TO A TECHNOLOGY SHOCK (INTEREST RATE RULE)



RESPONSES TO A TECHNOLOGY SHOCK (INTEREST RATE RULE)



- persistent decline in inflation and output gap
- output and employment response ambiguous:

Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{13}$$

Money market clearing

$$\widehat{l}_t = \widehat{y}_t - \eta \, \widehat{i}_t \tag{14}$$

$$= \widetilde{y}_t + \widehat{y}_t^n - \eta \, \widehat{i}_t \tag{15}$$

where $l_t \equiv m_t - p_t$ denotes (log) real money balances.

Substituting (14) into (10):

$$(1 + \sigma\eta) \; \widetilde{y}_t = \sigma\eta \; E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta \; E_t\{\pi_{t+1}\} + \eta \; \widehat{r}_t^n - \widehat{y}_t^n \tag{16}$$

Furthermore, we have

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t \tag{17}$$

Equilibrium dynamics

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_{t-1} \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}^n_t \\ \widehat{y}^n_t \\ \Delta m_t \end{bmatrix} \tag{18}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness $\Longleftrightarrow$ $\mathbf{A_M} \equiv \mathbf{A}^{-1}_{\mathbf{M,0}}\mathbf{A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = y_t^n = 0$ (no real shocks).

Money growth process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{19}$$

where $\rho_m \in [0, 1)$

Figure 3 (based on $\rho_m = 0.5$)

Effects of a Technology Shock

Set $\Delta m_t = 0$ (no monetary shocks).

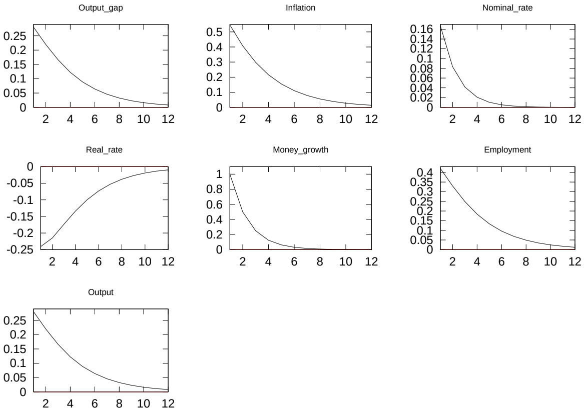
Technology process:

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a.$$

Figure 4 (based on $\rho_a = 0.9$).

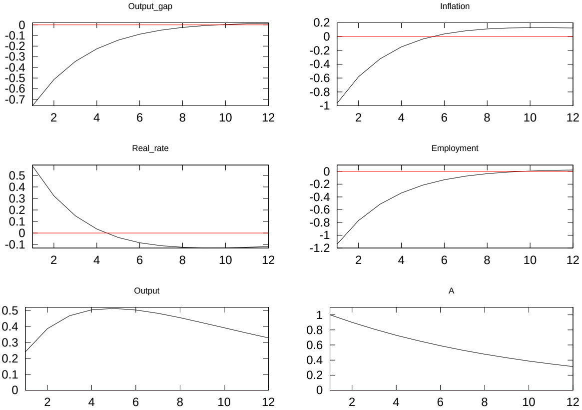
Empirical Evidence

RESPONSES TO A MONETARY POLICY SHOCK (MONEY GROWTH RATE RULE)



Note that liquidity effect is not present (due to the calibration)!

Responses to a technology shock (money growth rule)



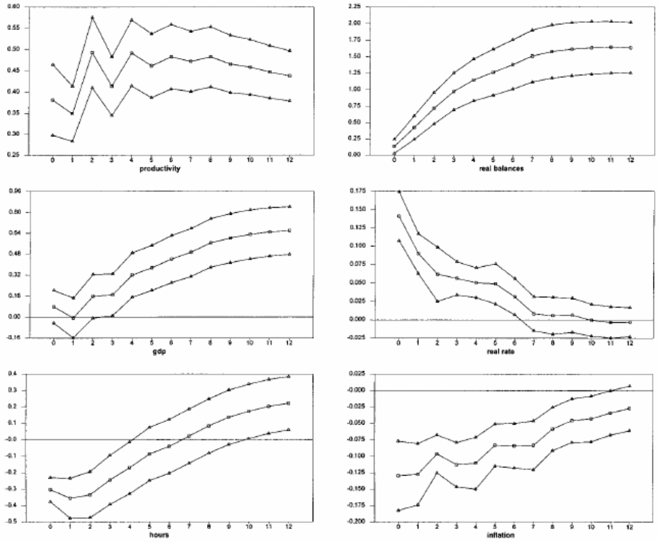


Figure 4. Estimated Impulse Responses from a Five-Variable Model: U.S. Data, First-Differenced Hours (Point Estimates and ±2 Standard Error Confidence Intervals)

Source: Galí (1999)

Outline

The Efficient Allocation

$$\max U\left(C_t, N_t\right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t\ N_t(i)^{1-\alpha},\ \ all\ \ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i)\ di$$

Optimality conditions:

$$C_t(i) = C_t,\ \ all\ \ i \in [0,1]$$

$$N_t(i) = N_t,\ \ all\ \ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha)\ A_t N_t^{-\alpha}$.

Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities:

- *Monopolistic competition*: $P_t = \mathcal{M}\,\frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Solution: employment subsidy τ. Under flexible prices, $P_t = \mathcal{M}\,\frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

- *Transactions friction* (economy with valued money): assumed to be negligible

2. Distortions associated with the presence of nominal rigidities:

- *Markup variations* resulting from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$ (assuming optimal subsidy)

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \, \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

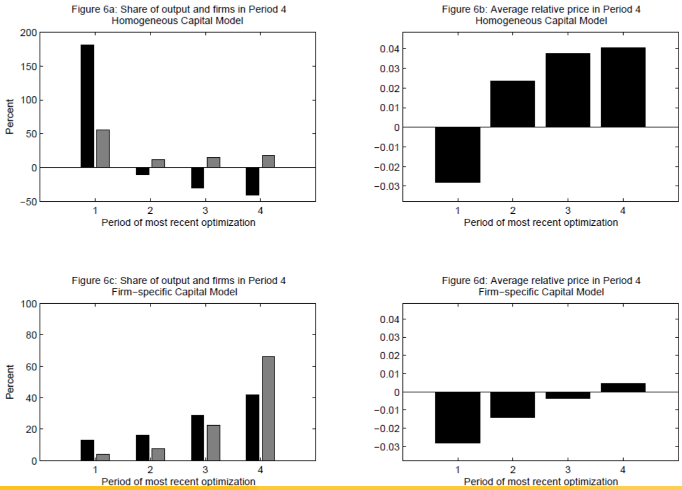
Optimality requires that the average markup be stabilized at its frictionless level.

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods. Accordingly, markups should be identical across firms/goods at all times.

Discuss asymmetries generated by Calvo. Relative price distortions!

PRICE AND OUTPUT DISPERSION

Figure 6: Features of the Distribution of Output and Prices Across Firms



Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$

$\Longrightarrow$ the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$As a result the aggregate price level is fully stabilized and no relative price distortions emerge.
- equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

Equilibrium under the Optimal Policy

$$\widetilde{y}_t = 0$$
$$\pi_t = 0$$
$$i_t = r_t^n$$

for all t.

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$
$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \kappa\, \widetilde{y}_t$$

An Exogenous Interest Rate Rule

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one.

→ indeterminacy. (real and nominal).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

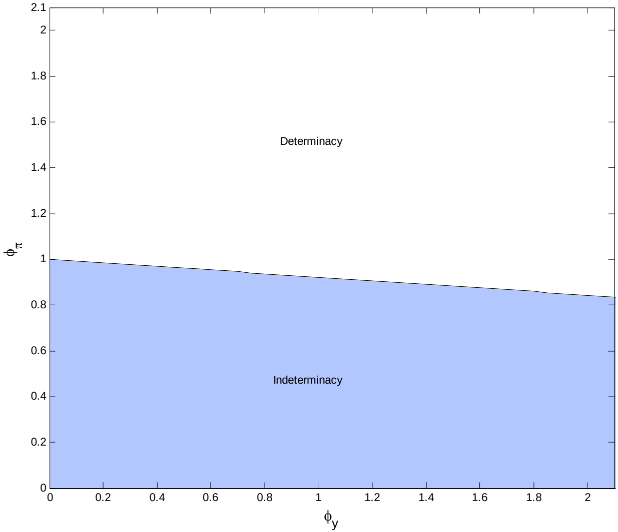
Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa\ (\phi_\pi - 1) + (1-\beta)\ \phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned} di &= \phi_\pi\ d\pi + \phi_y\ d\widetilde{y} \\ &= \left(\phi_\pi + \frac{\phi_y\ (1-\beta)}{\kappa}\right)\ d\pi \end{aligned}$$

Figure 4.1


2.1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
ϕ_π
Determinacy
Indeterminacy
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ϕ_y

A Forward-Looking Interest Rate Rule

$$i_t = r_t^n + \phi_\pi \; E_t\{\pi_{t+1}\} + \phi_y \; E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

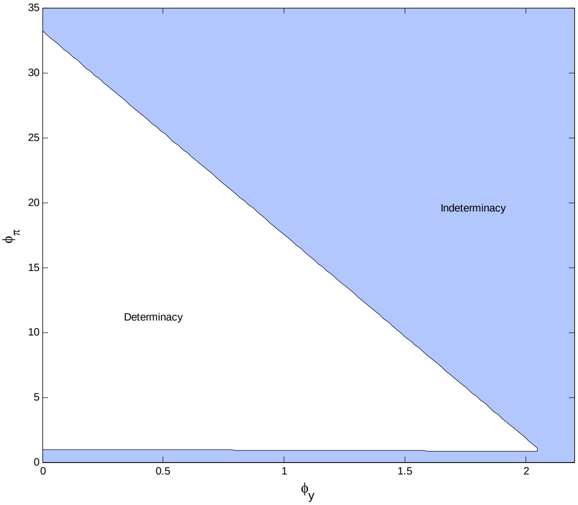
where

$$\mathbf{A}_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}$$

Existence and Uniqueness conditions: (Bullard and Mitra (2002):

$$\begin{aligned} \kappa \; (\phi_\pi - 1) + (1 - \beta) \; \phi_y &> 0 \\ \kappa \; (\phi_\pi - 1) + (1 + \beta) \; \phi_y &< 2\sigma(1 + \beta) \\ \phi_y &< \sigma(1 + \beta^{-1}) \end{aligned}$$

Figure 4.2


35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
ϕ_π
ϕ_y
Indeterminacy
Determinacy

Shortcomings of Optimal Rules

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

Alternative: "simple rules" , i.e. rules that meet the following criteria:

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

Simple Monetary Policy Rules

Welfare-based evaluation:

$$\mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left(\kappa \, \widetilde{y}_t^2 + \epsilon \, \pi_t^2 \right)$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2\lambda} \left[\kappa \, var(\widetilde{y}_t) + \epsilon \, var(\pi_t) \right]$$

See Appendix for Derivation.

A Taylor Rule

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \, \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \, (\widehat{r}_t^n - \phi_y \, \widehat{y}_t^n)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$. Note that $\widehat{r}_t^n - \phi_y \, \widehat{y}_t^n = -\psi_{ya}^n \, [\sigma(1 - \rho_a) + \phi_y] \, a_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \Delta y_t$

Money Growth Peg

$$\Delta m_t = 0$$

money market clearing condition

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta\, \widehat{i}_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ and ξ_t is a money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta\, \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

Define $l_t^+ \equiv l_t - \zeta_t$. $\Longrightarrow$

$$\widehat{i}_t = \frac{1}{\eta}\left(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+\right)$$

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1}^{+} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t^{+} \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where

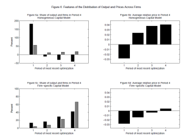
$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simulations and Evaluation of Simple Rules

Table 4.1: Evaluation of Simple Monetary Policy Rules

	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

PRICE AND OUTPUT DISPERSION



Kerro inflation forecast targeting!

OUTLINE

Discretion vs Commitment: time-consistency of optimal plans

Commitment

Commitment is the ability to deliver on past promises no matter what the particular current situation is.

Discretion

Under discretion, a policymaker is allowed to change policy depending on current circumstances and to disregard any past promises.

Time-consistency

Economists refer to the desire to alter previously made plans as the time-consistency problem because, at each date, an individual or policymaker finds it tempting to deviate from what an earlier plan dictated.

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Kerro joulupukkitarina!

Background

Time-consistency problem arises only if central bank faces a temptation to lower the output gap by rising inflation.

Divine coincidence: inherent feature of many New-Keynesian Models

Stabilizing inflation is equivalent to stabilizing the welfare-relevant output gap! This generalizes to models with wage frictions. $\longrightarrow$ More profound *real frictions* are needed.

Is monetary policy too easy? Are there no trade-offs?

Flexible inflation targeting

Monetary policy makers claim that — at least in the short-run — there is trade-off between stabilizing inflation or ouput/employment. Hence, a central bank should avoid too much instability in output while committing to a medium term inflation target.

The Monetary Policy Problem: the Case of an Efficient Steady State

- When nominal rigidities coexist with real imperfections, the flexible price equilibrium allocation is generally inefficient.
- When the possible inefficiencies associated with the flexible price equilibrium do not affect the steady state, which remains efficient.
- Here we analyze the optimal monetary policy problem under that assumption.
- Short run deviations between the natural and efficients levels of output.
- Cost-push, or markup shocks: assume gap between the two follows a shocks stationary process, with a zero mean, due to presence of some real imperfections that generate a time-varying gap between output and its efficient counterpart, even in the absence of price rigidities.

Breaking divine coincidence

Easy candidates

Markup shock

Suppose that the elasticity of substitution contains exogenous variation, ϵ_t. Then the markup

$$\mu_t = \log\left(\frac{\epsilon_t}{\epsilon_t - 1}\right)$$

enters to New-Keynesian Phillips curve as follows

$$\pi_t = \mathrm{E}_t\, \pi_{t+1} + \lambda\left(\hat{mc}_t + \hat{\mu}_t\right)$$

Later, we denote $u_t \equiv \lambda \hat{\mu}_t$.

Breaking divine coincidence. . .

Wage markup shock

Analoguous to product market markup. Creates wedge between the marginal rate of substitution and marginal product of labour.

Presence of markup shocks leads to

- Differences between natural and efficient level of output
- allows for short run deviations between the natural and efficient levels of output.
- assume that the gap between the two follows a stationary process, with zero mean:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \quad \varepsilon_t^u \sim \text{iid}(0, \sigma_u^2).$$

- time variations in gap between efficient and natural levels of output reflected in markup generate a tradeoff for monetary policy, since make it impossible to attain simultaneously zero inflation and efficient

The Monetary Policy Problem

$$\min \ E_0\{\sum_{t=0}^{\infty} \beta^t \ [\alpha_y \ \widetilde{y}_t^2 + \pi_t^2]\} \tag{1}$$

subject to:

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ \widetilde{y}_t + u_t$$

where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u \ u_{t-1} + \varepsilon_t$$

In addition:

$$\widetilde{y}_t = -\frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{2}$$

Note: utility based criterion requires $\alpha_y = \frac{\kappa}{\epsilon}$

Monetary policy with discretion

Period-by-period optimization

Discretionary monetary policy relies on period-by-period optimization of the central bank. It is a sequence of unrelated decisions.

Discretionary policy

Sequential optimization, policy that is, decision is optimal each period without commitment to future actions.

Optimal Policy with Discretion

Each period CB chooses (x_t, π_t) to minimize

$$\alpha_y \, \widetilde{y}_t^2 + \pi_t^2$$

subject to

$$\pi_t = \kappa \, \widetilde{y}_t + v_t$$

where $v_t \equiv \beta \, E_t\{\pi_{t+1}\} + u_t$ is taken as given.

Optimality condition:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha_y} \, \pi_t \tag{3}$$

Equilibrium

$$\pi_t = \alpha_y q \, u_t \tag{4}$$

$$\widetilde{y}_t = -\kappa q \, u_t \tag{5}$$

$$i_t = r_t^n + q \left[\kappa\sigma(1-\rho_u) + \alpha_y \rho_u\right] u_t \tag{6}$$

where $q \equiv \frac{1}{\kappa^2 + \alpha_y(1-\beta\rho_u)}$

Implementation:

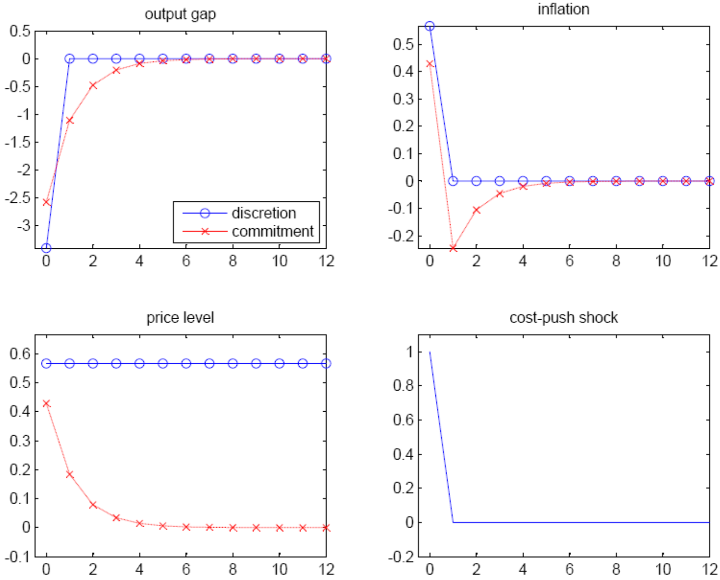
$$i_t = r_t^n + \left[(1-\rho_u)\frac{\kappa\sigma}{\alpha_y} + \rho_u\right]\pi_t$$

uniqueness condition: $\frac{\kappa\sigma}{\alpha_y} > 1$ (likely if utility-based: $\sigma\epsilon > 1$)

Alternatively,

$$i_t = r_t^n + q\left[\kappa\sigma(1-\rho_u) + \alpha_y\rho_u\right]u_t + \phi_\pi(\pi_t - \alpha_y q\, u_t)$$

uniqueness condition: $\phi_\pi > 1$.


output gap
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
2
4
6
8
10
12
discretion
commitment
inflation
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
price level
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
cost-push shock
1
0.8
0.6
0.4
0.2
0
-0.2

Degree of policy accommodation

- Under discretion, the central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock, and thus let inflation increase.
- However, the increase in inflation is smaller than the one that would obtain if the output gap remained unchanged:

$$\pi_t = \frac{1}{1-\beta\rho_u} u_t.$$

Commitment

Central bank is assumed to be able to commit, with full credibility, to a *policy plan*

In the case of our model, the plan consists of a specification of the desired levels of inflation and output gap at

1. all possible dates, and
2. states of nature,
3. current and future.

Optimal Policy with Commitment

State-contingent policy $\{\widetilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_y \, \widetilde{y}_t^2 + \pi_t^2)$$

subject to the sequence of constraints:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t + u_t$$

Lagrangean:

$$\mathcal{L} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_y \, \widetilde{y}_t^2 + \pi_t^2 + 2\gamma_t \left(\pi_t - \kappa \, \widetilde{y}_t - \beta \, \pi_{t+1}\right)\right]$$

First order conditions:

$$\alpha_y \, \widetilde{y}_t - \kappa \, \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

for $t = 0, 1, 2, \ldots$and where $\gamma_{-1} = 0$.

Eliminating multipliers:

$$\widetilde{y}_0 = -\frac{\kappa}{\alpha_y}\,\pi_0 \tag{7}$$

$$\widetilde{y}_t = \widetilde{y}_{t-1} - \frac{\kappa}{\alpha_y}\,\pi_t \tag{8}$$

for $t = 1, 2, 3, \ldots$..

Alternative representation:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha_y}\,\widehat{p}_t \tag{9}$$

for $t = 0, 1, 2, \ldots$where $\widehat{p}_t \equiv p_t - p_{-1}$.

Equilibrium

$$\widehat{p}_t = a\ \widehat{p}_{t-1} + a\beta\ E_t\{\widehat{p}_{t+1}\} + a\ u_t$$

for $t = 0, 1, 2, ...$where $a \equiv \frac{\alpha_y}{\alpha_y(1+\beta)+\kappa^2}$

Stationary solution:

$$\widehat{p}_t = \delta\ \widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}\ u_t \qquad (10)$$

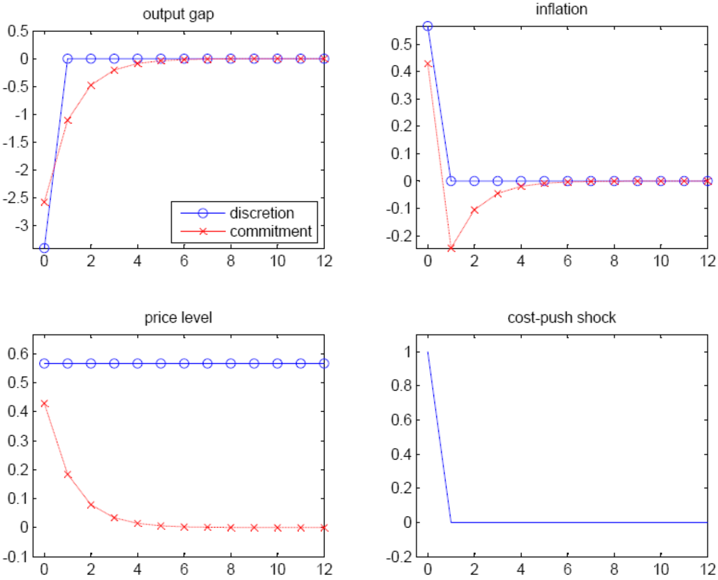
for $t = 0, 1, 2, ...$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$.

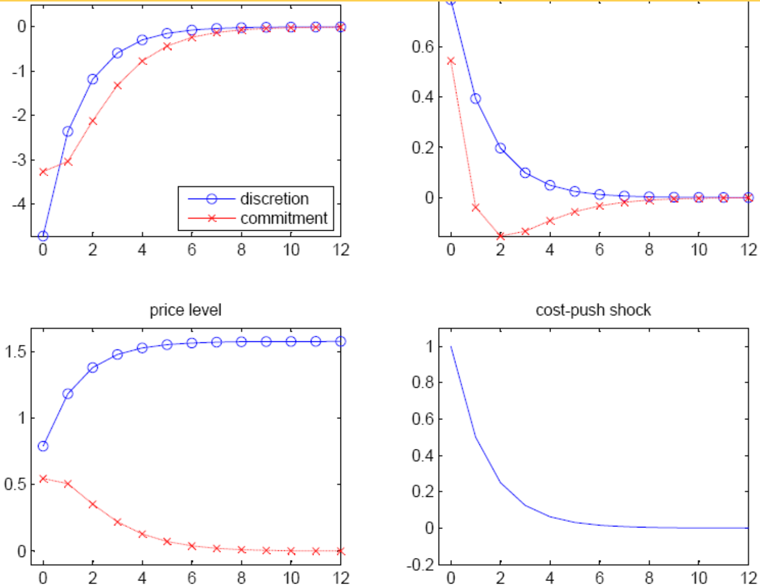
$\rightarrow$ *price level targeting !*

$$\widetilde{y}_t = \delta\ \widetilde{y}_{t-1} - \frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_t \qquad (11)$$

for $t = 1, 2, 3, ...$as well as

$$\widetilde{y}_0 = -\frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_0$$


output gap
discretion
commitment
inflation
price level
cost-push shock


discretion
commitment
price level
cost-push shock

Comparing discretion and commitment

- Note, that the loss function is quadratic! Large deviations results relatively higher losses than small deviations.
- In both policies, both output gap and inflation return to zero. In discretionary policy this is reached immediately after the first period and in the case of commitment gradually.

Commitment: why persistently negative output gap and inflation?

By committing to such a response, the central bank manages to improve the output gap/inflation tradeoff in the period when shock occur.

- A credible central bank is able tie his hands and smooth the losses over time. Discretionary central bank reoptimizes every period (that is known by the agents) and does not have this luxury.

Summary

Given the convexity of the loss function inflation and output gap deviations, the dampening of those dviations in the period of shock brings about an improvement in overall welfare relative to the case of discretion, because the implied benefits are not offset by the (relatively small) losses generated by the deviations in the subsequent periods (and which are absent in the discretionary case).

Stabilization bias

Discretionary policy attempts to stabilize output gap in the medium term more thatn the optimal policy under commitment calls for, without *internalizing* the benefits in terms of short term stability that results fro from allowing larger deviations of the output gap at future horizons.

DISTORTED STEADY-STATE

- Assume that there exist (unmodeled) real distortions/imperfections that generate permanent gap between the natural and the efficient levels of output, which is reflected in an inefficient steady-state

$$-\frac{U_n}{U_c} = MPN(1 - \Phi)$$

- Example: A non-zero steady-state markup (that is NOT corrected by a subsidy).
- Losses are given by

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \hat{x}_t^2 \right) - \Lambda \hat{x}_t \right]$$

, where $\hat{x}_t$ is the deviation of a welfare relevant output gap from its (negative) steady-state.

Distorted steady-state...

- New-Keynesian Phillips curve is as before, except that

$$u_t \equiv \kappa(\hat{y}_t^e - \hat{y}_t^n).$$

- Furthermore, the steady-state distortion has the same order magnitude as fluctuation in the output gap and inflation, ie "small". We need to be able analyze behavior in the neighborhood of the zero inflation steady-state.

Summarizing

Discretion

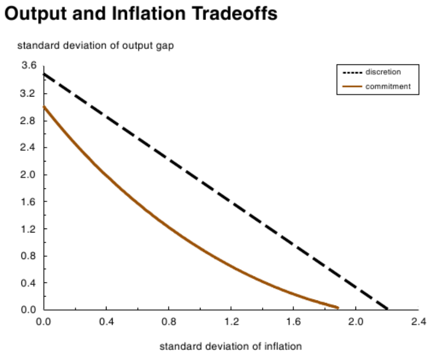
The response to cost-push shock is not affected (ie previous impulse responses are valid). Stabilization bias remains. It has effect to the steady-state around which the economy fluctuates.

Inflation bias: Due to inefficiently low level of output, central bank has a desire to increase output by inflating economy. This is known by the agents, and it results positive inflation. (See the constant term in the optimality conditions.)

Commitment

Having a committing "technology" (ability) central bank may (asymptotically) get rid of the *inflation bias*. This is a result of additional channel where price level converges to a constant (defined by the

TRADE-OFF



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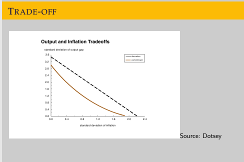


Kerro Maastrictin sopimuksesta! Kerro tarina 1979 öljykriisistä ja 2000 luvun öljykriisistä!

Outline

Motivation

- The basic new Keynesian model for the closed economy

  - equilibrium dynamics: simple three-equation representation
  - ability to match much of the evidence on the effects of monetary policy and technology shocks
  - monetary policy: optimality of inflation targeting

- How does the introduction of open economy elements affect that analysis and prescriptions?

- Can a model with nominal rigidities account for the volatility of nominal and real exchange rates?

- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?

Some References

- Kollmann (JIE 01): nominal and real exchange rates, SOE version of EHL, pricing to market, many shocks
- Chari et al. (RES 02): two country model, Taylor type contracts, MP shocks
- Benigno and Benigno (RES 03): one-period contracts, two country, conditions for optimality of price stability
- Svensson (JIE 00): not-fully-optimizing model, strict vs. flexible CPI inflation targeting
- Benigno (JIE 04): staggered, currency union, heterogeneity
- Galí and Monacelli (RES 05): staggered, small open economy, equivalence result, optimal policy.
- Monacelli (JMCB 05): staggered, GM with limited pass-through
- Benigno and Benigno (JME 06): staggered, two countries, optimal policy
- de Paoli (LSE dissertation): generalization of GM

Modeling choices

New concepts

Open economy aspect brings new concepts: exchange rate, the terms of trade, exports, imports, international financial markets.

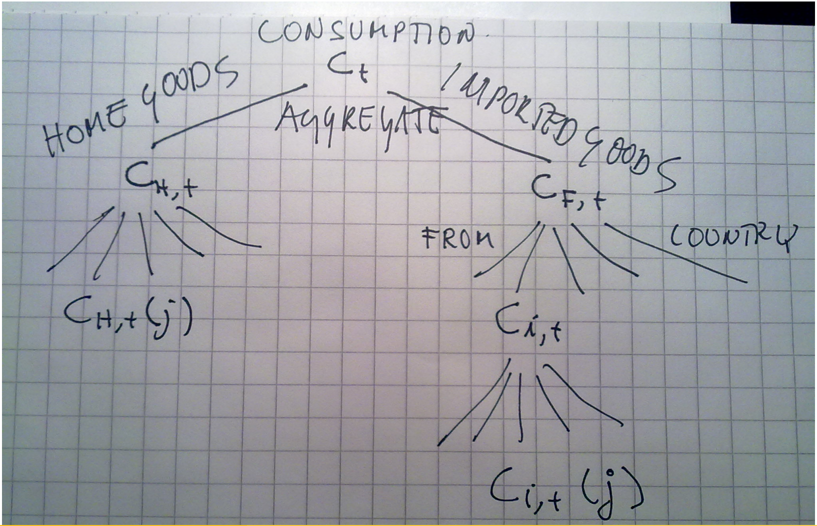
Choose from

1. Large or small economy
2. Nature of international asset markets: autarky or complete markets.
3. Discrimination between domestic and foreign markets.
4. Tradeables vs. nontradeables,
5. Trading costs,
6. International policy coordination
7. Exchange rate regimes

Small open economy

- World economy is a continuum of small open economies represented by the unit interval. $\longrightarrow$ performance of a single economy does not have any impact on the rest of the world.
- Productivity shocks are imperfectly correlated across the economies.
- Identical preference, technology and market structure.
- Notation: no i-index refers to domestic (home) economy, $i \in [0, 1]$ subscript refers to economy i, one in the continuum. Superscript * correspond the world economy as a whole.

Goods structure



Households

Representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \qquad \text{eq:sec7:disc} \quad (7.1)$$

where (as before) N_t denotes hours of labour, and C_t is the CES composite

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad (7.2)$$

where α is a measure of *openness* and $1-\alpha$ the *degree of home bias*.
$\eta > 0$ is elasticity of substitution between domestic and foreign goods.

HOUSEHOLDS...

$C_{H,t}$ is index of consumption domestic goods (Home goods) give by the CES aggregator

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $j \in [0,1]$ denotes the good variety.

Households. . . . . .

$C_{F,t}$ is an index of imported goods given aggregegate from countries i by

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

with elasticity of substitution $\gamma > 0$ between importing countries. Imports from each country i is a bundle varieties j

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Hence, the varieties in each country are produced by similar technology given by elasticity of substitution $\varepsilon > 0$.

Budget constraint

$$\int_0^1 P_{H.t}(j)\, C_{H,t}(j)\, dj + \int_0^1 \int_0^1 P_{i,t}(j)\, C_{i,t}(j)\, djdi$$
$$+ \mathrm{E}_t\, Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t, \quad t = 0, 1, 2, \ldots \quad (7.3)$$

eq:sec7:budgetconstraint

where

- $P_{H,t}(j)$ is the price of domestic variety j,
- $P_{i,t}(j)$ is price of variety j imported from country i.
- D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t.
- W_t is nominal wage, and
- T_t denotes lump-sum transfers/taxes
- $Q_{t,t+1}$ is stochastic discount factor for one period payoff of the household's portfolio.

Complete set of contingent claims traded internationally!

Prices and demand functions

Aggregate price index of home goods, $P_{H,t}$, and imported goods from country $i \in [0,1]$ (in domestic currency)

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$

Demand for domestic good (home good) of variety j and imported good from country i of variety j

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} C_{H,t} \quad \text{eq:sec7:chtj} \quad (7.4)$$

$$C_{i,t}(j) = \left[\frac{P_{i,t}(j)}{P_{i,t}}\right]^{-\varepsilon} C_{i,t}$$

Prices and demand functions...

Aggregate price index of imported goods, $P_{F,t}$, and, finally, the aggregate consumption price index

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

Prices and demand functions. . . . . .

Demand for imported good from country i and demand form domestic and imported aggregate goods respectively

$$C_{i,t} = \left[\frac{P_{i,t}}{P_{F,t}}\right]^{-\gamma} C_{F,t} \quad \text{eq:sec7:cit} \quad (7.4)$$

$$C_{H,t} = (1-\alpha)\left[\frac{P_{H,t}}{P_t}\right]^{-\eta} C_t \quad \text{eq:sec7:cHt} \quad (7.5)$$

$$C_{F,t} = \alpha\left[\frac{P_{F,t}}{P_t}\right]^{-\eta} C_t$$

Prices and demand functions. . . . . .

Demand for imported good from country i and demand form domestic and imported aggregate goods respectively

$$C_{i,t} = \left[\frac{P_{i,t}}{P_{F,t}}\right]^{-\gamma} C_{F,t} \tag{7.4}$$

eq:sec7:cit

$$C_{H,t} = (1-\alpha)\left[\frac{P_{H,t}}{P_t}\right]^{-\eta} C_t \tag{7.5}$$

eq:sec7:cHt

$$C_{F,t} = \alpha\left[\frac{P_{F,t}}{P_t}\right]^{-\eta} C_t$$

Tell the Cobb-Douglas case!!!

HOUSEHOLD BUDGET CONSTRAINT REVISITED

Given the market equilibrium (for all of these aggregators), the total consumption expenditure is as follows

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}.$$

The argument is similar to the beginning of the chapter 3.
The budget constraint (7.3) can be written in the following form

$$P_t C_t + \mathsf{E}_t\, Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t. \qquad \texttt{eq:sec7:bc} \quad (7.5)$$

Optimality conditions

Optimality conditions are as before (suppose the familiar utility function, $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \qquad \text{eq:sec7:ls} \quad (7.6)$$

and

$$\mathrm{E}_t\, \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = \mathrm{E}_t\, Q_{t,t+1} \qquad \text{eq:sec7:Q} \quad (7.8)$$

and loglinearized as

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho\right) \qquad \text{eq:sec7:q} \quad (7.8)$$

Terms of trade

Important relative price that deserve a special term! Price of something in terms of price of home good!
Between home and country i:

$$\mathcal{S}_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$$

and (with aggregate) *effective terms of trade*

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} .$$

Terms of trade...

Symmetric steady-state $\mathcal{S}_{i,t} = 1$ for all $i \in [0, 1]$. The loglinearized effective terms of trade is given by

$$s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t} = \int_0^1 s_{i,t} di \tag{7.9}$$

eq:sec7:s

Other useful loglinearizations

- CPI

$$p_t \equiv (1 - \alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t. \tag{7.10}$$

eq:sec7:cpi

- CPI inflation π_t and domestic inflation $\pi_{H,t}$

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{7.11}$$

eq:sec7:inflation

Exchange rates

$\mathcal{E}_{i,t}$ is the *bilateral nominal exchange rate*, ie price of country i's currency in terms of domestic currency, eg how many euros (domestic country) one US (country i) dollar is worth.
$P^i_{i,t}(j)$ is the price of country i's good j expressed in terms of its own currency, eg iPhone (j) in US (i) dollars.

Law of one price

$P_{i,t}(j) = \mathcal{E}_{i,t} P^i_{i,t}(j)$ for all $i, j \in [0.1]$. Assume it holds at all times for all internationally traded goods. Aggregating results $P_{i,t} = \mathcal{E}_{i,t} P^i_{i,t}$, where $P^i_{i,t} = \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$

Exchange rates...

$p^i_{i,t} = \int_0^1 p^i_{i,t} dj$ is the loglinerized domestic price index for country i (in country i's own currency).

$e_t = \int_0^1 e_{i,t} di$ is the log of *effective nominal exchange rate*. Note that it is an index.

$p^*_t = \int_0^1 p^i_{i,t} di$ is the log *world price index*.

Then $P_{F,t}$ may be loglinearized around symmetric steady-state as

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p^i_{i,t} \right) di = e_t + p^*_t,$$

and when combined with the loglinearized terms of trade results

$$s_t = e_t + p^*_t - p_{H,t}. \tag{7.12}$$

Exchange rates. . . . . .

Bilateral real exchange rate with country i is defined as

$$\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t},$$

is the ratio of the two countries' CPIs, both expressed in terms of domestic currency.
$q_t = \int_0^1 q_{i,t} di$ is the log *effective real exchange rate*

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t = (1-\alpha) s_t$$

International risk-sharing

An analogous optimality condition to (7.6) holds also for the country i. When expressed in the domestic currency, it can be written as

$$\mathrm{E}_t \, \beta \left(\frac{C^i_{t+1}}{C^i_t} \right)^{-\sigma} \left(\frac{\mathcal{E}^i_t P^i_t}{\mathcal{E}^i_{t+1} P^i_{t+1}} \right) = \mathrm{E}_t \, Q_{t,t+1}. \qquad \text{eq:sec7:lsi} \quad (7.13)$$

Combining this with (7.6) (and assuming zero net foreign asset holdings and an *ex ante* identical environment) results

$$C_t = C^i_t \mathcal{Q}^{\frac{1}{\sigma}}_{i,t}. \qquad \text{eq:sec7:CQ} \quad (7.14)$$

Loglinearizing and aggregating ($c^*_t = \int_0^1 c^i_t di$) over i gives

$$c_t = c^*_t + \frac{1}{\sigma} q_t = c^*_t + \left(\frac{1-\alpha}{\sigma} \right) s_t. \qquad \text{eq:sec7:ccast} \quad (7.15)$$

Complete international asset markets equalizes consumption levels between countries up terms of trade

Uncovered interest parity

Allow households to invest both domestic B_t and foreign B_t^* one-period bonds. The budget constraint may be written as

$$P_t C_t + Q_{t,t+1} B_{t+1} + Q^*_{t,t+1} \mathcal{E}_{i,t} B^*_{t+1} \leq B_t + \mathcal{E}_{i,t} B_t^* + W_t N_t + T_t. \quad \text{eq:sec7:bcuip} \quad (7.16)$$

The optimality conditions wrt to these assets are

$$\beta \, \mathrm{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1},$$

$$\beta \, \mathrm{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \right) = Q^*_{t,t+1}.$$

Combining these results

$$\mathrm{E}_t\left(\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}\right) = \frac{Q^*_{t,t+1}}{Q_{t,t+1}},$$

whose loglinear form is the familiar

$$i_t = i^*_t + \mathrm{E}_t\,\Delta e_{t+1}$$

Combining this with the definition of terms of trade, we find that

$$s_t = \mathrm{E}_t \sum_{k=0}^{\infty}\left[\left(i^*_{t+k} - \pi^*_{t+k+1}\right) - \left(i_{t+k} - \pi_{H,t+k+1}\right)\right].$$

That express the terms of trade as the expected sum of real interest rate differential. Note, however, that this is not an equilibrium condition. It simply combines the two from previous slide with the definition of the terms of trade.

Firms and technologies

The supply side follows the very same structure as in the basic New-Keynesian model. We assume constant-returns-to-scale, $\alpha = 0$ (according to notation of the chapter 3).
Firms price is the domestic price $P_{H,t}(j)$ (price index of domestic production). Therefore, the marginal costs area

$$mc_t = -\nu + w_t - p_{H,t} - a_t,$$

where $\nu \equiv -\log(1-\tau)$ (employment subsidy).
The Calvo-pricing applies here too:

$$\bar{p}_{H,t} = \mu + (1-\beta\theta)\,\mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k (mc_{t+k} + p_{H,t+k}), \quad \text{eq:sec7:barpH} \quad (7.17)$$

where $\bar{p}_{H,t}$ denotes the price of firms allowed to optimize.

Goods market clearing

Domesticly produced goods

$$\underbrace{Y_t(j)}_{\text{Domestic output}} = \underbrace{C_{H,t}(j)}_{\text{Domestic demand for home goods}} + \underbrace{\int_0^1 C^i_{H,t}(j)\,di}_{\text{Foreign demand for homegoods, =export}} \quad (7.18)$$

eq:sec7:homegoods

Notice, that due to nested structure the demand for domestic good j in country i is given by

$$C^i_{H,t}(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P_{F,t}}{P^i_t}\right)^{-\eta} C^i_t$$

EXPRESSING (7.8) IN TERMS OF OUTPUT

Aggregate (7.18) and express in terms of terms of trade

$$
\begin{aligned}
Y_t &\equiv \left[\int_0^1 Y_t(j)^{1.\frac{1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma}\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha\int_0^1 \left(\frac{\mathcal{E}_{i,t}P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}^{\eta}_{i,t} C^i_t di\right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha\int_0^1 (\mathcal{S}^i_t \mathcal{S}_{i,t})^{\gamma-\eta} \mathcal{Q}^{\eta-\frac{1}{\sigma}}_{i,t} di\right] \qquad \text{eq:sec7:Y} \quad (7.19)
\end{aligned}
$$

In the Cobb-Douglas case, ie $\sigma = \eta = \gamma = 1$ this aggregates as

$$Y_t = \mathcal{S}_t^{\alpha} C_t. \qquad \text{eq:sec7:CDY} \quad (7.20)$$

Note that at the world level, the terms of trade is unity, ie $\int_0^1 s_t^i di = 0$. The loglinear approximation of (7.19) around the symmetric steady state is the following

$$y_t = c_t + \alpha\gamma s_t + \alpha \left(\eta - \frac{1}{\sigma} \right) q_t = c_t + \frac{\alpha\omega}{\sigma} s_t, \qquad \text{eq:sec7:y} \quad (7.21)$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Aggregating the country i counterparts of (7.21) over all countries results

$$y_t^* \equiv \int_0^1 y_t^i di = \int_0^1 c_t^i di \equiv c_t^*. \qquad \text{eq:sec7:yast} \quad (7.22)$$

Combine (7.21), (7.15) and (7.22) to express output y_t in terms of world demand and terms of trade as

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t, \tag{7.23}$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.
Finally, combining (7.8) with (7.21) gives

$$\begin{aligned} y_t &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho\right) - \frac{\alpha\omega}{\sigma}\,\mathrm{E}_t\, \Delta s_{t+1} \\ &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{H,t+1} - \rho\right) - \frac{\alpha\Theta}{\sigma}\,\mathrm{E}_t\, \Delta s_{t+1} \\ &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - \mathrm{E}_t\, \pi_{H,t+1} - \rho\right) + \alpha\Theta\,\mathrm{E}_t\, \Delta y_{t+1}^* \end{aligned} \tag{7.24}$$

eq:sec7:is

Coefficients in (7.24)

- Real rate sensitivity: $\sigma_\alpha < \sigma$ in the case $\omega > 1$, ie when η and γ are high.
- Foreign output growth sensitivity:

$$\Theta \equiv (\sigma\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = \omega - 1$$

is positive if η and γ are high (relative to σ).

Intuition: direct negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced ral appreaciation (and the resulting switch toward foreign goods). It is dampened by expected real depreciation (CPI inflation is higher than domestic inflation) which dampens the change in the consumption based real rate $i_t - \mathrm{E}_t\, \pi_{t+1}$ (relative to $i_t - \mathrm{E}_t\, \pi_{H,t+1}$).

The trade balance

In the pure Cobb-Dougals (eg $\omega = 0$) case (7.19) results

$$P_{H,t} Y_t = P_t C_t, \quad t > 0.$$

which means that the *trade balance*

$$nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}}\right)$$

is zero all the time. Loglinearize to obtain

$$nx_t = y_t - c_t - \alpha x_t = \alpha\left(\frac{\omega}{\sigma} - 1\right) s_t. \qquad \text{eq:sec7:nx} \quad (7.25)$$

Hence, the sign of net exports is ambiguous and depends on (as usual) the elasticities of substitution!

Output and employment

Labour market clearing condition is

$$N_t \equiv \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj.$$

Loglinearizing

$$y_t = a_t + n_t \quad \text{eq:sec7:productionfunction} \tag{7.26}$$

Calvo-pricing implies

$$\pi_{H,t} = \beta \, \mathrm{E}_t \, \pi_{H,t+1} + \underbrace{\frac{(1-\beta\theta)(1-\theta)}{\theta}}_{\equiv \lambda} \hat{mc}_t. \quad \text{eq:sec7:NPC} \tag{7.27}$$

Marginal cost is given by

$$
\begin{aligned}
mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\
&= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
&= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
&= -\nu + \sigma y_t^* + \varphi y_t + s - (1+\varphi) a_t \qquad \text{eq:sec7:mc1} \quad (7.28) \\
&= -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1+\varphi) a_t \qquad \text{eq:sec7:mcs} \quad (7.29)
\end{aligned}
$$

Channels

φ: output via employment

σ_α: output via terms of trade

σ: world output via consumption (real wage)

σ_α: world output via terms of trade

Marginal cost is given by

$$
\begin{aligned}
mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\
&= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
&= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
&= -\nu + \sigma y_t^* + \varphi y_t + s - (1+\varphi) a_t && \text{eq:sec7:mc1} \quad (7.28) \\
&= -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1+\varphi) a_t && \text{eq:sec7:mcs} \quad (7.29)
\end{aligned}
$$

Channels

- φ: output via employment
- σ_α: output via terms of trade
- σ: world output via consumption (real wage)
- σ_α: world output via terms of trade

Note that in the high elasticity of substitution case ($\Theta > 0$, $\sigma > \sigma_\alpha$, implying that an increase in the world output rises the marginal cost

Equilibrium Dynamics in the SOE: A Canonical Representation

$$\pi_{H,t} = \beta\, E_t\{\pi_{H,t+1}\} + \kappa_\alpha\, \widetilde{y}_t$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\,(i_t - E_t\{\pi_{H,t+1}\} - r_t^n)$$

where

$$\begin{aligned} \widetilde{y}_t &= y_t - y_t^n \\ y_t^n &= \Omega + \Gamma\, a_t + \alpha\Psi\, y_t^* \\ r_t^n &\equiv \rho - \sigma_\alpha \Gamma(1-\rho_a)\, a_t + \alpha\sigma_\alpha(\Theta+\Psi)\, E_t\{\Delta y_{t+1}^*\} \end{aligned}$$

$$\kappa_\alpha \equiv \lambda\,(\sigma_\alpha + \varphi) \quad ; \quad \sigma_\alpha \equiv \frac{\sigma}{(1-\alpha)+\alpha\omega} \quad ; \quad \omega \equiv \sigma\gamma + (1-\alpha)\,(\sigma\eta - 1)$$

$$\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} \quad ; \quad \Psi \equiv -\frac{\Theta\,\sigma_\alpha}{\sigma_\alpha + \varphi}$$

Role of openness: assuming high substitutability (high η, γ)

$$\frac{\partial \sigma_\alpha}{\partial \alpha} < 0 \qquad ; \qquad \frac{\partial \kappa_\alpha}{\partial \alpha} < 0$$

Open-economy distortions

First of all: this is model dependent!

1. Imperfect competition (monopoly power): corrected with employment subsidy $1-\tau$.
2. Staggered (Calvo) price setting, resulting fluctuations in the markup: corrected by the inflation stabilization.
3. Terms of trade distortion arising from imperfect substitutability of domestic and foreign goods: increase employment subsidy.

Policy that stabilizes domestic inflation maximizes welfare. Nominal exchange rate, CPI inflation are adjusting to replicate the response of the terms of trade that would obtained under flexible prices.

Optimal Monetary Policy

Background and Strategy

A Special Case

$$\sigma = \eta = \gamma = 1$$

Optimality of Flexible Price Equilibrium:

$$(1-\tau)(1-\alpha) = 1 - \frac{1}{\epsilon}$$

Implied Monetary Policy Objectives

$$y_t = y_t^n$$
$$\pi_{H,t} = 0$$

for all t.

Implementation

$$i_t = r_t^n + \phi_\pi \, \pi_{H,t} + \phi_y \, \widetilde{y}_t$$

Evaluation of Alternative Monetary Policy Regimes

Welfare Losses (special case)

$$\mathbb{W} = -\frac{(1-\alpha)}{2} \sum_{t=0}^{\infty} \beta^t \left[\frac{\epsilon}{\lambda} \pi_{H,t}^2 + (1+\varphi) \, \widetilde{y}_t^2 \right]$$

Average period losses

$$\mathbb{V} = -\frac{(1-\alpha)}{2} \left[\frac{\epsilon}{\lambda} \, var(\pi_{H,t}) + (1+\varphi) \, var(\widetilde{y}_t) \right]$$

Three Simple Rules

Domestic inflation-based Taylor rule (DITR)

$$i_t = \rho + \phi_\pi \, \pi_{H,t}$$

CPI inflation-based Taylor rule (CITR):

$$i_t = \rho + \phi_\pi \, \pi_t$$

Exchange rate peg (PEG)

$$e_t = 0$$

Impulse Responses and Welfare Evaluation

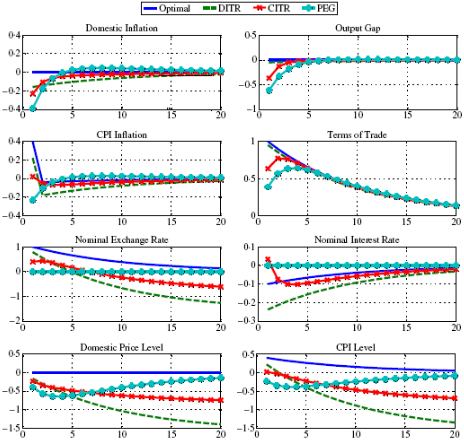
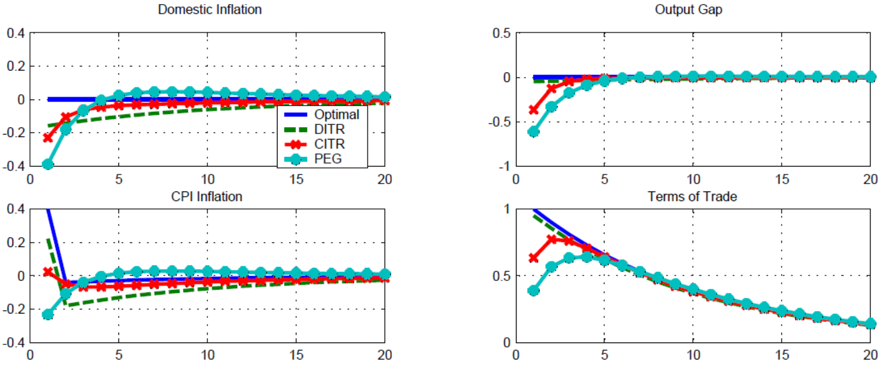
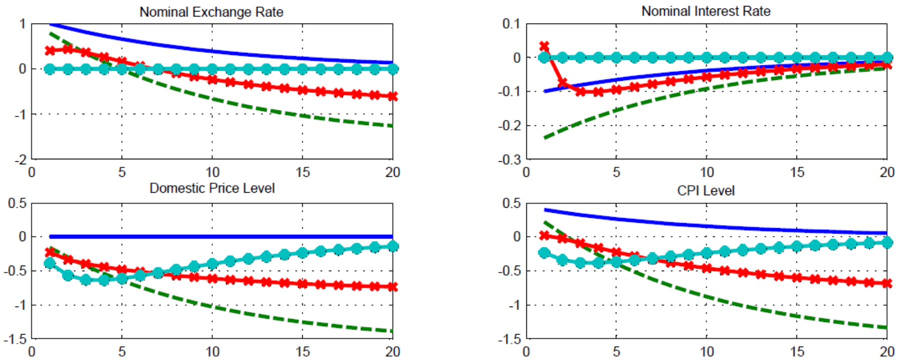


FIGURE 1
Impulse responses to a domestic productivity shock under alternative policy rules

Impulse responses to positive technology shock



IMPULSE RESPONSES TO POSITIVE TECHNOLOGY SHOCK



Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal I. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

Note: Sd denotes standard deviation in %.

TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(domestic infl)	0·0157	0·0151	0·0268
Var(output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

An Extension with Imperfect Pass-Through (Monacelli JIE 05)

Setup as in GM, with rest of the world modelled as a single economy.

Key Assumption:

- imports sold through retail firms
- price at the dock: $e_t + p^*_{F,t}(j)$
- staggered price setting by retailers $\Longrightarrow$ in general, $p_{F,t}(j) \neq e_t + p^*_{F,t}(j)$

Law of One Price Gap:

$$\psi_{F,t} \equiv e_t + p^*_t - p_{F,t}$$

Consistent with the evidence (Campa and Goldberg (REStat 05):

- partial pass-through in the short run
- full pass through in the long run (for most industries).

Imported Goods Inflation:

$$\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \lambda_F \; \psi_{F,t}$$

Domestic Goods Inflation

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda_H \; \widehat{mc}_t$$

$\Longrightarrow$ impossibility of replicating flexible price allocation

$\Longrightarrow$ emergence of a policy trade-off

$\Longrightarrow$ gains from commitment

References

Bils, Mark, and Peter J. Klenow (2004) 'Some evidence on the importance of sticky prices.' *Journal of Political Economy* 112(5), 947–985

Calvo, Guillermo A. (1983) 'Staggered prices in a utility-maximizing environment.' *Journal of Monetary Economics* 12, 983–998

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005) 'Nominal rigidities and the dynamic effects of a shock to monetary policy.' *Journal of Political Economy* 113(1), 1–43

Dhyne, Emmanuel, Luis J. Alvarez, Herve Le Bihan, Giovanni Veronese, Daniel Dias, Johannes Hoffmann, Nicole Jonker, Patrick Lunnemann, Fabio Rumler, and Jouko Vilmunen (2006) 'Price changes in the euro area and the united states: Some facts from individual consumer price data.' *Journal of Economic Perspectives* 20(2), 171–192

Dickens, William T., Lorenz Goette, Erica L. Groshen, Steinar Holden, Julian Messina, Mark E. Schweitzer, Jarkko Turunen, and Melanie E. Ward (2007) 'How wages change: Micro evidence from the international