Advanced Macroeconomics 4 (ECOM-R319)

Monetary Policy and Business Cycles

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Outline

We follow chapters 1–5, 7 in Galí's excellent book:

1. Introduction
2. Monetary Policy in Classical Model
3. The Basic New Keynesian Model
4. Monetary Policy Design in the Basic New Keynesian Model
5. Monetary policy at the zero lower bound of nominal interest rate; Debt and develerageing (material listed in syllabus)
6. Time-Consistency in Monetary Policy: Discretion vs. Commitment
7. Monetary Policy in Open Economy
8. Fiscal theory of price level: Interaction of monetary and fiscal policy (material listed in syllabus)

We will, in addition, look at the solution methods behind our computational tool `dynare`. Note: many of the slides are reproduced from Galí's book's

Outline

OUTLINE

MOTIVATION

- Monetary policy has been a central area of macroeconomic research
- The macroeconomics of monetary policy studies the interaction between monetary policy, inflation and business cycles (fluctuations in economic activity)
- The modern models of monetary policy build on — as most of the recent macroeconomic research — the real business cycle models by Kydland and Prescott (1982) and Prescott (1986).

Real Business Cycle Revolution

Methodological revolution

Intertemporally optimizing agents. Budget and technology constraints.

Conceptual revolution

- In a frictionless markets under perfect competition business cycles are efficient: no need for stabilization; stabilization may be counter-productive.
- Economic fluctuations are caused by technology shocks: they are the main source of fluctuation.
- Monetary factors (price level) has a limited (or no) role: money (price level) has no effect on the real economy, real wages, relative prices, consumption, investments, employment, ...

New Keynesian Model

- Methodologically similar to RBC models.
- Builds on the following features:
  - monopolistic competition
  - nominal rigidities

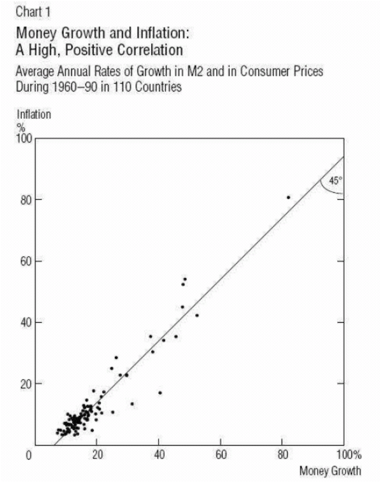
  $\rightarrow$ short-run non-neutrality of money.
- Leads to differences w.r.t RBC models: economy's response to shocks is generally inefficient.
- Removing the effects of non-neutrality is potentially welfare improving $\rightarrow$ role for monetary policy.

OUTLINE

Empirical evidence on money, inflation and output I

- Help to judge theoretical models: theoretical models should be consistent with the empirical data
- Help to evaluate the effects of money and monetary policy

Long-run correlations: money growth and inflation



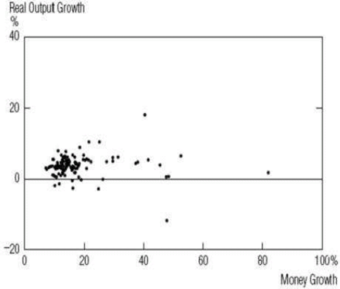
- long-run correlations between money growth and inflation close to one
- quantitative theory of money predicts: $\Delta m \longrightarrow \pi$. Causality?

Long-run correlations: money growth and output growth

Chart 2

Money and Real Output Growth: No Correlation in the Full Sample . . .

Average Annual Rates of Growth in M2 and in Nominal Gross Domestic Product, Deflated by Consumer Prices During 1960–90 in 110 Countries



Source: International Monetary Fund

No long-run trade-off between inflation and unemployment ⟶Phillips curve vertical (in the long-run).

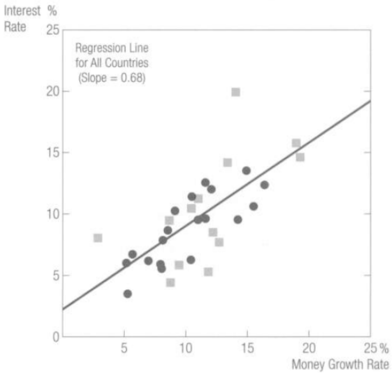
FISHER EQUATION

Fisher equation: $i_t = r_t + \mathrm{E}_t\, \pi_{t+1}$
$\longrightarrow i_{SS} = r_{SS} + \pi_{SS}$

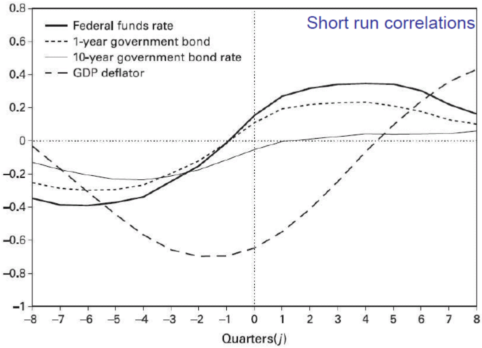
- Fisher equation: nominal interest rates should be positively related to expected inflation, suggesting that the level of nominal interest rate positively link to average inflation in the long run. Then nominal interest rates and average money growth rates should also be positively correlated
- Monnet and Weber (2001): higher correlation between money growth and long-term interest rates. Consistent with Fisher equation
- Mishkin (1992): test for the presence of a long-run relationship between the interest rate and money growth rate. Support Fisher equation in the long run (trend together)

MONEY GROWTH AND INTEREST RATES

Chart 1 Money Growth vs. Money Market Rates



SHORT-RUN CORRELATIONS: INTEREST RATES, INFLATION, OUTPUT



Interest rates are negatively correlated with GDP at lags and positively at leads
Price *level* is negatively correlated with GDP at lags (and contemporaneously) and positively at (long) leads

Correlation and causality

When investigating the joint behaviour of money, prices, interest rates, and output, one of the challenges is to determine the degree to which these data reveal causality relationship

Correlation is a statistical measure of co-movement between two variables but tells little about causality

Lots of econometric works have thus been conducted to assess the effects of money on output

Natural experiments non-existent/rare in empirical macroeconomics

⟶modelling the joint probability distribution of data

⟶to obtain causality, *identifying* assumptions needed

⟶robustness checks (wrt identification)!

SIMS'S VAR MODELING FRAMEWORK

Step 1 Construct statistical forecasting model (VAR) to separate expected from unexpected effects

Step 2 Identify causal links and extract fundamental shocks
(VAR $\longrightarrow$ SVAR)

Step 3 Trace effects of fundamental shocks on economy over time — Impulse response analysis

SIMS' STEP 1: CONSTRUCT VAR

Forecasting model for a collection of macro variables without *ad hoc* identifying restrictions

Each variable is forecast using lagged values of itself and all other variables:

$$\text{Variable} = \underbrace{\text{Forecast (expected)}}_{\text{systematic part}} + \text{error (unexpected)}$$

Short-term forecasting accuracy is typically quite good

SIMS' STEP 2: INDENTIFY CAUSAL LINKS

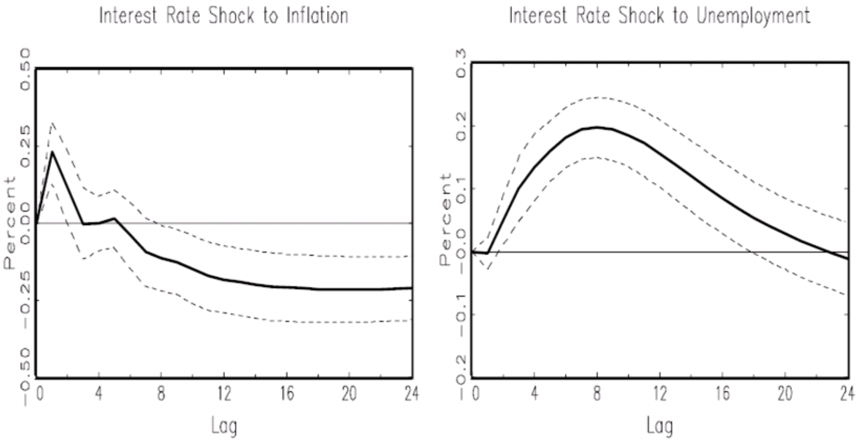
Identify causal links and extract fundamental shocks using theory-based and non-ad hoc assumptions

- Some variables react to others with a lag (e.g., M only responds to Y with a lag)
- Long-run neutrality arguments (e.g., M has no long-run impact on Y)

Process transforms errors in VAR equations into fundamental shocks

SIMS'S STEP 3: IMPULSE RESPONSE ANALYSIS

How do macro variables respond to unexpected fundamental shocks?



OUTCOME

- Sims and others were able to show — using SVAR's — that influences of monetary policy were detectable in the data.
- But at the same time, they showed that most movements in both money stock and interest rates represented systematic reactions of monetary authorities to the state of the economy.
- Only a small part of macroeconomic fluctuations could be attributed to erratic monetary policy (= monetary policy shocks).

Identification in CEE (2005)

Variables in the VAR, $Y_t = [Y'_{1t}\ Y'_{2t}]'$:

$$Y_{1t} = \{GDP, C, P, I, W/P, Y/N\}$$

and

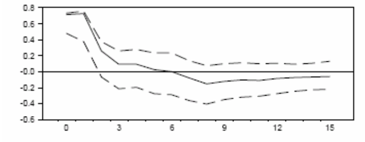
$$Y_{2t} = \{\text{real profits}, \Delta M\}$$

Monetary policy shock

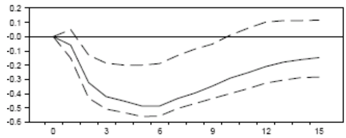
1. *no contemporaneous impact to* Y_{1t}
2. monetary authority does know Y_{1t} and only *past values* of Y_{2t}

RESPONSE TO MONETARY POLICY SHOCK

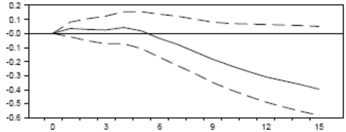
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



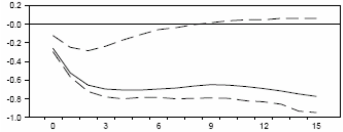
Federal funds rate



GDP



GDP deflator



M2

OTHER APPROACHES

VARs without recursive assumption
E.g.: equations for banking reserves
Romer and Romer's Narrative Approach (Fed's record of policy actions)
Narrow time window (GÃ¼rgaynak et al; Gertler et al)
Sign restrictions
Large information set

Summary of the effects of monetary policy

Contractionary MP shocks lead to a reduction in inflation (possibly with initial increase, price puzzle)

Contractionary MP shocks lead to hump-shaped response in output

Sluggish response of macroeconomic variables to MP shocks (peak after 1–2 years)

Small overall contribution of MP shocks on BC fluctuations (5–30 %)

[Systematic vs. non-systematic MP]

Cross-section / microeconomics evidence

Supportive to price rigidity

4–6 quarters by Bils and Klenow (2004), 8–11 quarters by Nakamura and Steinsson (2008), 8-11 by Álvarez et al. (2006); Dhyne et al. (2006).
Wages are also rigid: 1 year, downward rigidity by Dickens et al. (2007)

OUTLINE

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This section builds mostly on Sargent (1987).

Outline

LAG OPERATORS

The backward shift or lag *operator* is defined as

$$Lx_t = x_{t-1}$$

$$L^n x_t = x_{t-n}, \quad \text{for } n = \ldots, -2, -1, 0, 1, 2, \ldots$$

Note that if $n < 0$ the operator shifts x_t forward.
This language is loose, since we start with the sequence

$$\{x_t\}_{t=-\infty}^{\infty},$$

where x_t is a real number. We *operate on* $\{x_t\}_{t=-\infty}^{\infty}$ *by* L.
This section builds mostly on Sargent (1987).

Operations with lag operator I

Lag operator and multiplication operator are *commutative*

$$L(\beta x_t) = \beta L x_t.$$

It is *distributive* over the addition operator

$$L(x_t + w_t) = Lx_t + Lw_t.$$

Hence, we are free to use the standard commutative, associative, and distributive algebraic laws for multiplication and addition to express the compound operator in an alternative form.
Examples

$$y_t = (a + bL)Lx_t = (aL + bL^2)x_t = ax_{t-1} + bx_{t-2}$$

Operations with lag operator II

or

$$\begin{aligned}(1-\lambda_1 L)(1-\lambda_2 L)x_t &= (1-\lambda_1 L-\lambda_2 L+\lambda_1\lambda_2 L^2)x_t \\ &= [1-(\lambda_1+\lambda_2)L+\lambda_1\lambda_2 L^2]x_t \\ &= x_t-(\lambda_1+\lambda_2)x_{t-1}+\lambda_1\lambda_2 x_{t-2}.\end{aligned}$$

and

$$Lc = c$$

and

$$L^0 = 1$$

Polynomials in the lag operator I

Polynomial

$$A(L) = a_0 + a_1 L + a_2 L^2 + \cdots = \sum_{j=0}^{\infty} a_j L^j,$$

where a_j's are constants.

$$\begin{aligned} A(L)x_t &= (a_0 + a_1 L + a_2 L^2 + \cdots)x_t \\ &= a_0 x_t + a_1 x_{t-1} + a_2 x_{t-2} + \cdots = \sum_{j=0}^{\infty} a_j x_{t-j}. \end{aligned}$$

Rational $A(L)$

$$A(L) = \frac{B(L)}{C(L)},$$

POLYNOMIALS IN THE LAG OPERATOR II

where

$$B(L) = \sum_{j=0}^{m} b_j L^j, \qquad C(L) = \sum_{j=0}^{n} c_j L^j,$$

where b_j, c_j, m and n are constants.
Simple example

$$A(L) = \frac{1}{1-\lambda L} = 1 + \lambda L + \lambda^2 L^2 + \cdots,$$

which is "useful" only if $|\lambda| < 1$. Why?
Then

$$\frac{1}{1-\lambda L} x_t = (1 + \lambda L + \lambda^2 L^2 + \cdots) x_t = \sum_{i=0}^{\infty} \lambda^i x_{t-i}.$$

POLYNOMIALS IN THE LAG OPERATOR III

Consider the case $|\lambda| > 1$, the following "trick" is useful:

$$\begin{aligned} \frac{1}{1-\lambda L} &= \frac{-(\lambda L)^{-1}}{1-(\lambda L)^{-1}} \\ &= -\frac{1}{\lambda L}\left[1+\frac{1}{\lambda}L^{-1}+\left(\frac{1}{\lambda}\right)^{2}L^{-2}+\cdots\right] \\ &= -\frac{1}{\lambda}L^{-1}-\left(\frac{1}{\lambda}\right)^{2}L^{-2}-\left(\frac{1}{\lambda}\right)^{3}L^{-3}+\cdots . \end{aligned}$$

Then

$$\frac{1}{1-\lambda L}x_t = -\frac{1}{\lambda}x_{t+1}-\left(\frac{1}{\lambda}\right)^{2}x_{t+2}+\cdots = -\sum_{i=1}^{\infty}\lambda^{-i}x_{t+i},$$

which is geometrically declining weighted sum of *future* values of x_t.

Outline

USEFUL PROOFS

For any $a \neq 1$,

$$\sum_{i=1}^{T} a^i = a\frac{1-a^T}{1-a}, \quad \sum_{i=0}^{T} a^i = a\frac{1-a^{T+1}}{1-a}$$

and

$$\sum_{i=1}^{T} ia^i = \frac{1}{1-a}\left(a\frac{1-a^T}{1-a} - Ta^{T+1}\right)$$

OUTLINE

DIFFERENCE EQUATION I

Consider the following difference equation

$$y_t = \lambda y_{t-1} + bx_t + a, \quad t = \ldots, -1, 0, 1, \ldots, \tag{3.1}$$

where y_t is an *endogenous* variable and x_t is an *exogenous* variable and sequences of real number and $\lambda \neq 1$. It can be written as

$$(1 - \lambda L)y_t = a + bx_t.$$

Multiply both sides by $(1 - \lambda L)^{-1}$ to obtain

$$\begin{aligned} y_t &= \frac{a}{1-\lambda L} + \frac{b}{1-\lambda L} x_t + c\lambda^t \\ &= \frac{a}{1-\lambda} + b\sum_{i=0}^{\infty} \lambda^i x_{t-i} + c\lambda^t. \end{aligned} \tag{3.2}$$

DIFFERENCE EQUATION II

The reason why we have the last term is that for any constant c

$$(1 - \lambda L)c\lambda^t = c\lambda^t - c\lambda\lambda^{t-1} = 0.$$

By multiplying both sides by $1 - \lambda L$, gives the original difference equation! This the "general solution"!
To get the "particular solution" we must be able to tie down the constant c *with an additional bit of information*.
An example of an additional bit of information is

$$\lim_{n \to \infty} \sum_{i=n}^{\infty} \lambda^i x_{t-i} = 0 \quad \text{for all } t.$$

It simply says that $\lambda^i x_{t-i}$ must be "small" for large i. If x_t were constant, say $\bar{x}$, all time, this requires $|\lambda| < 1$. This also results bounded constant term in the solution.

DIFFERENCE EQUATION III

Assume $t > 0$ to see the impact of an arbitrary initial condition. Solution (3.2) may be written as

$$\begin{aligned} y_t &= a\sum_{i=0}^{t-1}\lambda^i + a\sum_{i=t}^{\infty}\lambda^i + b\sum_{i=0}^{t-1}\lambda^i x_{t-i} + b\sum_{i=t}^{\infty}\lambda^i x_{t-i} + c\lambda^t \\ &= a\frac{1-\lambda^t}{1-\lambda} + a\frac{\lambda^t}{1-\lambda} + b\sum_{i=0}^{t-1}\lambda^i x_{t-i} + b\lambda^t\sum_{i=0}^{\infty}\lambda^i x_{0-i} + c\lambda^t, \end{aligned}$$

$$y_t = a\frac{1-\lambda^t}{1-\lambda} + b\sum_{i=0}^{t-1}\lambda^i x_{t-i} + \lambda^t\left(\frac{a}{1-\lambda} + b\sum_{i=t}^{\infty}\lambda^i x_{0-i} + c\right), \quad t \geq 1.$$

DIFFERENCE EQUATION IV

The term in braces is equals y_0! Hence

$$\begin{aligned} y_t &= a\frac{1-\lambda^t}{1-\lambda} + b\sum_{i=0}^{t-1}\lambda^i x_{t-i} + \lambda^t y_0 \\ &= \frac{a}{1-\lambda} + \lambda^t\left(y_0 - \frac{a}{1-\lambda}\right) + b\sum_{i=0}^{t-1}\lambda^i x_{t-i}, \quad t \geq 1. \end{aligned}$$

If $x_t = 0$ $(\forall t)$, we obtain the typical text book case. If $y_0 = a/(1-\lambda)$, $y_t = y_0$ and $a/(1-\lambda)$ is the "stationary point" or long-run equilibrium value of y.

Difference equation V

The original difference equation (3.1) may be solved "forward" by applying the "forward inverse".

$$
\begin{aligned}
y_t &= \frac{-(\lambda L)^{-1}}{1-(\lambda L)^{-1}} a + b \frac{-(\lambda L)^{-1}}{1-(\lambda L)^{-1}} x_t + d\lambda^t, \\
&= \frac{a}{1-\lambda} - b \sum_{i=0}^{\infty} \left(\frac{1}{\lambda}\right)^{i+1} x_{t+i+1} + d\lambda^t,
\end{aligned} \tag{3.3}
$$

where d is constant to be determined by some side condition. An example of such side condition is

$$
\lim_{n\to\infty} \sum_{i=n}^{\infty} \left(\frac{1}{\lambda}\right)^{i} x_{t+i} = 0.
$$

Difference equation VI

Equivalence of (3.2) and (3.3)

If $a = 0$, then for any value of $\lambda \neq 1$ both (3.2) and (3.3) represent solution to the difference equation (3.1). They are simply alternative representation of the solution! The equivalence will hold whenever

$$\frac{b}{1-\lambda L}x_t \tag{3.4}$$

and

$$b\frac{-(\lambda L)^{-1}}{1-(\lambda L)^{-1}}x_t \tag{3.5}$$

are both finite for all t.

It often happens that one of them fails to be finite.

Difference equation VII

- If the sequence $\{x_t\}$ is bounded and $|\lambda| < 1$, the (3.4) is a convergent sum for all t.
- If the sequence $\{x_t\}$ is bounded and $|\lambda| > 1$, the (3.5) is a convergent sum for all t.

Since our desire is to impose that the sequence $\{y_t\}$ is bounded, and we do not have sufficient side conditions, then we must set $c = 0$ in (3.2) or $d = 0$ (3.3). To see that

- If $\lambda > 1$ and $c > 0$, then

$$\lim_{t\to\infty} c\lambda^t = \infty.$$

- If $\lambda < 1$ and $d < 0$, then

$$\lim_{t\to\infty} d\lambda^t = \infty.$$

All of the above stuff means that we need to solve "stable roots" $|\lambda| < 1$ backward and "unstable roots" $|\lambda| > 1$ forward.

GRAPHICAL ANALYSIS OF DIFFERENCE EQUATIONS I

Simple difference equation:

$$x_{t+1} = ax_t + b$$

in a *phase* diagram.
The phase diagram tells the consecutive values of x_t when we start from arbitrary point x_0. There is 45° line to equate x_{t+1} and x_t. Phase diagram is very useful for illustrating the dynamics — in particular in the case of nonlinear difference equation.

GRAPHICAL ANALYSIS OF DIFFERENCE EQUATIONS II

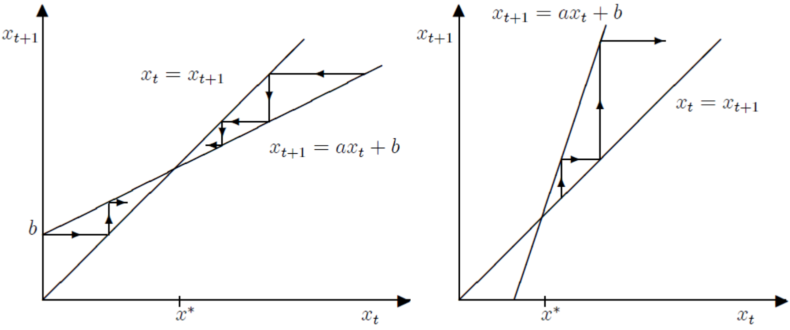


Figure 2.5.3 *Phase diagrams of (2.5.6) for positive (left panel) and negative b (right panel) and higher a in the right panel*

OUTLINE

(LOG)LINEARIZING

The *stationarized* decision rules, budget constraints and equilibrium conditions can typically be written in the following form

$$\mathrm{E}_t \, \Psi(Z_{t+1}, Z_t) = 0, \tag{3.6}$$

where Z_t and 0 are $n \times 1$ vectors. The conditional expectation operator E_t uses information up to and including period t. Note, that it is not restrictive to use the first order system, i.e. having only one lead. The higher order leads/lags may be introduced by augmenting the state vector Z_t (google "companion form").

The goal is to approximate (3.6) with a linear system, which can then be solved using the methods that will be described in the following chapters.

Notation

Denote the steady state by a variable without time index, Z. Small letter denotes log of original capital letter variable, $z_t \equiv \log(Z_t)$.

The *deterministic* steady state of (3.6) is

$$\Psi(Z, Z) = 0,$$

Note, that Z, the deterministic steady state of the model is a nonlinear function of the model's parameters μ.

Compute the first-order Taylor approximation around the steady-state Z.

$$0 \approx \mathrm{E}_t \left\{ \Psi(Z,Z) + \underbrace{\frac{\partial \Psi}{\partial Z_t}(Z,Z)}_{\equiv A(\mu)} \times (Z_t - Z) + \underbrace{\frac{\partial \Psi}{\partial Z_{t+1}}(Z,Z)}_{B(\mu)} \times (Z_{t+1} - Z) \right\}$$

where $Z_t - Z$ is $n \times 1$, $\partial \Psi(Z,Z)/\partial Z_t$ denotes the Jacobian of $\Psi(Z_{t+1}, Z_t)$ wrt Z_{t+1} evaluated at (Z,Z). To shorten

$$A(\mu)(Z_t - Z) + B(\mu)\,\mathrm{E}_t(Z_{t+1} - Z) = 0.$$

The coefficient matrices are function of deep (model's) parameters.
Note that this is a mechanical step that is typically done by the software (e.g. Dynare, Iris). Often the Jacobian may be computed analytically.

LOGARITHMIC APPROXIMATION

Logarithmic approximation is a special case of above. Note that $Z_t = \exp(\log(Z_t))$, and, as denoted above, $Z_t = \exp(z_t)$. Suppose we have

$$f(X_t, Y_t) = g(Z_t), \tag{3.7}$$

with strictly positive X, Y, Z (ie the linearization point). The steady state counterpart is $f(X, Y) = g(Z)$.

This simple summarization is, for example, in the slides by Jürg Adamek).
(http://www.vwl.unibe.ch/studies/3076_e/linearisation_slides.pdf)

Start from replacing $X_t = \exp(\log(X_t))$ in (3.7),

$$f\left(e^{\log(X_t)}, e^{\log(Y_t)}\right) = g\left(e^{\log(Z_t)}\right),$$

i.e.

$$f\left(e^{x_t}, e^{y_t}\right) = g\left(e^{z_t}\right),$$

Taking first-order Taylor approximations from both sides:

$$\begin{aligned} f(X,Y) + f_1'(X,Y)X(x_t - x) + f_2'(X,Y)Y(y_t - y) \\ = g(Z) + g'(Z)Z(z_t - z) \end{aligned} \tag{3.8}$$

Often we denote $\hat{x}_t \equiv x_t - x$.

Divide both sides of (3.8) by $f(X,Y) = g(Z)$ to obtain

$$1 + f_1'(X,Y)X(x_t - x)/f(X,Y) + f_2'(X,Y)Y(y_t - y)/f(X,Y) \\ = 1 + g'(Z)Z(z_t - z)/g(Z) \quad (3.9)$$

Note that

$$f_1'(X,Y)X/f(X,Y)$$

is the elasticity of $f(X_t, Y_t)$ with respect to X_t at the steady-state point.

Also note, that $100 \times (x_t - x)$ tells X_t's relative deviation from the steady-state point.

WARNING!

You may only loglinearize strictly positive variable. Typical example of a variable that may obtain negative values is the net foreign asset. This needs to be linearized!

USEFUL LOG-LINEARIZATION RULES

Denote $\hat{x}_t \equiv \log(X_t) - \log(X)$

$$X_t \approx X(1 + \hat{x}_t)$$

$$X_t^{\rho} \approx X^{\rho}(1 + \rho\hat{x}_t)$$

$$aX_t \approx aX(1 + \hat{x}_t)$$

$$X_t Y_t \approx XY(1 + \hat{x}_t + \hat{y}_t)$$

$$Y_t(a + bX_t) \approx Y(1a + bX) + aY\hat{y}_t + bXY(\hat{x}_t + \hat{y}_t)$$

$$Y_t(a + bX_t + cZ_t) \approx Y(a + bX + cZ) + Y(a + bX + cZ)\hat{y}_t + bXY\hat{x}_t + cZY\hat{z}$$

$$\frac{X_t}{aY_t} \approx \frac{X}{aY}(1 + \hat{x}_t - \hat{y}_t)$$

$$\frac{X_t}{Y_t + aZ_t} \approx \frac{X}{Y + aZ}\left[1 + \hat{x}_t - \frac{Y}{Y + aZ}\hat{y}_t - \frac{aZ}{Y + aZ}\hat{z}_t\right]$$

OUTLINE

Blanchard and Kahn method I

Blanchard and Kahn (1980) develop a solution method based on the following setup of a model

$$\begin{bmatrix} x_{1t+1} \\ \mathrm{E}_t\, x_{2t+1} \end{bmatrix} = \tilde{A} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + Ef_t, \tag{3.10}$$

where

- x_{1t} is $n_1 \times 1$ vector of endogenous predetermined variables $=$ variables for which $\mathrm{E}_t\, x_{1t+1} = x_{1t+1}$.
  - These are typically backward-looking variables.
  - (Do not exist with $t+1$.)
  - For example k_{t+1} in the standard RBC model.
- x_{2t} is $n_2 \times 1$ vector of endogenous nonpredetermined variables $=$ for which $x_{2t+1} = \mathrm{E}_t\, x_{2t+1} + \eta_{t+1}$, where η_{t+1} represents an expectational error.

Blanchard and Kahn method II

- forward-looking variables
- jump-variables

- f_t contains $k \times 1$ vector of exogenous forcing variables: e.g. shock innovations.
- $\tilde{A}$ has full rank.

Use spectral decomposition[1] of the matrix $\tilde{A}$ as follows

$$\tilde{A} = \Lambda^{-1} J \Lambda,$$

where J is a *diagonal matrix* consisting of *eigenvalues* of $\tilde{A}$ that are ordered from in increasing value. It is partitioned as follows

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix},$$

where

- the eigenvalues in J_1 lie on or within the unit circle (stable eigenvalues)
- the eigenvalues in J_2 lie outside the unit circle (unstable eigenvalues)

[1] See also Jordan normal form or Eigendecomposition or factorization into canonical form

Matrix Λ contains the corresponding *eigenvectors*. It is partitioned accordingly (and E too)

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \quad E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

STABILITY CONDITION

Saddle-path stability aka "Blanchard-Kahn Condition"

If the number of unstable eigenvalues is equal to the number of nonpredetermined variables, the system is said to be saddle-path stable and an unique solution exists.

Other cases

1. If the number of unstable eigenvalues exceeds the number of nonpredetermined variables, no solution exists.
2. If the number of unstable eigenvalues is smaller than the number of nonpredetermined variables, there are infinite solutions

SADDLE-PATH CASE I

Rewrite (3.10) as

$$\begin{bmatrix} x_{1t+1} \\ \mathrm{E}_t\, x_{2t+1} \end{bmatrix} = \Lambda^{-1} J \Lambda \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} f_t \qquad (3.11)$$

and premultiply by Λ to obtain

$$\begin{bmatrix} \acute{x}_{1t+1} \\ \mathrm{E}_t\, \acute{x}_{2t+1} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \acute{x}_{1t} \\ \acute{x}_{2t} \end{bmatrix} + \begin{bmatrix} \acute{E}_1 \\ \acute{E}_2 \end{bmatrix} f_t,$$

where

$$\begin{bmatrix} \acute{x}_{1t} \\ \acute{x}_{2t} \end{bmatrix} \equiv \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}$$

$$\begin{bmatrix} \acute{E}_1 \\ \acute{E}_2 \end{bmatrix} \equiv \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}.$$

Saddle-path case II

- The system is now "de-coupled" in the sense that the nonpredetermined variables are related only to the unstable eigenvalues J_2 of $\tilde{A}$.
- Hence, we have two "seemingly unrelated" set of equations.
- As in the univariate case, we derive the solution of the nonpredetermined variables by forward iteration and predetermined variables by backward iteration.
- We start by analysing the lower block, ie the system of the nonpredetermined variables by performing the forward interation.
- Denote f_{2t} of those f_ts that are conformable with $\acute{E}_2$.
- The lower part of (3.11) is as follows

$$\acute{x}_{2t} = J_2^{-1} \, \mathrm{E}_t \, \acute{x}_{2t+1} - J_2^{-1} \acute{E}_2 f_{2t} \tag{3.12}$$

Saddle-path case III

- Shift it one period and use the law-of-iterated-expectations $(\mathrm{E}_t(\mathrm{E}_{t+1}\, x_t) = \mathrm{E}_t\, x_t)$

$$\mathrm{E}_t\, \acute{x}_{2t+1} = J_2^{-1}\, \mathrm{E}_t\, \acute{x}_{2t+2} - J_2^{-1} \acute{E}_2\, \mathrm{E}_t\, f_{2t+1}$$

and substitute it back to (3.12) to obtain

$$\acute{x}_{2t} = J_2^{-2}\, \mathrm{E}_t\, \acute{x}_{2t+2} - J_2^{-2} \acute{E}_2\, \mathrm{E}_t\, f_{2t+1} - J_2^{-1} \acute{E}_2 f_{2t} \tag{3.13}$$

- Because diagonal matrix J_2 contains the eigenvalues above the unit disc, J_2^{-n} will asymptotically wanish. The iteration results

$$\acute{x}_{2t} = -\sum_{i=0}^{\infty} J_2^{-(i+1)} \acute{E}_2\, \mathrm{E}_t\, f_{2t+i}$$

Saddle-path case IV

- Using the definition $\acute{x}_{2t}$, we may write in to the form

$$x_{2t} = -\Lambda_{22}^{-1}\Lambda_{21}x_{1t} - \Lambda_{22}^{-1}\sum_{i=0}^{\infty} J_2^{-(i+1)}\acute{E}_2 \,\mathrm{E}_t f_{2t+i} \tag{3.14}$$

Finally, the upper part of (3.11) is given by

$$x_{1t+1} = \tilde{A}_{11}x_{1t} + \tilde{A}_{22}x_{2t} + E_1 f_t, \tag{3.15}$$

where $\tilde{A}_{11}$ and $\tilde{A}_{22}$ are reshuffled $\tilde{A}$ according to the above ordering.

AN EXAMPLE WITH AR(1) SHOCK

Suppose

$$f_{2t+1} = \Phi f_{2t} + \varepsilon_{t+1}, \quad \varepsilon_t \sim \mathrm{IID}(0, \Sigma)$$

and Φ is full rank and its roots are within the unit disc (ie stationary VAR(1)).
Then

$$\mathrm{E}_t f_{2t+i} = \Phi^i f_{2t}, \quad i \geq 0$$

and (3.14) becomes

$$x_{2t} = -\Lambda_{22}^{-1}\Lambda_{21}x_{1t} - \Lambda_{22}^{-1}(I - \Phi J_2^{-1})^{-1}\acute{E}_2 f_{2t} \tag{3.16}$$

"VAR" FORM

Stacking sets of equations (3.14) and (3.15) gives the following

$$\underbrace{\begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix}}_{\equiv x_{t+1}} = \underbrace{\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{22} \\ -\Lambda_{22}^{-1}\Lambda_{21} & 0 \end{bmatrix}}_{\equiv F_1} \underbrace{\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix}}_{\equiv x_t} + \underbrace{\begin{bmatrix} E_1 & 0 \\ 0 & -\Lambda_{22}^{-1}(I - \Phi J_2^{-1})^{-1}\acute{E}_2 \end{bmatrix}}_{\equiv F_\Gamma} \underbrace{\begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix}}_{\equiv f_t} \tag{3.17}$$

ie

$$x_{t+1} = F_1 x_t + F_\Gamma f_t$$

Issues

- The model variables has to be classified either predetermined or nonpredetermined. $\longrightarrow$ model-specific system reduction may be required.
- $\tilde{A}$ has to be a full rank matrix. Hence, for example, identities are not allowed!

Klein's method I

Klein (2000) proposes a method that overcome some of the drawbacks of Blanchard and Kahn (1980) by allowing singular $\tilde{A}$. It is also computationally fast. The system has to be in the form

$$\tilde{A}\, \mathrm{E}_t\, x_{t+1} = \tilde{B} x_t + E f_t, \tag{3.18}$$

where f_t ($n_z \times 1$ vector) follows the VAR(1)[2]. $\tilde{A}$ and $\tilde{B}$ are $n \times n$ matrices, and E $n \times n_z$ matrix.

$$f_t = \Phi f_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{IID}(0, \Sigma)$$

and $\tilde{A}$ may be singular (reduced rank). This mean that we may have static equilibrium conditions (like identities) in the system.

Klein's method II

Decompose x_t to predetermined[3] x_{1t} and nonpredetermined x_{2t} variables as before. Then

$$E_t\, x_{t+1} = \begin{bmatrix} x_{1t+1} \\ E_t\, x_{2t+1} \end{bmatrix}.$$

As before the system will be de-coupled according to x_{1t} $(n_1 \times 1)$ and x_{2t} $(n_2 \times 1)$. Generalized Schur decomposition, that allows singularity is used instead of standard spectral decomposition. Applying it to $\tilde{A}$ and $\tilde{B}$ gives

$$Q\tilde{A}Z = S \tag{3.19}$$

$$Q\tilde{B}Z = T, \tag{3.20}$$

KLEIN'S METHOD III

where Q, Z are *unitary*[4] and S, T *upper triangular* matrices with diagonal elements containing the generalized eigenvalues of $\tilde{A}$ and $\tilde{B}$. Eigenvalues are ordered as above.
Z is partitioned accordingly

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}.$$

Z_{11} is $n_1 \times n_1$ and corresponds the stable eigenvalues of the system and, hence, conforms with x_1, ie predetermined variables.
Next we *triangularize* the system (3.18) to stable and unstable blocks

$$z_t \equiv \begin{bmatrix} s_t \\ u_t \end{bmatrix} = Z^H x_t,$$

KLEIN'S METHOD IV

where ^{H} denotes the Hermitian transpose. and (3.19) and (3.20) can be written as

$$\tilde{A} = Q'SZ^H$$
$$\tilde{B} = Q'TZ^H.$$

Premultiplying the partitioned system by Q we obtain

$$S\,\mathrm{E}_t\,z_{t+1} = Tz_t + QEf_t \tag{3.21}$$

and since S and T are upper triangular, (3.18) may be written as

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \mathrm{E}_t \begin{bmatrix} s_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} s_t \\ u_t \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} Ef_t. \tag{3.22}$$

KLEIN'S METHOD V

Due to block-diagonal (recursive) structure, the process u_t is unrelated s_t. We iterate this forward to obtain[5]

$$u_t = -T_{22}^{-1} \sum_{i=0}^{\infty} [T_{22}^{-1} S_{22}]^i Q_2 E \, \mathrm{E}_t f_{t+i}.$$

Since f_t is VAR(1), we obtain

$$u_t = M f_t$$
$$\operatorname{vec} M = [(\Phi' \otimes S_{22}) - I_n \otimes T_{22}]^{-1} \operatorname{vec}[Q_2 E].$$

This solution of the unstable component is used to solve the stable block, resulting

$$s_{t+1} = S_{11}^{-1} T_{11} s_t + S_{11}^{-1} [T_{12} M - S_{12} M \Phi + Q_1 E] f_t - Z_{11}^{-1} Z_{12} M \varepsilon_{t+1}.$$

Given the definition of u_t and s_t, we may express the solution in terms of original variables.

[2]We drop the constant term (and other stationary deterministic stuff) to simplify algebra.

Method of undetermined coefficients

Following Anderson and Moore (1985) (AiM) and Zagaglia (2005), DSGE model may be written in the form

$$H_{-1} z_{t-1} + H_0 z_t + H_1 \operatorname{E}_t z_{t+1} = D\eta_t, \tag{3.23}$$

where z_t is vector of endogenous variables and η_t are pure innovations with zero mean and unit variance.
The solution to (3.23) takes the form

$$z_t = B_1 z_{t-1} + B_0 \eta_t,$$

where

$$B_0 = S_0^{-1} D,$$
$$S_0 = H_0 + H_1 B_1.$$

B_1 satisfies the identity

$$H_{-1} + H_0 B_1 + H_1 B_1^2 = 0.$$

SUMMARIZING I

The general feature of the solution methods is that they result a "VAR(1)" representation of the model

$$\xi_t = F_0(\mu) + F_1(\mu)\xi_{t-1} + F_\Gamma(\mu)v_t, \tag{3.24}$$

where

- Variable vector ξ_t are the variables in the model.
- matrices $F_i(\mu)$ are complicated (and large) matrices whose exact form depends on the solution method (See, eg, previous slide)
- They are also highly nonlinear function of the "deep" parameters μ of the economic model
  - The parameters μ include, among others, the parameters specifying the stochastic processes of the model: shock variances, for example

Summarizing II

- We may use this representation to analytically calculate various model moments for given parameter values.
- We may use this also for simulating the model.
- Note that we do not know anything about the data at this stage.

OUTLINE

OUTLINE

HOUSEHOLD PROBLEM

Households

- Decide how much they consume, C_t, in each period and
- how much they work, N_t.
- They know the *current and historical* values of nominal wages W_t,
- and their savings in bonds B_t, with bond price Q_t, and
- their lump-sum taxes, T_t.
- Price level is P_t.

They cannot accumulate infinite debt (transversality condition):

$$\lim_{T \to \infty} \mathrm{E}_t \, \beta^T \frac{U_{C,T}}{U_{C,0}} \frac{B_T}{P_T} \geq 0$$

Households maximize the expected present value of utility

$$\max_{\{C_t,N_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to the following (flow) budget constraint

$$\underbrace{P_t C_t + Q_t B_t}_{\text{where they allocate their money}} \leq \underbrace{B_{t-1} + W_t N_t - T_t}_{\text{where they receive their money}} . \tag{4.1}$$

Q_t is the period t price of a bond that gives unit amount in the period $t+1$.

Optimality conditions

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t}$$

Marginal rate of substitution between consumption and leisure (equaling real wages).

$$Q_t = \beta \, \mathrm{E}_t \left(\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right)$$

Intertemporal marginal rate of substitution in nominal terms.

Parametric version

Assume the following functional form

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\sigma \geq 0$ and $\varphi \geq 0$.
Then

$$U_{N,t} = -N_t^{\varphi} \quad U_{C,t} = C_t^{-\sigma}$$

and the optimality conditions as follows

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{4.2}$$

$$Q_t = \beta \, \mathrm{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \quad \text{for } t = 0, 1, 2 \ldots . \tag{4.3}$$

Log-linearization of (4.3) around non-zero inflation and growth

In a growing economy $C_{t+1} > C_t$ and in a non-zero-inflation economy $P_{t+1} > P_t$. Hence, neither C nor P is a point. This means that the model is nonstationary: here growing in consumption and price level. The growth rates are, however, stationary. Hence, we may express the model in a stationary form using growth rates!
Let's denote $\dot{X}_t \equiv X_t / X_{t-1}$. Then (4.3) is

$$Q_t = \beta \, \mathrm{E}_t \left(\dot{C}_{t+1}^{-\sigma} \dot{P}_{t+1}^{-1} \right).$$

In the steady state: $Q = \beta \dot{C}^{-\sigma} / \dot{P}$. Denote $\rho \equiv -\log \beta$, then $\rho = -q - (\sigma \Delta c + \pi)$, where $\pi_t \equiv \log \dot{P}_t$ and, generally, $\Delta x_t \equiv \log \dot{X}_t$. Note also that $i_t = -q_t$.

Apply mechanically (3.8)

$$Q + Q(q_t - q) = \underbrace{\beta \dot{C}^{-\sigma} / \dot{P}}_{=Q} - \sigma \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{C}} \dot{C} (\mathrm{E}_t \, \Delta c_{t+1} - \Delta c)$$
$$+ (-1) \underbrace{\beta \frac{\dot{C}^{-\sigma}}{\dot{P}}}_{=Q} \frac{1}{\dot{P}} \dot{P} (\mathrm{E}_t \, \pi_{t+1} - \pi).$$

and divide by Q and get rid of constants to obtain

$$q_t - q = -\sigma(\mathrm{E}_t \, \Delta c_{t+1} - \Delta c) - (\mathrm{E}_t \, \pi_{t+1} - \pi)$$

and combine with log of steady-state to obtain

$$i_t - \mathrm{E}_t \, \pi_{t+1} - \rho = \sigma \, \mathrm{E}_t \, \Delta c_{t+1} \tag{4.4}$$

Loglinear versions

$$w_t - p_t = \sigma c_t + \varphi n_t$$
$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho\right),$$

where $i_t \equiv \log Q_t$ and $\rho \equiv -\log\beta$.
Non-stochastic steady-state

$$i = \pi + \rho$$

and the implied real rate

$$r \equiv i - \pi = \rho.$$

OUTLINE

Firms

Firms use only labour N to produce output Y:

$$Y_t = A_t N_t^{1-\alpha}. \tag{4.5}$$

Log-linearized as $y_t = a_t + (1-\alpha)n_t$. A_t is exogenously given stationary technology process (a shock process). Firms maximize profits

$$P_t Y_t - W_t N_t$$

subject to production function (4.5) and obtain the following FOC:

$$(1-\alpha)\underbrace{A_t N_t^{-\alpha}}_{=Y_t/N_t} = \frac{W_t}{P_t}. \tag{4.6}$$

which gives labour demand schedule and tells us how much labour the firm is willing to hire for given real wages and technological process A_t.

OUTLINE

EQUILIBRIUM

Goods market clearing

$$y_t = c_t$$

Labour market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha)$$

Asset market clearing

$$y_t = \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho\right)$$

Aggregate production

$$y_t = a_t + (1 - \alpha) n_t$$

Solution I

$$n_t = \psi_{na} a_t + \vartheta_n$$

$$y_t = \psi_{ya} a_t + \vartheta_y$$

$$\omega_t \equiv w_t - p_t = y_t - n_t + \log(1-\alpha) = \psi_{\omega a} a_t + \log(1-\alpha)$$

$$r_t \equiv i_t - \mathrm{E}_t\, \pi_{t+1} = \rho + \sigma\, \mathrm{E}_t(y_{t+1} - y_t) = \rho + \sigma \psi_{ya}\, \mathrm{E}_t(a_{t+1} - a_t).$$

where

$$\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} \quad \vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$$

$$\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \quad \vartheta_y \equiv (1-\alpha)\vartheta_n \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}.$$

SOLUTION II

- ⟶ real variables determined *independently of monetary policy* (super neutrality)
- ⟶ *optimal policy* is indetermined.
- ⟶ Specification of monetary policy needed to determine the nominal variables

Discussion on the parameters

Technology shock has always positive impact on output

$$\psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

is always positive given reasonable values of $\varphi > -1$.

Technology shock may reduce or increase employment

$$\psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} = \begin{cases} < 0 & \text{if } \sigma > 1 \\ = 0 & \text{if } \sigma = 1 \\ > 0 & \text{if } \sigma < 1. \end{cases}$$

A closer look at employment response

Let's rewrite (4.2) as follows

$$N_t^{\varphi} = \frac{W_t}{P_t} C_t^{-\sigma}$$

The elasticity of substitution is unity (or $1/\varphi$), and wealth elasticity is $-\sigma$ (or $-\sigma/\varphi$).

- The substitution effect dominates the negative wealth effect if $0 < \sigma < 1$ and
- *vice versa* if $\sigma > 1$.
- They cancel each other when $\sigma = 1$ (logarithmic utility).

Technology process defines properties of real interest rate

r_t will go down if $E_t\, a_{t+1} < a_t$ and go up if $E_t\, a_{t+1} > a_t$ (like in the growing economy).

Summary of equilibrium

In equilibrium, output, consumption, employment, real wages and real rate of return are function of productivity shock only — not of anything else! Hence monetary factors play no role in real economy, ie monetary policy is neutral w.r.t. real variables.

Outline

MONETARY POLICY RULES

1. Fixed interest rate rule
2. Inflation based interest rate rule
3. Money growth rule

FIXED INTEREST RATE RULE

Next we move to study various monetary policy rules!

Consider standard Fisher equation that we have derived above

$$i_t = \mathrm{E}_t\, \pi_{t+1} + r_t = \mathrm{E}_t\, p_{t+1} - p_t + r_t. \tag{4.7}$$

Its solution should be of the form

$$p_t = -\,\mathrm{E}_t \sum_{i=0}^{\infty} (i_{t+i} - r_{t+i}).$$

It is easy to see that this does not converge in general.

SIMPLE INFLATION BASED INTEREST RATE RULE I

Consider an interest rate rule

$$i_t = \rho + \phi_\pi \pi_t$$

and combine it with the Fisher equation to obtain

$$\rho + \phi_\pi \pi_t = \mathrm{E}_t \, \pi_{t+1} + r_t.$$

or

$$\phi_\pi \pi_t = \mathrm{E}_t \, \pi_{t+1} + x_t. \tag{4.8}$$

by denoting $x_t \equiv r_t - \rho$.
Let's further assume that x_t follows the first-order autoregressive process:

$$x_t = \theta x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim \mathrm{iid}(0, \sigma_\varepsilon^2), \quad |\theta| < 1,$$

SIMPLE INFLATION BASED INTEREST RATE RULE II

Equation (4.8) has many solutions. We can write the equilibria of this model as

$$\pi_{t+1} = \phi_\pi \pi_t - x_t + \delta_{t+1}, \quad \mathrm{E}_t\, \delta_{t+1} = 0, \tag{4.9}$$

where δ_{t+1} is any conditionally mean zero random variable. Multiple equilibria are indexed by arbitrary initial inflation π_0 and by the arbitrary random variables (called often "sunspots") δ_{t+1}.

If $|\phi_\pi| > 1$ all of these equilibria except one eventually explode. If we rule out such solutions, the a unique locally bounded solution remains.

We may iterate (4.8) forward to obtain solution of the form

$$\pi_t = \mathrm{E}_t \sum_{i=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^i x_{t+i} = \frac{x_t}{\phi_\pi - \theta}.$$

SIMPLE INFLATION BASED INTEREST RATE RULE III

Equivalently, we may select π_0 and $\{\delta_{t+1}\}$ in (4.9) appropriately

$$\pi_0 = \frac{x_0}{\phi_\pi - \theta}; \quad \delta_{t+1} = \frac{\varepsilon_{t+1}}{\phi_\pi - \theta}$$

Taylor principle

$\phi_\pi > 1$

It is attractive to study only this locally bounded equilibria. However, hyperinflations are historical realities.

MONEY GROWTH RULE I

Substitute (4.7) into the money demand equation to obtain

$$m_t - p_t = y_t - \eta(\mathrm{E}_t\,\pi_{t+1} + r_t).$$

Solve price level forward

$$p_t = \frac{1}{1+\eta}\mathrm{E}_t\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i (m_{t+i} + \eta r_{t+i} - y_{t+i})$$

$$= m_t + \mathrm{E}_t\sum_{i=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i \Delta m_{t+i} + \mathrm{E}_t\sum_{i=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^i (\eta r_{t+i} - y_{t+i})$$

MONEY GROWTH RULE II

and the implied nominal interest rate

$$i_t = \eta^{-1}\left[y_t - (m_t - p_t)\right]$$
$$= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t(\Delta m_{+k}) + r_t.$$

SUMMARY

- Real variables are *independent* of monetary policy.
- Monetary policy has an important impact on nominal variables.
- Since the household's utility depends only on real variables, monetary policy has no welfare implications.
- No monetary policy rule is better that any other.
- The non-existence of the interaction between nominal and real variables is in contrast to empirical evidence.

Outline

Various approaches to motivate money

In the above classical models, money had a role of unit of account: *cashless economy, cashless limit*.
Money provides liquidity services. They can be modelled, for example, as

- Real balances generate utility:
  Money-in-the-utility-function (MIUF)
- The transaction cost approach
  - Explicit microfounded matching models starting from double coincidence of wants
  - Cash-in-advance (CIA) constraint
  - Shopping-time model

We study MIUF and leave CIA as an exercise. First item can be found from micro courses.

OUTLINE

A MODEL WITH MONEY IN THE UTILITY FUNCTION I

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

Budget constraint

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t$$

Define $A_t \equiv B_{t-1} + M_{t-1}$, then

$$P_t C_t + Q_t A_{t+1} + (1 - Q_t) M_t \leq A_t + W_t N_t - T_t.$$

Note

$$(1 - Q_t) = 1 - \exp(-i_t) \simeq i_t,$$

which is the opportunity cost of holding money.

A MODEL WITH MONEY IN THE UTILITY FUNCTION II

In addition, the following transversality condition holds:

$$\lim_{T\to\infty} \mathrm{E}_t \, \beta^T \frac{U_{C,T}}{U_{C,0}} \frac{A_T}{P_T} = 0$$

Optimality conditions

$$-\frac{U_{N,t}}{U_{C,t}} = \frac{W_t}{P_t} \tag{4.10}$$

$$Q = \beta \, \mathrm{E}_t \left(\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right) \tag{4.11}$$

$$\frac{U_{m,t}}{U_{C,t}} = 1 - \exp(-i_t), \tag{4.12}$$

where marginal utilities are evaluated at $(C_t, M_t/P_t, N_t)$.
Two interesting cases:

A MODEL WITH MONEY IN THE UTILITY FUNCTION III

- Utility separable in real balances $\longrightarrow$ neutrality
- Utility non-separable in real balances (eg $U_{C,m} > 0$) $\longrightarrow$ non-neutrality

Non-separable utility

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X\left(C_t, \frac{M_t}{P_t}\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where

$$X\left(C_t, \frac{M_t}{P_t}\right) \equiv \begin{cases} \left[(1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-\nu}}, & \text{for } \nu \neq 1 \\ C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta}, & \text{for } \nu = 1. \end{cases}$$

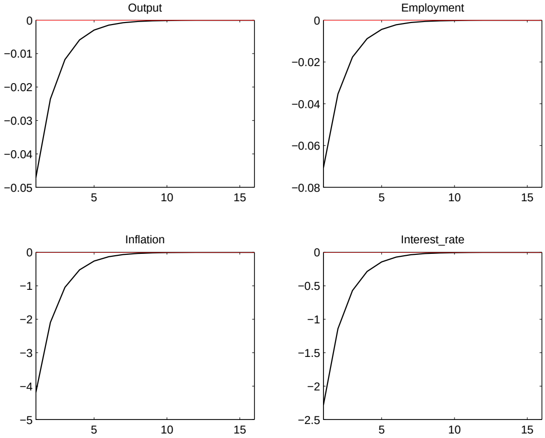
A MODEL WITH MONEY IN THE UTILITY FUNCTION IV

Impulse responses, when policy rule is

$$i_t = \rho + \phi_\pi \pi_t + v_t, \quad \text{where } v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

with calibrated values: $\nu = 2.56$, $\sigma = 2$ (resulting $U_{C,m} > 0$)

RESPONSES TO A POSITIVE MONETARY POLICY SHOCK (INTEREST RATE RULE)



Inflation and interest rate responses are annualized.

OUTLINE

OPTIMAL MONETARY POLICY IN A CLASSICAL SETTING I

Assume a hypothetical social planner that maximize the utility of representative household, that contains real money. Social planner faces a static problem, since only an individual household (not the society as a whole) can smooth its consumption over time. The planner's problem is to maximize

$$\max U(C_t, \frac{M_t}{P_t}, N_t)$$

subject to resource constraint

$$C_t = A_t N_t^{1-\alpha}$$

Optimal monetary policy in a classical setting II

The optimality conditions are given by

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha} \tag{4.13}$$

$$U_{m,t} = 0 \tag{4.14}$$

(4.13) First corresponds the labour market equilibrium that is independent of monetary policy (except in the non-separable MIUF case)

(4.14) Second condition equates marginal utility of real balances to the "social" marginal cost of producing them (zero!).

Optimal monetary policy in a classical setting III

From household's problem we know

$$\frac{U_{m,t}}{U_{c,t}} = 1 - e^{-i_t}.$$

RHS can be zero only if $i_t = 0$. This is called Friedman rule. (In steady-state) this results $\pi = -\rho(\equiv -\log(\beta)) < 0$, ie in the steady state, the price level declines at the rate of time preference.

Implementation

$$i_t = \phi(r_{t-1} + \pi_t)$$

for $\phi > 1$. Combined with the definition of the real rate results

$$\mathrm{E}_t\, i_{t+1} = \phi i_t,$$

Optimal monetary policy in a classical setting IV

whose only stationary solution is

$$i_t = 0, \text{ for all } t.$$

The implied equilibrium inflation is

$$\pi_t = -r_{t-1}.$$

OUTLINE

Outline

CES AGGREGATOR

In modern macro models with imperfect competition, the Dixit-Stiglitz, or Constant-Elasticity-of-Substitution aggregator plays an important role.

Consider a static optimization problem of a firm that buy infinite number of intermediate products $C(i)$, puts them together using technology

$$C = \left[\int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{5.1}$$

where ϵ is the elasticity of substitution (and also the price elasticity of the demand function). Its optimization problem is

$$\max_{C(i)} C \cdot P - \int_0^1 C(i)P(i)di$$

subject to the production technology (5.1).

The optimality conditions are given by

$$C(i) = \left[\frac{P(i)}{P}\right]^{-\epsilon} C \quad \forall i \in [0,1].$$

This is also the demand function of a good $C(i)$. (You must work out the details by yourself.) Plug this to the profits and use zero profit constraint to get the aggregate price level (=price index=marginal costs):

$$P = \left[\int_0^1 P(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}. \tag{5.2}$$

SOME SUMS

$$(1-\beta\theta)E_t\sum_{k=0}^{\infty}(\beta\theta)^k(p_{t+k}-p_t)=$$

$$(1-\beta\theta)E_t((p_t-p_t)+\beta\theta(p_{t+1}-p_t)+(\beta\theta)^2(p_{t+2}-p_t)+\cdots)=$$

$$E_t(\beta\theta(p_{t+1}-p_t)-(\beta\theta)^2(p_{t+1}-p_t)+(\beta\theta)^2(p_{t+2}-p_t)+\cdots)=$$

$$E_t(\beta\theta\pi_{t+1}+(\beta\theta)^2(-p_{t+1}+p_t+p_{t+2}-p_t)+\cdots)$$

$$=E_t(\beta\theta\pi_{t+1}+(\beta\theta)^2\pi_{t+2}+\cdots)$$

$$=E_t[\sum_{k=1}^{\infty}(\beta\theta)^k\pi_{t+k}]=E_t[\sum_{k=0}^{\infty}(\beta\theta)^k\pi_{t+k}]-\pi_t$$

OUTLINE

Introduction

The basic new Keynesian model consists of two key ingredients:

Imperfect competition

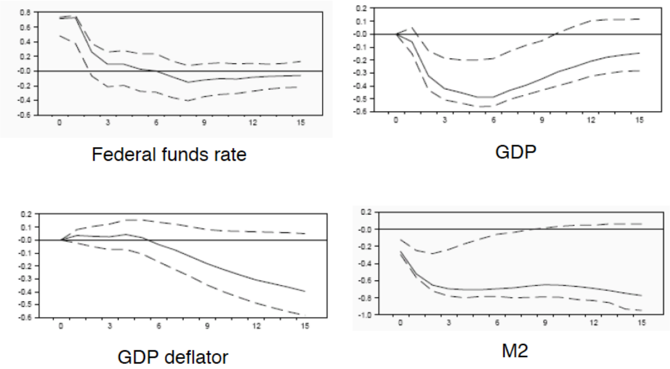
We assume that there is a continuum of firms and each produce a differentiated intermediate good for which it sets the price.

Price rigidities

We assume (*a la* Calvo (1983)) that, in each period, only a fraction of firms can change their price.

Empirical evidence I

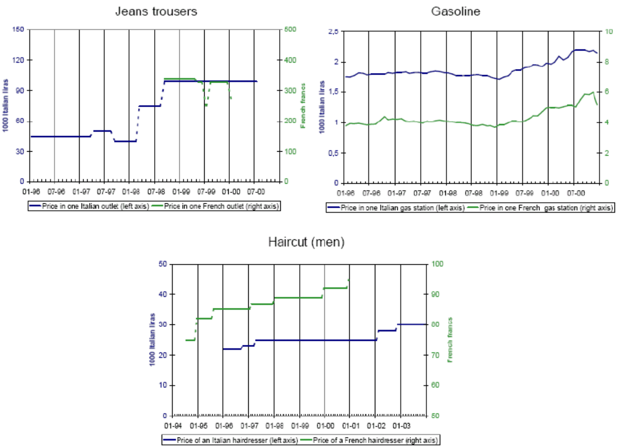
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



Source: Christiano, Eichenbaum and Evans (1999)

EMPIRICAL EVIDENCE II

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. WP 05

OUTLINE

HOUSEHOLDS

Household problem is the same as in the case of classical model except that the aggregate consumption consists of continuum of goods:

$$C_t = \left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

Household must allocate its consumption to different goods according to their relative price

$$C_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon} C_t, \quad \forall i \in [0,1], \tag{5.3}$$

where the aggregate price index is as in (5.2).

The optimality condition are as before and assuming same functional form of utility function, and loglinearizing, we obtain

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{5.4}$$

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho) \tag{5.5}$$

$$m_t - p_t = y_t - \eta i_t. \tag{5.6}$$

OUTLINE

FIRMS

Assume continuum of identical firms indexed by $i \in [0, 1]$ that use the following common production technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}.$$

All firms face an identical demand curve (5.3) and take aggregate price index P_t and aggregate consumption index C_t as given.

Calvo Fairy

A firm may change price of its product only when Calvo Fairy visits.

The probability of a visit is $1 - \theta$.

It is independent of the length of the time and the time elapsed since the last adjustment. Hence, in each period the $1 - \theta$ share of firms may change their price and rest, θ, keep their price unchanged.

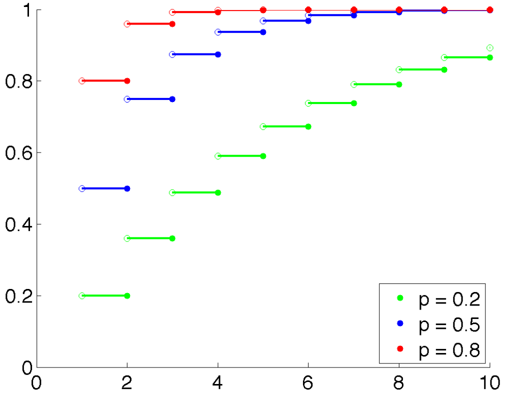
Mathematically, Calvo Fairy's visits follows Bernoulli process (discrete version of Poisson process).

The probability distribution of the number of periods between the visits of Calvo Fairy is geometric distribution.

The expected value of geometric distribution and, hence, the average number of periods between the price changes (of a firm) is

$$\frac{1}{1 - \theta}.$$

Cumulative distribution function of geometric distribution

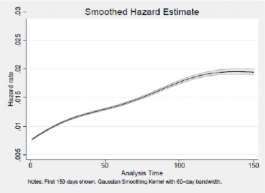


Source: http://en.wikipedia.org/wiki/Geometric_distribution

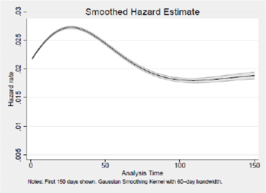
Hazard rate (of price changes)

Hazard rate is defined as the event rate (probability of price change) at time t conditional on survival (no price changes) until time t or later. In price setting this means the event rate of price changes.

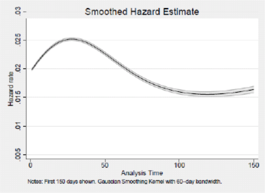
In Calvo setting, the hazard function is flat. This means that the event rate of price changes is not influenced by the (temporal) distance from the last price change.



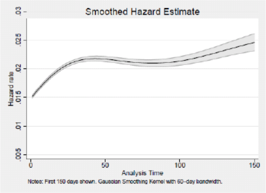
(a) Argentina



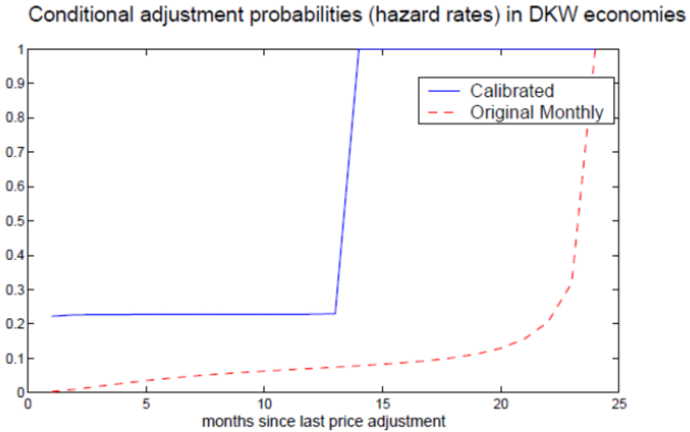
(b) Brazil



(c) Chile (with sales)



(d) Colombia


Conditional adjustment probabilities (hazard rates) in DKW economies
Calibrated
Original Monthly
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
25
months since last price adjustment

Stylized facts of price changes in the USA and the Euroa Area I

4-5 quarters

The average duration of a price spell in the euro area ranges from four to five quarters, which is about twice as long as in the United States.

Variation across products

The frequency of price changes varies substantially across products, with very frequent change for energy products and unprocessed food, and relatively infrequent changes for processed food, non-energy industrial goods and, particularly, services.

STYLIZED FACTS OF PRICE CHANGES IN THE USA AND THE EUROA AREA II

No important cross-country heterogeneity

Heterogeneity in price changes across countries in the euro area is relevant but less important than cross-sector heterogeneity in price changes. Differences in price changes across countries are partly related to the consumption structure and to the statistical treatment of sales.

Symmetric rigidity

There is no evidence of general downward price rigidity. Price decreases are not uncommon, except in services.

Stylized facts of price changes in the USA and the Euroa Area III

Changes larger than inflation

Price changes are sizeable compared to the prevailing inflation rate. Price reductions and price increases have a similar order of magnitude, though price reductions are on average larger.

No syncronization

Price changes are not synchronized across products, even within the same country.

Optimal price setting I

Let $P_t^\star$ denote the price level of the firm that receives price change signal. This is the price level of the firm that Calvo Fairy visits.

When making its pricing decision, the firm takes into account that it can change is price with the probability $1-\theta$, ie the chosen price remains the same with probability θ.

$$\max_{P_t^\star} E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left\{ P_t^\star Y_{t+k}(P_t^\star) - \Psi_{t+k}\left[Y_{t+k}(P_t^\star) \right] \right\},$$

Optimal price setting II

where $\Psi_{t+k}\left[Y_{t+k}(P_t^\star)\right]$ is firm's total costs that depends on the demand function $Y_{t+k}(P_t^\star)$ with a relative price of $P_t^\star/P_{t+k}$. Let $\psi_{t+k|t} \equiv \Psi'_{t+k}\left[Y_{t+k}(P_t^\star)\right]$ and

$$Y_{t+k}(P_t^\star) = \left(\frac{P_t^\star}{P_{t+k}}\right)^{-\epsilon} C_{t+k}$$

The first one is the demand function that the firm faces and is due to the households' consumption index.

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right).$$

The second equation is the nominal *stochastic discount factor* (pricing kernel), that household use to price any financial asset. Note, that in our standard household's optimisation problem,

Optimal price setting III

the household price one-period bond using the very same pricing kernel.
The total costs are

$$\Psi_{t|t}(i) = W_t N_t(i) = W_t \underbrace{\left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}}_{\text{from prod.func.}}$$

and the marginal costs

$$\psi_{t|t}(i) \equiv \Psi_t'(i) = \frac{\partial W_t \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}}{\partial Y_t(i)} = \frac{W_t N_t(i)}{(1-\alpha) Y_t(i)}.$$

The optimality condition

$$\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left\{ Y_{t+k}(P_t^\star) + P_t^\star \frac{\partial Y_{t+k}(P_t^\star)}{\partial P_t^\star} - \Psi'(\cdot) \frac{\partial Y_{t+k}(P_t^\star)}{\partial P_t^\star} \right\} = 0$$

where

$$\begin{aligned}
\frac{\partial Y_{t+k}(P_t^\star)}{\partial P_t^\star} &= \frac{\partial \left(\frac{P_t^\star}{P_{t+k}} \right)^{-\epsilon} C_{t+k}}{\partial P_t^\star} \\
&= -\epsilon \left(\frac{P_t^\star}{P_{t+k}} \right)^{-\epsilon-1} C_{t+k} \frac{1}{P_{t+k}} \\
&= -\epsilon \frac{P_{t+k}}{P_t^\star} \frac{1}{P_{t+k}} \underbrace{\left(\frac{P_t^\star}{P_{t+k}} \right)^{-\epsilon} C_{t+k}}_{=Y_{t+k}(P_t^\star)} \\
&= -\epsilon \frac{1}{P_t^\star} Y_{t+k}(P_t^\star).
\end{aligned}$$

SOME INTERMEDIATE RESULTS

Then

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \Big\{ Y_{t+k}(P_t^\star) + P_t^\star(-\epsilon)\frac{1}{P_t^\star} Y_{t+k}(P_t^\star) - \Psi'(\cdot)(-\epsilon)\frac{1}{P_t^\star} Y_{t+k}(P_t^\star) \Big\} = 0.$$

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left\{ 1 - \epsilon + \epsilon \frac{1}{P_t^\star} \psi_{t+k} \right\} = 0.$$

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left\{ -P_t^\star(\epsilon - 1) + \epsilon \psi_{t+k} \right\} = 0.$$

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left\{ P_t^\star - \frac{\epsilon}{(\epsilon - 1)} \psi_{t+k} \right\} = 0.$$

STATIONARIZE

Divide by P_t to obtain

$$\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left[\frac{P_t^\star}{P_t} - \mathcal{M} \underline{MC}_{t+k|t} \frac{P_{t+k}}{P_t} \right] = 0,$$

where

$$\underline{MC}_{t+k|t} = \frac{\psi_{t+k}\left[Y_{t+k}\left(P_t^\star\right)\right]}{P_{t+k}} \text{ and } \mathcal{M} \equiv \frac{\epsilon}{(\epsilon - 1)}$$

and

$$\frac{P_{t+k}}{P_t} = \frac{P_{t+k}}{P_{t+k-1}} \frac{P_{t+k-1}}{P_{t+k-2}} \cdots \frac{P_{t+1}}{P_t} = \prod_{i=1}^{k} \Pi_{t+i},$$

where $\Pi_t \equiv P_t / P_{t-1}$. Denote $\underline{P}_t^\star = P_t^\star / P_t$. Then

$$\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left[\underline{P}_t^\star - \mathcal{M} \underline{MC}_{t+k|t} \prod_{i=1}^{k} \Pi_{t+i} \right] = 0. \quad (5.7)$$

Zero inflation and zero growth steady-state

$\Pi = 1$,

$$Q_k = \beta^k \left(\frac{C}{C}\right)^{-\sigma} \frac{P}{P} = \beta^k$$

$$Y(P^\star) = \underbrace{\left(\frac{P^\star}{P}\right)^{-\epsilon}}_{=1} Y = Y$$

Then

$$\sum_{k=0}^{\infty} (\theta\beta)^k Y(\underline{P}^\star - \mathcal{M}\underline{MC}) = 0$$

ie $1 = \mathcal{M}\underline{MC}$. Hence

$$\underline{MC} = \frac{1}{\mathcal{M}}.$$

Loglinearize I

Plug the definition of Q_{t+k} into (5.7).

$$E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) Y_{t+k}(P_t^\star) \left[\underline{P_t^\star} - \mathcal{M}\underline{MC}_{t+k|t} \prod_{i=1}^{k} \Pi_{t+i}\right] = 0$$

$$E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\prod_{i=1}^{k} \Pi_{t+i}\right)^{-1} Y_{t+k}(P_t^\star)\underline{P_t^\star}$$

$$= E_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\prod_{i=1}^{k} \Pi_{t+i}\right)^{-1} Y_{t+k}(P_t^\star)\mathcal{M}\underline{MC}_{t+k|t} \prod_{i=1}^{k} \Pi_{t+i}$$

LOGLINEARIZE II

$$
\begin{aligned}
&\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C}{C}\right)^{-\sigma} \underbrace{(\prod_{i=1}^{k} \Pi)^{-1}}_{=1} Y \underbrace{\underline{P}^\star}_{=1} \Big[1 - \sigma \hat{c}_{t+k} + \sigma \hat{c}_t \\
&\qquad - \sum_{i=1}^{k} \hat{\pi}_{t+i} + \hat{y}_{t+k}(P_t^\star) + \underline{\hat{p}}_t^\star \Big] \\
&= \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \left(\frac{C}{C}\right)^{-\sigma} \underbrace{(\prod_{i=1}^{k} \Pi)^{-1}}_{=1} Y \underbrace{\mathcal{M}\underline{MC}}_{=1} \underbrace{(\prod_{i=1}^{k} \Pi)}_{=1} \Big[1 - \sigma \hat{c}_{t+k} + \sigma \hat{c}_t \\
&\qquad - \sum_{i=1}^{k} \hat{\pi}_{t+i} + \hat{y}_{t+k}(P_t^\star) + \underline{\hat{mc}}_{t+k|t} + \sum_{i=1}^{k} \hat{\pi}_{t+i} \Big]
\end{aligned}
$$

LOGLINEARIZE III

$$\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \underline{\hat{p}}_t^\star = \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (\underline{\hat{mc}}_{t+k|t} + \sum_{i=1}^{k} \hat{\pi}_{t+i})$$

$$\frac{1}{1-\theta\beta} \underline{\hat{p}}_t^\star = \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (\underline{\hat{mc}}_{t+k|t} + \sum_{i=1}^{k} \hat{\pi}_{t+i}).$$

$$\begin{aligned} \underline{\hat{p}}_t^\star &= (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \underline{\hat{mc}}_{t+k|t} \\ &\quad + \underbrace{(1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \sum_{i=1}^{k} \hat{\pi}_{t+i}}_{=\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{\pi}_{t+k} - \hat{\pi}_t} \\ &= (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \underline{\hat{mc}}_{t+k|t} + \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{\pi}_{t+k} - \hat{\pi}_t \qquad (5.8) \end{aligned}$$

LOGLINEARIZE IV

Shift $t \longrightarrow t+1$ and condition on the period t information

$$\mathrm{E}_t\, \underline{\hat{p}}^{\star}_{t+1} = (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \underline{\hat{mc}}_{t+1+k|t+1} + \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{\pi}_{t+1+k} - \mathrm{E}_t\, \hat{\pi}_{t+1}$$

LOGLINEARIZE V

From the previous slides

$$\underline{\hat{p}}_t^\star = (1-\theta\beta)\underline{\hat{mc}}_{t|t} + (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^{k+1}\beta^{k+1}\underline{\hat{mc}}_{t+1+k|t+1}$$

$$\hat{\pi}_t + \mathrm{E}_t \sum_{k=0}^{\infty} \theta^{k+1}\beta^{k+1}\hat{\pi}_{t+1+k} - \hat{\pi}_t$$

$$= (1-\theta\beta)\underline{\hat{mc}}_{t|t} + \theta\beta\Big[(1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^{k}\beta^{k}\underline{\hat{mc}}_{t+1+k|t+1}$$

$$+ \mathrm{E}_t \sum_{k=0}^{\infty} \theta^{k}\beta^{k}\hat{\pi}_{t+1+k} - \mathrm{E}_t\,\hat{\pi}_{t+1}\Big] + \theta\beta\,\mathrm{E}_t\,\hat{\pi}_{t+1}.$$

Note, that the term in brackets is $\mathrm{E}_t\,\underline{\hat{p}}_{t+1}^\star$. Hence,

$$\underline{\hat{p}}_t^\star = \theta\beta\,\mathrm{E}_t\,\underline{\hat{p}}_{t+1}^\star + \theta\beta\,\mathrm{E}_t\,\hat{\pi}_{t+1} + (1+\theta\beta)\underline{\hat{mc}}_{t|t} \tag{5.9}$$

TO SUMMARIZE

The first-order condition

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} Y_{t+k}(P_t^\star) \left(P_t^\star - \underbrace{\frac{\epsilon}{\epsilon - 1}}_{\equiv \mathcal{M}} \psi_{t+k|t} \right) = 0,$$

where

$$\psi_{t+k|t} = \Psi'_{t+k}\left(Y_{t+k}(P_t^\star)\right)$$

is nominal marginal costs and $\mathcal{M}$ is the desired or frictionless markup. In the case of no frictions $(1 - \theta = 1)$:

$$P_t^\star = \mathcal{M}\psi_{t,t}.$$

Connection to Galí's book: write (5.8) as follows

$$\underline{\hat{p}}_t^\star = (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \underline{\hat{m}c}_{t+k|t} + \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{\pi}_{t+k} - \hat{\pi}_t \quad \Longleftrightarrow$$

$$\begin{aligned} p_t^\star - p_t &= (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (\hat{m}c_{t+k|t} - p_{t+1}) \\ &+ \mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} - p_{t+k-1} - p_t + p_{t-1} \\ &= (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (\hat{m}c_{t+k|t} - p_{t+1}) \\ &+ (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k (p_{t+k} - p_t) \end{aligned}$$

$$= (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{m}c_{t+k|t} - p_t \quad \Longleftrightarrow$$

$$p_t^{\star} = (1-\theta\beta)\,\mathrm{E}_t \sum_{k=0}^{\infty} \theta^k \beta^k \hat{m}c_{t+k|t}.$$

Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{5.10}$$

for $k = 0, 1, 2, ...$where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k} / U_{c,t}$
Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0$$

where $\Psi_{t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$.

Flexible price case ($\theta = 0$):

$$P_t^* = \mathcal{M}\Psi_{t|t}$$

Zero inflation steady state

$\Lambda_{t,t+k} = \beta^k$; $P_t^*/P_{t-1} = P_t/P_{t+k} = 1 \Rightarrow Y_{t+k|t} = Y$; $\Psi_{t+k|t} = \Psi_t$; $P_t =$

Linearized optimal price setting condition:

$$p_t^* = \mu + (1 - \beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu \equiv \log \mathcal{M}$

AGGREGATE PRICE LEVEL

Note that all those firms that may set their price level (ie where Calvo Fairy visits) will choose identical price $P_t^\star$. Let $S(t) \subset [0,1]$ represent the set of firms that do not reoptimize their price in period t. Note that "the distribution of prices among firms not adjusting in period t corresponds to the distribution effective prices in period $t-1$, though with total mass reduced to θ". Remember the aggregate price index P_t in eq (5.2). The aggregate price level has to follow this functional form

$$P_t = \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1-\theta)(P_t^\star)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}$$

$$= \left[\theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^\star)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \quad (5.11)$$

LOG-LINEARIZATION OF THE AGGREGATE PRICE LEVEL

Stationarize equation (5.11), ie express it in inflation rates and in relative prices: Divide (5.11) by P_{t-1} and rise it power $1-\epsilon$:

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^\star}{P_t}\Pi_t\right)^{1-\epsilon},$$

where $\Pi_t = P_t/P_{t-1}$.
Zero inflation steady-state implies that $\underline{P}_t^\star \equiv P_t^\star/P_t = 1$.
Log-linearizing it around zero steady state inflation results:

$$\pi_t = (1-\theta)(p_t^\star - p_{t-1}).$$

(As in Gali's book). Or (remember that $\pi = 0$, hence $\hat{\pi}_t = \pi_t$)

$$\hat{\pi}_t = (1-\theta)(p_t^\star - p_t + p_t - p_{t-1}) = (1-\theta)(\hat{\underline{p}}_t^\star + \hat{\pi}_t)$$

or

$$\hat{\underline{p}}_t^\star = \frac{\theta}{1-\theta}\pi_t \qquad (5.12)$$

PHILLIPS CURVE

Plug (5.12) into (5.9) to obtain

$$\begin{aligned}\frac{\theta}{1-\theta}\hat{\pi}_t &= \theta\beta\frac{\theta}{1-\theta}\,\mathrm{E}_t\,\hat{\pi}_{t+1} + \theta\beta\,\mathrm{E}_t\,\hat{\pi}_{t+1} + (1-\beta\theta)\underline{\hat{mc}}_t \\ &= \theta\beta\frac{\theta+1-\theta}{1-\theta}\,\mathrm{E}_t\,\hat{\pi}_{t+1} + (1-\beta\theta)\underline{\hat{mc}}_t\end{aligned}$$

$$\frac{\theta}{1-\theta}\hat{\pi}_t = \frac{\theta\beta}{1-\theta}\,\mathrm{E}_t\,\hat{\pi}_{t+1} + (1-\beta\theta)\underline{\hat{mc}}_t$$

And finally

$$\hat{\pi}_t = \beta\,\mathrm{E}_t\,\hat{\pi}_{t+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\underline{\hat{mc}}_t$$

RETURNS-TO-SCALE AND MARGINAL COSTS I

Under *constant returns to scale*[6] production function, the marginal costs does not depend on the scale of production.

$$mc_t = (w_t - p_t) - \text{marginal product of labour},$$

where the marginal product of labour is given by

$$\log(1-\alpha) + a_t - \alpha n_t \underset{\text{production function}}{=} \log(1-\alpha) + \frac{1}{1-\alpha}(a_t - \alpha y_t)$$

$\alpha = 0 \quad \longrightarrow$ constant returns to scale. $0 < \alpha < 1$ corresponds decreasing returns to scale.
Particular Case: $\alpha = 0$ (constant returns)

$$\Longrightarrow \psi_{t+k|t} = \psi_{t+k}$$

Returns-to-scale and marginal costs II

Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\widehat{\mu}_t$$

where $\mu_t \equiv p_t - \psi_t$ and $\widehat{\mu}_t \equiv \mu_t - \mu$

Combined with price dynamics equation yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

General case: $\alpha \in [0,1)$

Returns-to-scale and marginal costs III

$$\begin{aligned}\psi_{t+k|t} &= w_{t+k} - mpn_{t+k|t} \\ &= w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1-\alpha))\end{aligned}$$

$$\psi_{t+k} \equiv w_{t+k} - (a_{t+k} - \alpha n_{t+k} + \log(1-\alpha))$$

$$\begin{aligned}\psi_{t+k|t} &= \psi_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ &= \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k})\end{aligned}$$

Optimal price setting equation:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{p_{t+k} - \Theta\widehat{\mu}_{t+k}\right\}$$

Returns-to-scale and marginal costs IV

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \in (0,1]$.
Recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1-\beta\theta)p_t - (1-\beta\theta)\Theta\widehat{\mu}_t$$

Combined with price dynamics equation yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\widehat{\mu}_t$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$$

[6] The production function is a linear homogenous (the first degree) function of inputs.

OUTLINE

EQUILIBRIUM I

Goods market

$$Y_t(i) = C_t(i) \quad \forall i \in [0,1] \text{ and } \forall t.$$

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$:

$$Y_t = C_t. \quad \forall t$$

Combined with Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

EQUILIBRIUM II

Labour markets

$$N_t = \int_0^1 N_t(i)di$$
$$= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di$$
$$= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \underbrace{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} di}_{\equiv \text{price dispersion}}.$$

Up to a first order approximation:

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

EQUILIBRIUM III

Average price markup and output

$$
\begin{aligned}
\mu_t &\equiv p_t - \psi_t \\
&= -(w_t - p_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1-\alpha)) \\
&= -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)
\end{aligned}
$$

Under flexible prices:

$$\mu = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)$$

implying

$$y_t^n = \psi_{ya} a_t + \psi_y$$

Equilibrium IV

where $\psi_y \equiv -\frac{(1-\alpha)(\mu-\log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. Thus,

$$\widehat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n)$$

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ and $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$.

The Non-Policy Block of the Basic New Keynesian Model

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t \tag{5.13}$$

EQUILIBRIUM V

Dynamic IS equation

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) \tag{5.14}$$

where r_t^n is the *natural rate of interest*, given by

$$r_t^n = \rho - \sigma(1 - \rho_a)\psi_{ya} a_t \tag{5.15}$$

Missing block: description of monetary policy (determination of i_t).

OUTLINE

Monetary policy

Determination of nominal interest rate, i_t, gives the path for actual real rate. It is a description how monetary policy is conducted.

Monetary policy is non-neutral

When prices are sticky nominal interest rate path determines output gap.

DYNAMICS

To study the dynamics of a linear rational expectation model, we write is as follows

$$A_0 x_t = A_1 \mathrm{E}_t\, x_{t+1} + B z_t.$$

and study the properties of the transition matrix $A_T \equiv A_0^{-1} A_1$ of the following form

$$x_t = \underbrace{A_0^{-1} A_1}_{\equiv A_T} \mathrm{E}_t\, x_{t+1} + \underbrace{A_0^{-1} B}_{\equiv B_T} z_t.$$

A_0 should be of full rank (some solution algorithms allow reduced rank cases too).

SIMPLE INTEREST RATE RULE I

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$ and

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n + v_t$$

where $\widehat{y}_t^n \equiv y_t^n - y$.

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T\, u_t$$

Simple interest rate rule II

where

$$\begin{aligned} u_t &\equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t \\ &= -\psi_{ya}(\phi_y + \sigma(1-\rho_a))a_t - v_t \end{aligned}$$

and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

Uniqueness condition (Bullard and Mitra):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Exercise: analytical solution (method of undetermined coefficients).

Exogenous money growth rule I

Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Money demand

$$l_t \equiv m_t - p_t = \widetilde{y}_t - \eta i_t + y_t^n$$

Substituting into dynamic IS equation

$$(1 + \sigma\eta)\,\widetilde{y}_t = \sigma\eta E_t\{\widetilde{y}_{t+1}\} + \widehat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta\widehat{r}_t^n - \widehat{y}_t^n$$

Identity:

$$\widehat{l}_{t-1} = \widehat{l}_t + \pi_t - \Delta m_t$$

EXOGENOUS MONEY GROWTH RULE II

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta m_t \end{bmatrix} \qquad (5.16)$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv$$

Uniqueness condition:

$\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}} \mathbf{A_{M,1}}$ has two eigenvalues inside and one outside the unit circle.

Calibration

EXOGENOUS MONEY GROWTH RULE III

Households: $\sigma = 1$; $\varphi = 5$; $\beta = 0.99$; $\epsilon = 9$; $\eta = 4$; $\rho_z = 0.5$
Firms: $\alpha = 1/4$; $\theta = 3/4$; $\rho_a = 0.9$
Policy rules: $\phi_\pi = 1.5$, $\phi_y = 0.125$; $\rho_v = \rho_m = 0.5$
Dynamic Responses to Exogenous Shocks

- Monetary policy, discount rate, technology
- Interest rate rule vs. money growth rule

ANALYTICAL SOLUTION: UNDETERMINED COEFFICIENTS I

Let's use a bit simpler version of the above interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \tag{5.17}$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad \varepsilon_t^v \sim \text{IID}(0, \sigma_v^2)$$

The system is characterized by the equations (5.17), (5.14) and (5.13).

1. Guess that the solution takes the form $\tilde{y}_t = \psi_{yv} v_t$ and $\pi_t = \psi_{\pi v} v_t$ (why?)
2. Substitute (5.17) into (5.14),

ANALYTICAL SOLUTION: UNDETERMINED COEFFICIENTS II

3. Let's assume that $\hat{r}_t^n \equiv r_t^n - \rho = 0$ (because it is not affected by monetary policy shocks,
4. Impose these to (5.13) and (5.14) (note that $\mathrm{E}_t\, v_{t+1} = \rho_v v_t$) and
5. solve the unknown ψ_{yv} and $\psi_{\pi v}$ to obtain

$$\tilde{y}_t = -(1-\beta\rho_v)\Lambda_v v_t$$
$$\pi_t = -\kappa\Lambda_v v_t,$$

where $\Lambda_v = \{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y] + \kappa(\phi_\pi - \rho_v)\}^{-1}$

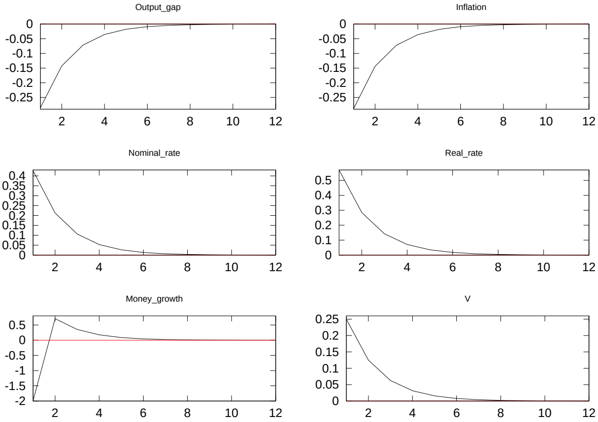
Analytical solution: undetermined coefficients...

6. Note, however, that $v_t = \rho_v v_{t-1} + \varepsilon_t^v = \frac{1}{1-\rho_v L}\varepsilon_t^v$, where L is lag-operator (or backshift operator), ie $Lx_t = x_{t-1}$.
7. Then, the solution ('VAR' representation) is

$$\tilde{y}_t = \rho_v \tilde{y}_{t-1} - (1-\beta\rho_v)\Lambda_v \varepsilon_t^v$$
$$\pi_t = \rho_v \pi_{t-1} - \kappa \Lambda_v \varepsilon_t^v.$$

Note, that the persistence (lagged endogenous variable) is *inherited* from the shock process v_t.

RESPONSES TO A MONETARY POLICY SHOCK (INTEREST RATE RULE)



Effects of a technology shock

Set $v_t =$ (no monetary policy shocks).
Technology process:

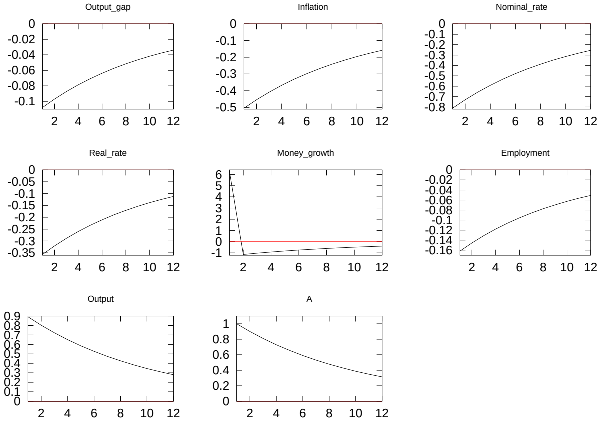
$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

Implied natural rate:

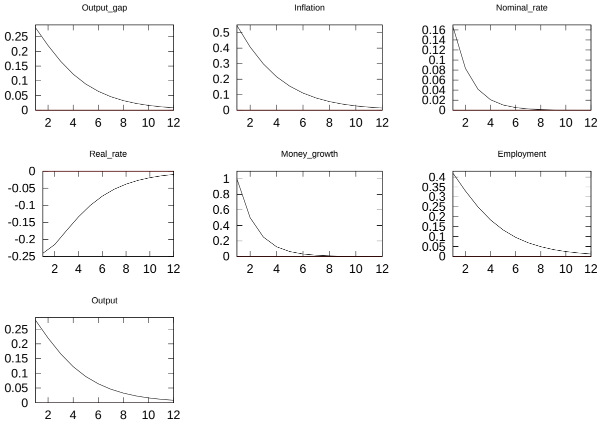
$$\hat{r}_t^n = \sigma \varphi_{ya}(1 - \rho_a) a_t.$$

Dynamic effects of a technology shock ($\rho_a = 0.9$)

Responses to a technology shock (interest rate rule)

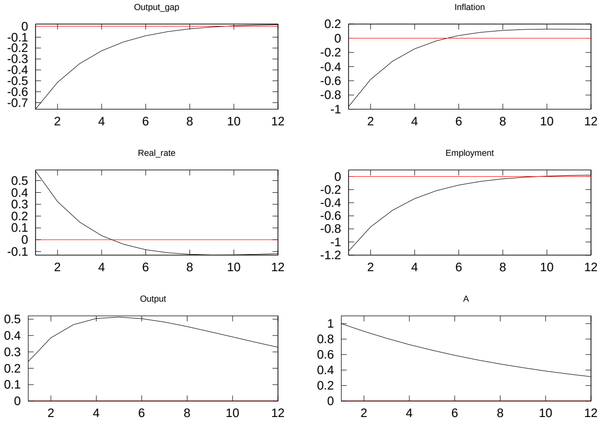


Responses to a monetary policy shock (money growth rate rule)



Note that liquidity effect is not present (due to the calibration)!

Responses to a technology shock (money growth rule)



ESTIMATED RESPONSES TO A TECHNOLOGY SHOCK

Figure 3.5: Estimated Effects of a Permanent Technology Shock

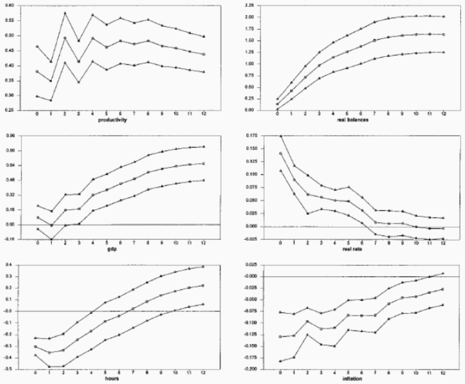


FIGURE 4. ESTIMATED IMPULSE RESPONSES FROM A FIVE-VARIABLE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

Source: Galí (1999)

Outline

Alternarive approaches I

Adjustment costs

Quadratic adjustment costs by Rotemberg (1982). Fixed costs, Gertler and Leahy (2006).

Staggered price setting

Deterministic time duration: price adjustments allowed (in a non-syncronized manner) only after each n^{th} period. Taylor (1980), Chari, Kehoe, McGrattan (2000).

Time-dependent and state-dependent pricing:

ALTERNARIVE APPROACHES II

Time-dependent pricing

Calvo structure assumes the price changes to be time independent (and also state-dependent). The hazard is flat; i.e. the propability of changing prices at time t does not depend on when the last price-change occurred. *This contradicts with micro evidence, where hazard is upward-sloping.* The Calvo models also predict bigger absolute price changes for older prices, when no such pattern exists in the data.

Alternarive approaches III

State-dependent pricing (SDP)

In SDP models, a positive monetary shock boosts the fraction of firms changing prices and/or the average size of those price changes. In Dotsey, King, and Wolman (1999),it is predominantly the fraction that responds, whereas in Golosov and Lucas (2007) it is almost wholly the average size of changes (more increases and fewer decreases). The DKW model produces no large price changes, and predicts far too big a role for the extensive margin in inflation movements. The GL model does not generate enough small price changes.

In reasonable stable macro environment, the state and time dependent results will be similar.

ALTERNARIVE APPROACHES IV

2nd-generation time-dependent pricing

Mash (2004) combines Calvo with overlapping-contracts of Taylor. Sheedy (2010) modifies Calvo model such the probability of the visit of a Calvo fairy increases with distance from the last visit. This generates inflation inertia.

Sticky information

Mankiw and Reis (2003) present a Phillips curve with a substantial amount of price setting based on Sticky information. Erceg and Levin (2000) builds on adaptive learning by agents in response to a change in policy regime.

Alternarive approaches V

2nd-generation state-dependent pricing

Gertler and Leahy (2006) propose a model with Poisson arrival of idiosyncratic shocks and small menu costs, so that many price changes are small and the size of price changes is unrelated to the time since the last change. Dotsey, King, and Wolman (2006) add idiosyncratic shocks to their earlier model, thereby generating large price changes andmitigating the importance of the extensive margin. Woodford (2009) presents a generalization of state-dependent pricing in which decisions about when to review a firm's existing price must be made on the basis of imprecise awareness of current market conditions.

Outline

The Efficient Allocation

$$\max U(C_t, N_t)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t\, N_t(i)^{1-\alpha},\ \ all\ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i)\, di$$

Optimality conditions:

$$C_t(i) = C_t,\ \ all\ i \in [0,1]$$

$$N_t(i) = N_t,\ \ all\ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha)\ A_t N_t^{-\alpha}$.

The Efficient Allocation...

- Produce and consume same quantity of all goods.
- Allocate same amount of labour to all firms.
- Due to
  - all goods enter symmetrically to utility function,
  - concavity of $U(\cdot)$,
  - identical technologies.
- Marginal rate of substitution equals marginal rate of transformation equals marginal product of labour: corresponds classical model!

NEXT: How this efficient allocation is violated!

Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities:

- *Monopolistic competition*: $P_t = \mathcal{M} \frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Solution: employment subsidy τ. Under flexible prices, $P_t = \mathcal{M} \frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

- *Transactions friction* (economy with valued money): assumed to be negligible

2. Distortions associated with the presence of nominal rigidities:

- *Markup variations* resulting from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$ (assuming optimal subsidy)

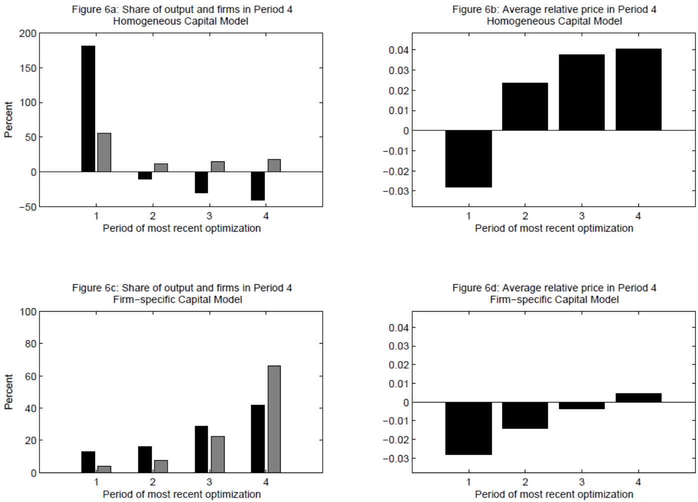
$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Optimality requires that the average markup be stabilized at its frictionless level.

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods. Accordingly, markups should be identical across firms/goods at all times.

PRICE AND OUTPUT DISPERSION

Figure 6: Features of the Distribution of Output and Prices Across Firms



Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$

$\Longrightarrow$ the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, \ldots$As a result the aggregate price level is fully stabilized and no relative price distortions emerge.
- equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

Equilibrium under the Optimal Policy

$$\widetilde{y}_t = 0$$
$$\pi_t = 0$$
$$i_t = r_t^n$$

for all t.

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$
$$\pi_t = \beta\, E_t\{\pi_{t+1}\} + \kappa\, \widetilde{y}_t$$

Two features of optimal policy

Stabilization of output is not desirable

It should vary one for one with the natural level of output.

Aggregate price stability is not a policy *target*

It arises making all firms content with their existing prices.

An Exogenous Interest Rate Rule

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one.

$\rightarrow$ indeterminacy. (real and nominal).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_\pi \, \pi_t + \phi_y \, \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

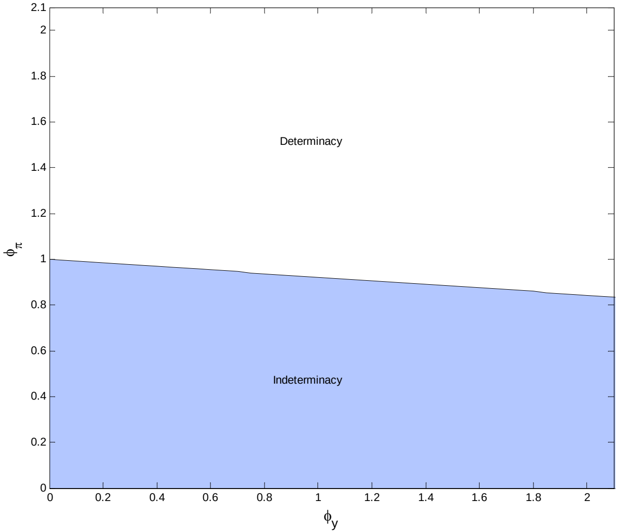
Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa\ (\phi_\pi - 1) + (1-\beta)\ \phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned} di &= \phi_\pi\ d\pi + \phi_y\ d\widetilde{y} \\ &= \left(\phi_\pi + \frac{\phi_y\ (1-\beta)}{\kappa}\right)\ d\pi \end{aligned}$$

Figure 4.1


Determinacy
Indeterminacy
φπ
φy
2.1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

A Forward-Looking Interest Rate Rule

$$i_t = r_t^n + \phi_\pi \; E_t\{\pi_{t+1}\} + \phi_y \; E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

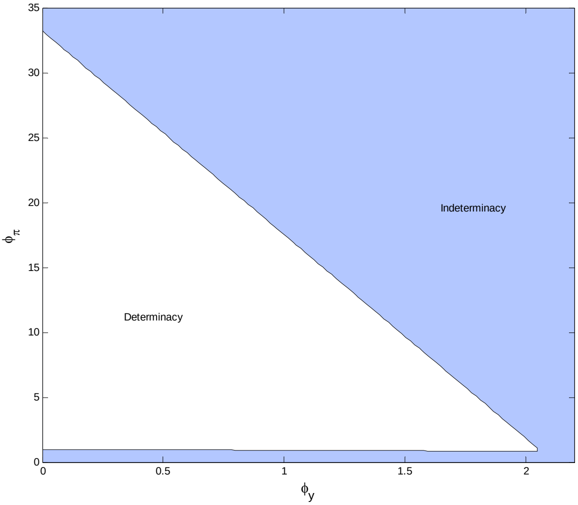
where

$$\mathbf{A}_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}\phi_\pi \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}\phi_\pi \end{bmatrix}$$

Existence and Uniqueness conditions: (Bullard and Mitra (2002):

$$\begin{aligned} \kappa \; (\phi_\pi - 1) + (1 - \beta) \; \phi_y &> 0 \\ \kappa \; (\phi_\pi - 1) + (1 + \beta) \; \phi_y &< 2\sigma(1 + \beta) \\ \phi_y &< \sigma(1 + \beta^{-1}) \end{aligned}$$

Figure 4.2


35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
ϕ_π
ϕ_y
Indeterminacy
Determinacy

Shortcomings of Optimal Rules

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

Alternative: "simple rules" , i.e. rules that meet the following criteria:

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

Simple Monetary Policy Rules

Welfare-based evaluation:

$$\mathbb{W} \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left(\kappa \, \widetilde{y}_t^2 + \epsilon \, \pi_t^2 \right)$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2\lambda} \left[\kappa \; var(\widetilde{y}_t) + \epsilon \; var(\pi_t) \right]$$

See Appendix for Derivation.

A Taylor Rule

$$i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \; \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \; (\widehat{r}_t^n - \phi_y \; \widehat{y}_t^n)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$. Note that $\widehat{r}_t^n - \phi_y \; \widehat{y}_t^n = -\psi_{ya}^n \; [\sigma(1 - \rho_a) + \phi_y] \; a_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \Delta y_t$

Money Growth Peg

$$\Delta m_t = 0$$

money market clearing condition

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta\,\widehat{i}_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ and ξ_t is a money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta\,\Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

Define $l_t^+ \equiv l_t - \zeta_t$. $\Longrightarrow$

$$\widehat{i}_t = \frac{1}{\eta}\left(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+\right)$$

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1}^{+} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t^{+} \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simulations and Evaluation of Simple Rules

Table 4.1: Evaluation of Simple Monetary Policy Rules

	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

Outline

Outline

Inflation dynamics

Consider the consumption Euler equation (4.3)

$$\frac{1}{1+i_t} = \beta \, \mathrm{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\}$$

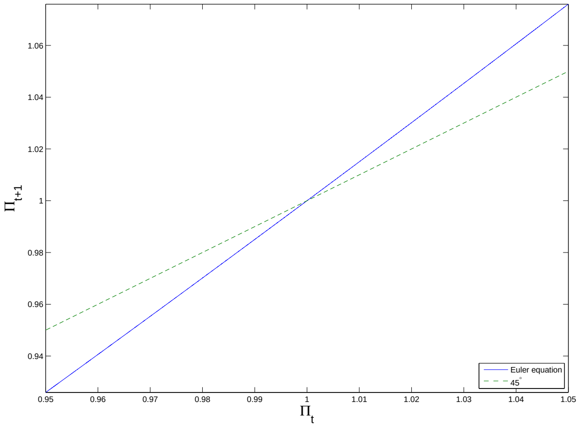
and the simple Taylor -type rule (without output gap) in the non-linearized form

$$1 + i_t = \beta^{-1} \Pi_t^{\phi_\pi}.$$

Consider the deterministic case (drop expectation operator) and substitute interest rate out from consumption Euler equation using the Taylor rule to obtain

$$\Pi_t^{\phi_\pi} = \left(\frac{C_{t+1}}{C_t} \right)^{\sigma} \Pi_{t+1}$$

PHASE DIAGRAM



$\sigma = 2$, $C_{t+1}/C_t = 1$, $\phi_\pi = 1.5$. The steady state is $\Pi = 1$

INFLATION DYNAMICS AND THE ZERO LOWER BOUND (NLB)

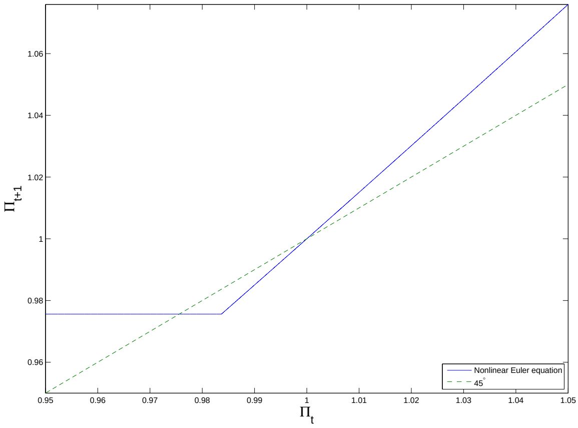
Consider the case where the nominal interest rate is bounded by zero. This is due to existence of paper money (that we do not model here).
The Taylor rule can be modified according to

$$1 + i_t = \max\left(1, \beta^{-1}\Pi_t^{\phi_\pi}\right).$$

and the resulting Euler equation

$$\max\left(1, \beta^{-1}\Pi_t^{\phi_\pi}\right) = \frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right)^{\sigma}\Pi_{t+1}$$

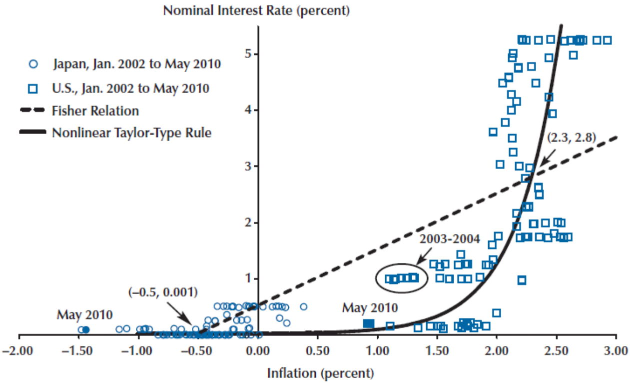
Phase diagram with two steady states



In the lower steady state $i = 0$: $\Pi = \beta \left(C_{t+1}/C_t\right)^{-\sigma} = \beta$, resulting $\pi = -\rho$.

INTEREST RATES AND INFLATION IN JAPAN AND US

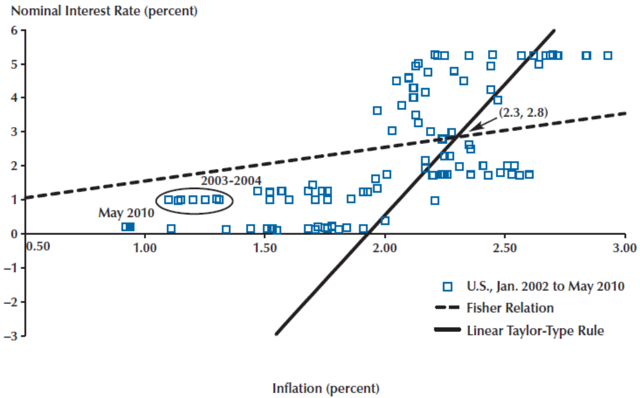
Interest Rates and Inflation in Japan and the U.S.



NOTE: Short-term nominal interest rates and core inflation rates in Japan and the United States, 2002-10.
SOURCE: Data from the Organisation for Economic Co-operation and Development.

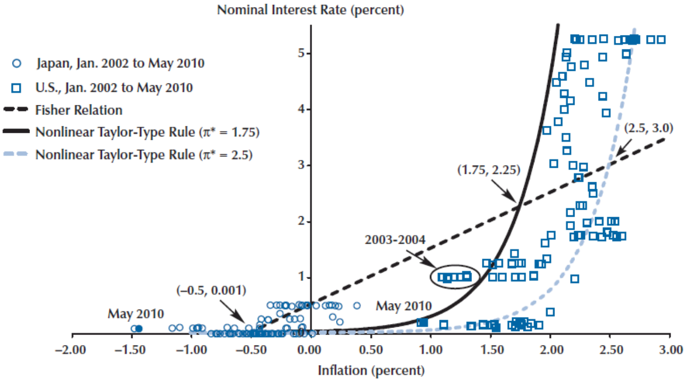
Source: Jim Bullard (2010)

Denial


Nominal Interest Rate (percent)
6
5
4
3
2
1
0
−1
−2
−3
0.50
1.00
1.50
2.00
2.50
3.00
(2.3, 2.8)
2003-2004
May 2010
U.S., Jan. 2002 to May 2010
Fisher Relation
Linear Taylor-Type Rule
Inflation (percent)

INCREASING INFLATION TARGET

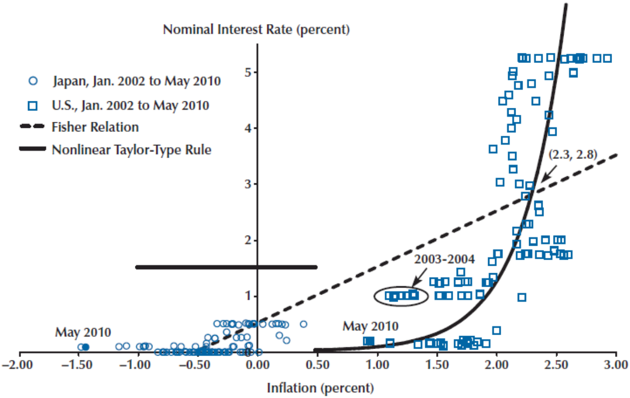
Inflation Expectations, Interrupted



NOTE: The 2003-04 episode. Thornton (2006, 2007) argues that FOMC communications increased the perceived inflation target of the Committee.

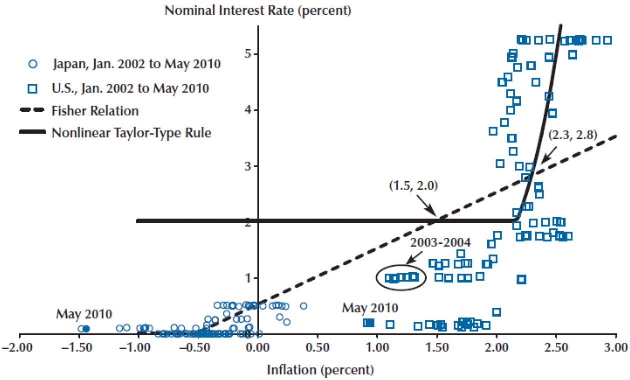
DISCONTINUITY

Discontinuity



NOTE: The discontinuous Taylor-type policy rule looks unusual but eliminates the unintended steady state.

HISTORIC POLICY



Other proposals

Government insolvency

- Government can threaten to behave in an insolvent manner in the case of lower steady-state.
- "Unsophisticated implementation".
- Japanese aggressive fiscal policy (debt-to-GDP ratio over 200 %)

Quantitative easing

- Long-term rates falling: US, UK, euro area
- Permanent or temporary asset purchases
- General problem: what is inflation expectations will anchor to the lower steady-state.

Forward guidance

- Commitment issues?
- Works in theory, not in practise?

OUTLINE

Introduction

Debt is mentioned as a central problem behind the Great Recession.
It has long tradition

- Fisherian debt-deflation: $P \downarrow \rightarrow$ real value of debt $\uparrow \rightarrow P \downarrow$
- Hyman Minsky.
- Richard Koo (2008): Japan's lost decade due to balance sheet distress.
- Hall (2011): tightening household borrowing constraint essential to understant the crisis
- Open-economy models by Krugman (1999) and Aghion, Bacchetta and Banerjee (2001): Debt in foreign currencies + massive devaluations lead to Fisherian debt-deflation.

In our benchmark model, level of debt plays no role!

Eggertsson and Krugman (2012)

- Level of debt matters only if the distribution of debt matters.
- High indebted households have different constraints to from households with low debt.
  $\longrightarrow$All debt is not created equal!
- Paper divides economy to patient and impatient agents. Latter ones are subject to an exogenous debt limit.
- If limit is reduced, impatient have to cut their spending. With nominal debt and high reduction in limit, this leads to Fisherian debt-deflation.
- Impatient consume all their current income
  $\longrightarrow$Fiscal policy may have a large effect.

LEVEL OF HOUSEHOLD GROSS DEBT

HOUSEHOLD GROSS DEBT AS PERCENT OF PERSONAL INCOME

	2000	2008
United States	96	128
United Kingdom	105	160
Spain	69	130

Source: McKinsey Global Institute (2010).

Preliminaries

Endowment economy, no aggregate savings nor investments.
All receive the same.
Two types of households

- Savers, s $\beta_s = \beta$

Borrowers, b $\beta_b < \beta$, rate of time preference $\rho(b)$
($\beta_b \equiv 1/(1+\rho(b))$) is higher
$\longrightarrow$appreciate future less than savers.

Each get endowment $\frac{1}{2}Y$ in each period.
Positive D means debt and negative D means positive asset holdings. (Opposite to B in previous section.)

Utility and budget constraints

Savers

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t^s)$$

budget constraint

$$D_t^s = (1 + r_{t-1})D_{t-1}^s - \frac{1}{2}Y + C_t^s$$

Borrowing limit

$$(1 + r_t)D_t^s \leq D^{\text{high}}$$

Borrowers

$$E_0 \sum_{t=0}^{\infty} \beta_b^t \log(C_t^b)$$

budget constraint

$$D_t^b = (1 + r_{t-1})D_{t-1}^b - \frac{1}{2}Y + C_t^b$$

Borrowing limit

$$(1 + r_t)D_t^b \leq D^{\text{high}}$$

Steady-state I

Assume that the borrowing limit is less than the *discounted value of output of each agent*

$$D^{\text{high}} \leq \frac{\beta}{1-\beta}\frac{1}{2}Y.$$

and that the borrower need to respect the following borrowing limit

$$(1+r_t)D_t^b \leq D^{\text{high}},$$

where we include the interest rate payment to borrowing limit. (This will simplify the algebra!)

STEADY-STATE II

Since borrower is more impatient than saver, the steady-state is the one in which the impatient agent (borrower) will borrow up her borrowing limit

$$D^b = \frac{1}{1+r} D^{\text{high}}.$$

Substitute this into the steady-state budget constraint of the borrower

$$C^b = \frac{1}{2} Y - r D^b$$

to obtain steady-state consumption

$$C^b = \frac{1}{2} Y - \frac{r}{1+r} D^{\text{high}}.$$

Steady-state III

All production (still exogenous) is consumed:

$$Y = C^s + C^b.$$

substituting C^b and solving for C^s gives

$$C^s = \frac{1}{2}Y + \frac{r}{1+r}D^{\text{high}}.$$

The first-order condition for saver is

$$\frac{1}{C_t^s} = (1+r_t)\beta \, \mathrm{E}_t \, \frac{1}{C_{t+1}^s}$$

and in the steady state

$$r = \frac{1-\beta}{\beta}. \tag{7.1}$$

STEADY-STATE IV

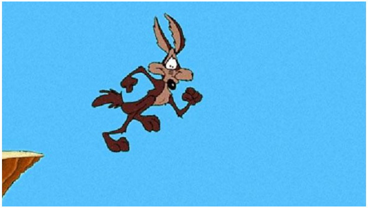
Remarks

- Borrower has negative wealth (=debt), and pays only the discounted interest on that, ie being at the borrowing limit all the time.
- Saver opposite
- Saver's discount rate β determines the real interest rate.

DELEVERAGING I

Unexpected fall in the debt constraint: change in the attitude.

D^{high} $> D^{\text{low}}$



Wile E. Coyote moment or Minsky moment

This will result a temporary fall in the real interest rate: large fall in the ceiling will result a *negative* real rate.
Assume, further, that this move $D^{\text{high}} \to D^{\text{low}}$ is rapid.

DELEVERAGING II

To simplify: S short-run, L long-run (steady-state).

Long-run consumption

Savers

$$C_L^s = \frac{1}{2}Y + \frac{r}{1+r}D^{\text{low}} = \frac{1}{2}Y + (1-\beta)D^{\text{low}}.$$

Borrowers

$$C_L^b = \frac{1}{2}Y - \frac{r}{1+r}D^{\text{low}} = \frac{1}{2}Y - (1-\beta)D^{\text{low}}.$$

The second inequality results from (7.1).

DELEVERAGING III

Let's assume that the borrower has to deleverage to new, lower debt limit within a single period such that

$$D_S = \frac{D^{\text{low}}}{1 + r_S}.$$

Substitute this to her budget constraint

$$\frac{D^{\text{low}}}{1 + r_S} = D^{\text{high}} - \frac{1}{2}Y + C_S^b$$

and solve the short-run consumption

$$C_S^b = \frac{1}{2}Y + \frac{D^{\text{low}}}{1 + r_S} - D^{\text{high}}.$$

Since $D^{\text{high}} \gg D^{\text{low}}$ the consumption will decline substantially.

Deleveraging IV

Since

$$Y = C_S^s + C_S^b$$

the short-run consumption of the saver is

$$C_S^s = \frac{1}{2}Y - \frac{D^{\text{low}}}{1+r_S} + D^{\text{high}}.$$

Her consumption Euler equation gives

$$C_L^s = (1+r_S)\beta C_S^s.$$

Substituting C_L^s and C_S^s above and solving for $1+r_S$ gives the level of real interest rate that balanances the bond market, ie makes savers to save less (=consume more):

$$1 + r_S = \frac{\frac{1}{2}Y + D^{\text{low}}}{\beta\frac{1}{2}Y + \beta D^{\text{high}}}.$$

DELEVERAGING V

To get negative real interest rate $r_S < 0$ $(1 + r_S < 1)$, we must have

$$\frac{\frac{1}{2}Y + D^{\text{low}}}{\beta\frac{1}{2}Y + \beta D^{\text{high}}} < 1$$

or

$$\beta D^{\text{high}} - D^{\text{low}} > \frac{1}{2}(1 - \beta)Y. \tag{7.2}$$

This condition is valid if $\beta D^{\text{high}} - D^{\text{low}}$ is high enough (in absolute terms not very high actually since $1 - \beta$ is very small). Negative real interest rate will make the saver to substitute the future consumption to current consumption, ie consume more today. Hence, the negative real interest rate would make *saver* to spend sufficiently more.

Price level I

Assume that there exist *nominal* government debt traded in zero supply, such that we get the nominal arbitrage equation (as previous section) for saver

$$\frac{1}{C_t^s} = \beta(1+i_t)\,\mathrm{E}_t \frac{1}{C_{t+1}^s}\frac{P_t}{P_{t+1}},$$

where P_t is the price level, and i_t is the nominal interest rate. (Cashless limit as before!)
Assumptions:

- Impose zero-lower-bound: $i_t \geq 0$.
- Monetary policy targets the price level $P^\star$.
- Fix long-run price level $P_L = P^\star$: monetary policy does its job in the long-run.

PRICE LEVEL II

Then the short-run Fisher equation

$$1 + r_S = (1 + i_S)\frac{P_S}{P^\star}.$$

The consumption (nominal) Euler equation results the same condition (7.2) for nominal interest rates as above. But this cannot happen due to ZLB.
If ZLB is binding, $i_t = 0$, the (from Fisher equation) $1 + r_S = P_S / P^\star$ and then — due to the condition (7.2) — we have deflation if

$$\frac{P_S}{P^\star} = \frac{\frac{1}{2}Y + D^{\text{low}}}{\beta\frac{1}{2}Y + \beta D^{\text{high}}} < 1$$

We need to have a drop in price level to have positive inflation in the future such that we achieve negative real interest rate.

FISHERIAN DEBT DEFLATION I

D^{high} is denominated in consumption goods (real debt).
Introduce nominal debt B_t but keep the debt ceiling in real terms $D^{\text{high}} = B^{\text{high}}/P_S$.
Then the short-run consumption of borrower

$$C_S^b = \frac{1}{2}Y + \frac{D^{\text{low}}}{1+r_S} - \frac{B^{\text{high}}}{P_S}.$$

If price level P_S drops, she must pay more debt down.
The natural rate is now endogenous

$$1 + r_S = \frac{\frac{1}{2}Y + D^{\text{low}}}{\beta\frac{1}{2}Y + \beta\frac{B^{\text{high}}}{P_S}}.$$

FISHERIAN DEBT DEFLATION II

When price level P_S drops, the real rate becomes even more negative and making price level (from Fisher equation)

$$\frac{P_S}{P^\star} = 1 + r_S$$

to drop even more.

ENDOGENOUS OUTPUT I

Assume Dixit-Stiglitz aggregate of continuum of goods as in the previous section.
The utility of each type i ($i = s$ or b) of household is given by

$$E_0 \sum_{t=0}^{\infty} \beta(i)^t \left[u^i \left(C_t^i \right) - v^i \left(h_t^i \right) \right],$$

where h_t is hours worked.
Households are continuum of measure 1 such that

- Fraction χ_s are savers
- Fraction $\chi_b = 1 - \chi_s$ are borrowers, and
- Borrowers and savers consume similar basket.

Endogenous output II

- Aggregate consumption is given by the weighted sum of the two groups

$$C_t = \chi_s C_t^s + \chi_b C_t^b.$$

Firms are also continuum of measure 1 such that

- each of which produce one type of the varieties the consumers like.
- Production function is linear in labour: $C_t(i) = h(i)$.
- Fraction $1 - \lambda$ keeps the prices fixed for one period.
- Fraction λ can change their prices all the time.
- Firms sell all the amount of goods that is demanded with the price they set, and
- hire the labour to satisfy this demand.

Endogenous output III

The appendix of Eggertsson and Krugman (2012) derives the complete model. The main differences to the model in the earlier sections (including this one) are the following

- utility is not parameterized. We have standard assumptions regarding first and second derivatives.
- instead of Calvo frictions we have the simplified friction. The logic is close to Calvo.
- note also the employment subsidy τ.
- it is assumed that borrowers borrow up to the limit, ie in the steady-state the borrowing is binding.

ENDOGENOUS OUTPUT IV

Since production is endogenous, agents choose how much they consume and how much they work. Hence for each type i ($i = s$ or b), the log-linearized first-order condition wrt to labour is given by

$$\widehat{W}_t = \omega \widehat{h}_t^i + \frac{1}{\sigma}\widehat{C}_t^i,$$

where

$$\sigma \equiv -\frac{\bar{u}_c^b}{\bar{u}_{cc}^b \widehat{Y}} \equiv -\frac{\bar{u}_c^s}{\bar{u}_{cc}^s \widehat{Y}} > 0$$

$$\omega \equiv \frac{\bar{v}_{hh}^b \bar{h}}{\bar{v}_h^b} \equiv \frac{\bar{v}_{hh}^s \bar{h}}{\bar{v}_h^s} > 0$$

$$\widehat{C}_t^i \equiv \frac{C_t^i - \bar{C}^i}{\bar{Y}}$$

$$\widehat{W}_t \equiv \log \frac{W_t}{\bar{W}}.$$

Endogenous output V

Borrower can deleverage by cutting her consumption or increasing hours worked. Or both.
Due to linear production function

$$\widehat{Y}_t = \chi_s \widehat{h}_t^s + \chi_b \widehat{h}_t^b,$$

where $\widehat{Y}_t \equiv \log \frac{Y_t}{\bar{Y}}$.
The Phillips curve

$$\pi_t = \kappa \widehat{Y} + \mathrm{E}_{t-1} \pi_t,$$

where

$$\begin{aligned} \kappa &\equiv \tfrac{\lambda}{1-\lambda} (\omega + 1/\sigma) \\ \pi_t &\equiv \log \tfrac{P_t}{P_{t-1}}. \end{aligned}$$

ENDOGENOUS OUTPUT VI

Monetary policy takes into account the zero lower bound:

$$i_t = \max\left(0, r_t^n + \phi_\pi \pi_t\right),$$

where $\phi_\pi > 1$ and r_t^n is the natural rate of interest, and $i_t = \log(1 + i_t)$.

$$\widehat{C}_t^s = \mathrm{E}_t\, \widehat{C}_{t+1}^s - \sigma(i_t - \mathrm{E}_t\, \pi_{t+1} - \bar{r}),$$

where $\bar{r} = \log \beta^{-1}$.
The loglinearized resource constraint

$$\widehat{Y}_t = \chi_s \widehat{C}_t^s + \chi_b \widehat{C}_t^b.$$

Endogenous output VII

The action is, as before, in the behaviour of the borrower. Borrower is up against the borrowing constraint. The budget constraint

$$B_t(i) = (1 + i_{t-1})B_{t-1}(i) - W_t P_t h_t(i) - \int_0^1 \Pi_t(i) + P_t C_t$$

and the debt constraint

$$(1 + r_t)\frac{B_t(i)}{P_t} \leq D_t(i).$$

Due to assumption, the debt constraint is binding and we may substitute it to the budget constraint and solve for borrower's consumption

$$C_t^b = -\left(\frac{1 + i_{t-1}}{1 + r_{t-1}}\right)\frac{P_{t-1}}{P_t}D_{t-1} + \frac{1}{1 + r_t}\frac{D_t}{P_t} + \underbrace{W_t h_t^b}_{\equiv I_t^b},$$

ENDOGENOUS OUTPUT VIII

Log-linearising this around $D_t = \overline{D} = \overline{D}^{\text{low}}$ gives

$$\widehat{C}_t^b = \widehat{I}_t^b + \beta \widehat{D}_t - \widehat{D}_{t-1} + \gamma_D \pi_t - \gamma_D \beta \left(i_t - \mathrm{E}_t\, \pi_{t+1} - \bar{r}\right),$$

where $\widehat{D}_t \equiv \frac{D_t - \overline{D}}{\overline{Y}}$ and $\gamma_D \equiv \overline{D}/\overline{Y}$.
Few observations:

- In the long run: $\widehat{Y}_L = 0$, $\widehat{C}_L^b = \widehat{C}_L^s = 0$, $i_L = r_L^n = \bar{r}$, and $\pi_L = 0$ (in the target).
- Price level will not revert to $P^{\star}$ since central bank targets inflation.

Endogenous output IX

To solve the model analytically, we distinguish the short-run and long-run behaviour, ie we solve the instantaneous decisions of the agents in the economy.
The borrower's consumption function can be written as

$$\widehat{C}_S^b = \widehat{I}_S^b - \widehat{D} + \gamma_D \pi_S - \gamma_D \beta(i_s - \pi_L - \bar{r}).$$

Note that in this formulation $\widehat{D} \equiv (D^{\text{high}} - \overline{D})/\overline{Y}$. Hence this term is positive under deleverageing!
This differs from standard consumption function:

- big bulk of consumption is determined by *current income* — not the life-cycle income!
- this is due to the binding borrowing constraint: the borrower cannot smooth its consumption *via* capital markets.

ENDOGENOUS OUTPUT X

- marginal propensity to consume out of current income is unity!

Saver's consumption function:

$$\widehat{C}^s_S = \widehat{C}^s_L - \sigma(i_S - \pi_L - \bar{r})$$

Note that $\hat{C}^b_L = \hat{C}^s_L = \pi_L = 0$ and, using aggregate resource constraint, saver's consumption and optimal labour decisions of each household type

$$\widehat{I}^b_S = \widehat{W}_S + \widehat{h}^b_S = \mu \widehat{Y}_S - \chi_s \omega^{-1} \chi_b^{-1} (i_S - \bar{r}),$$

where $\mu \equiv (1 + \omega^{-1})(\omega + \sigma^{-1}) - \sigma^{-1}\omega^{-1}\chi_b^{-1}$.

Endogenous output XI

Substitute the two consumption functions into the resource constraint and substitute the $\widehat{I}_S^b$ is in that of the borrower to obtain

$$\widehat{Y}_S = -\frac{\chi_s(1/\omega+\sigma)+\chi_b\gamma_D\beta}{1-\chi_b\mu}(i_S-\bar{r})-\frac{\chi_b}{1-\chi_b\mu}\widehat{D}+\frac{\chi_b\gamma_D}{1-\chi_b\mu}\pi_S.$$

Define

$$r_S^n \equiv \bar{r}-\frac{\chi_b}{\chi_s(1/\omega+\sigma)+\chi_b\gamma_D\beta}\widehat{D}+\frac{\chi_b\gamma_D}{\chi_s(1/\omega+\sigma)+\chi_b\gamma_D\beta}\pi_S$$

such that

$$\widehat{Y}_S = -\frac{\chi_s(1/\omega+\sigma)+\chi_b\gamma_D\beta}{1-\chi_b\mu}(i_S-r_S^n). \tag{7.3}$$

Natural rate r_S^n

ENDOGENOUS OUTPUT XII

- depends on the deleverageing (just as in the endowment economy)
- spending cuts lead to decline in real rate that lead to increase of the consumption of the saver.
- if $\omega \to 0$ (perfectly elastic labour), this effect disappear, since borrower supply more labour to finance the spending cuts.
- if $\omega \to \infty$ is equal to the endowment economy.

Aggregate demand $\widehat{Y}_S$

- looks like an IS curve

ENDOGENOUS OUTPUT XIII

- standard multiplier effect: if i_S falls, savers consume more, this leads to higher output and higher income of borrowers. Liquidity constraint borrowers will spend the additional income, which leads to second round effects, etc.
- When the IS curve (7.3) is combined with the (assumed) non-linear Taylor rule two steady-states ("regimes") might follow after deleverage shock:
  1. After small deleverage natural interest rate remains positive but will fall to offset the decline in output
  2. Large develerage will drive the economy to ZLB and output will fall below potential.

Endogenous output XIV

Since deleverage is unanticipated ($\mathrm{E}\,\pi_S = 0$), the aggregate supply (Phillips curve) will be

$$\pi_S = \kappa \widehat{Y}_S$$

which will result

$$\widehat{Y}_S = \Gamma - \frac{\chi_b}{1-\chi_b(\mu+\kappa\gamma_D)}\widehat{D} \leq 0 \tag{7.4}$$

$$\pi_S = \kappa\Gamma - \frac{\chi_b}{1-\chi_b(\mu+\kappa\gamma_D)}\widehat{D} \leq 0, \tag{7.5}$$

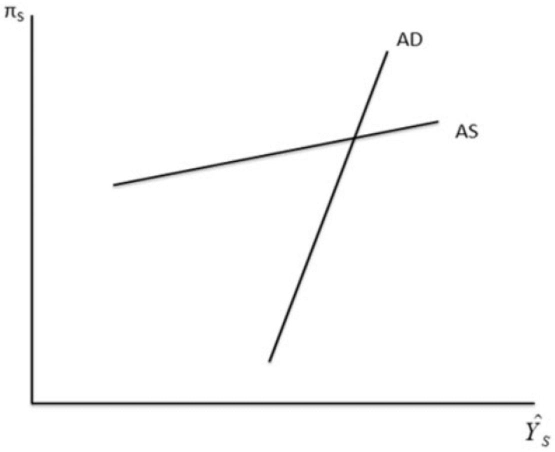
where $\Gamma \equiv \frac{\chi_s(\omega^{-1}+\sigma)+\chi_b\gamma_D\beta}{1-\chi_b(\mu+\kappa\gamma_D)}\bar{r}$

$\longrightarrow$AD and AS will be upward-sloping. AD will be steeper!

If $\chi_b = 0$ (no impatient, constraint households), AD becomes vertical

$\longrightarrow$The higher χ_b it starts sloping backwards.

TOPSY-TURVY ECONONOMY



Features of topsy-turvy

Shifting AS curve (to the right), ie positive supply shock ("paradox of toil")

- rise in the willingness to work, decline in labour income taxes, rise in productivity, etc.
- Results a decline in price level
- that is contractionary via the Fisher effect
- This result holds only to the pure supply shocks but not the ones who also shift AD.

Increased price flexibility ("paradox of flexibility")
$\longrightarrow$AS would be steeper

Monetary and fiscal policy I

Monetary policy:

- Rise in expected inflation would be cure
- Higher inflation target would do it (probably not credible)
- "Forward guidance": keeping interest rates longer lower
- Even partial accommodation would help (but not enough to close the output gap).

Fiscal policy:

- Multiplier is 1!
- Ricardian equivalence does not hold!
    - (in the develarageing proces) borrower's spending depends on their current income — not expected future income.
    - (Compare with rule-of-thumb-consumers)

MONETARY AND FISCAL POLICY II

- Multiplier can be higher than one, since it will increase inflation and, hence, loosen the real burden of debt.
- It also makes people to work more: due to an increase in marginal utility of consumption.
- Fiscal transfers (and tax-cuts) should be targeted to debt-constraint agents. Because they are the non-Ricardians.

Discussion I

The model has resulted a lot of discussion:

- Wieland (2014) shows that under nominal debt constraint the "paradox of toil" does not hold anymore. He also shows some empirical evidence (oil price shocks, Japanese earthquake) to support this finding.
- Braun, Körber and Waki (2012) show that "paradox of toil" results from log-linearization. Since they use Rotemberg (1983) quadratic adjustment costs (of prices) approach, that generates resource cost of price adjustment that is absent in the resource constraint of the log-linearized economy.

Discussion II

- Mertens and Ravn (2014) show that expectation driven ZLB equilibria (eg low confidence) results complete opposite fiscal policy consequences. This results upward-sloping AD curve at the ZLB, but it will be steeper than AS curve!
  $\longrightarrow$The paradox of toil disappears.

Outline

OUTLINE

Central Bank Incentives

No incentive to deviate

By stabilizing inflation, the central bank stabilizes output gap (*divine coincidence*). The resulting equilibrium is optimal (given the employment subsidy to compensate the imperfect competition). The *central bank has no incentive to deviate from this plan.* This is a peculiar situation.

If central bank had incentives to deviate

Modify the model such that there is a *trade-off* between inflation stabilization and closing the output gap. Then the inflation stabilization is not any more *incentive compatible*.

Discretion vs Commitment: time-consistency of optimal plans

Commitment

Commitment is the ability to deliver on past promises no matter what the particular current situation is.

Discretion

Under discretion, a policymaker is allowed to change policy depending on current circumstances and to disregard any past promises.

Time-consistency

A policy is time consistent if an action planned at time t for time $t+i$ remains optimal to implement when time $t+i$ actually arrives.

Why it is important to analyze time-consistency in monetary policy

Positive theories of observed rates of inflation

Forces one to examine the incentives faced by the central bank. Natural starting point in explaining the actual behaviour of central banks.

Normative task of designing policy-making institutions

If time-consistency is important, the models help us in designing how to set-up the policy institutions and how the institutional structure affect economy's outcome.

Background

Time-consistency problem arises only if central bank faces a temptation to lower the output gap by rising inflation.

Divine coincidence: inherent feature of many New-Keynesian Models

Stabilizing inflation is equivalent to stabilizing the welfare-relevant output gap! This generalizes to models with wage frictions. $\longrightarrow$ More profound *real frictions* are needed.

Is monetary policy too easy? Are there no trade-offs?

Flexible inflation targeting

Monetary policy makers claim that — at least in the short-run — there is trade-off between stabilizing inflation or output/employment. Hence, a central bank should avoid too much instability in output while committing to a medium term inflation target.

The Monetary Policy Problem in the Case of an Efficient Steady State

Combine two ingredients

1. When nominal rigidities coexist with *real*[7] imperfections, the flexible price equilibrium allocation is generally inefficient.
2. If economic activity deviates from the natural (flex price) level, it generates variations in inflation that generates relative price distortions.

to get a monetary policy *trade-off*!

[7] This is new to us!

Assume further that

- The possible inefficiencies associated with the flexible price equilibrium do not affect the steady state, which remains efficient. Hence:
- Short run deviations between the natural and efficients levels of output.

Here we analyze the optimal monetary policy problem under that assumption.

BREAKING DIVINE COINCIDENCE

Easy candidates

Markup shock

Suppose that the elasticity of substitution contains exogenous variation, ϵ_t. Then the markup

$$\mu_t = \log\left(\frac{\epsilon_t}{\epsilon_t - 1}\right)$$

enters to New-Keynesian Phillips curve as follows

$$\pi_t = \mathrm{E}_t\, \pi_{t+1} + \lambda\left(\widehat{mc}_t + \widehat{\mu}_t\right)$$

Later, we denote $u_t \equiv \lambda \hat{\mu}_t$.

Breaking divine coincidence. . .

Wage markup shock

Analogous to product market markup. Creates wedge between the marginal rate of substitution and marginal product of labour.

Markup shock and the natural level of output

- Note that under constant markup μ, the natural rate is

$$y_t^n = -\frac{(1-\alpha)[\mu - \log(1-\alpha)]}{\sigma(1-\alpha)+\varphi+\alpha} + \phi_{ya}^n a_t$$

- The existence of the markup, i.e. the imperfect competition, reduces the natural level of output.
- If the markup is time-varying, μ_t, then the natural rate will fluctuate according the fluctuation in markup.
- A rise in the markup will raise inflation and reduce the efficient output: monetary policy faces a trade-off.

Presence of markup shocks leads to

- Differences between natural y_t^n and efficient y_t^e level of output
  - Natural level y_t^n corresponds the flex price level of output, whereas
  - Efficient level y_t^e corresponds the output with flexible prices and absence of real rigidities.
  - Then

$$\tilde{y}_t \equiv (y_t - y_t^e) + (y_t^e - y_t^n)$$

- allows for short run deviations between the natural and efficient levels of output.
- assume that the gap between the two $u_t \equiv \kappa(y_t^e - y_t^n)$ follows a stationary process, with zero mean:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u, \quad \varepsilon_t^u \sim \text{iid}(0, \sigma_u^2).$$

- time variations in gap between efficient and natural levels of output (reflected in markup) generate a trade-off for monetary policy, since it is then impossible to attain simultaneously zero inflation and efficient level of activity.

The Monetary Policy Problem

$$\min \; E_0\{\sum_{t=0}^{\infty} \beta^t \, [\alpha_y \, \widetilde{y}_t^2 + \pi_t^2]\} \tag{1}$$

subject to:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t + u_t$$

where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u \; u_{t-1} + \varepsilon_t$$

In addition:

$$\widetilde{y}_t = -\frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\} \tag{2}$$

Note: utility based criterion requires $\alpha_y = \frac{\kappa}{\epsilon}$

Outline

Monetary policy with discretion

Period-by-period optimization

Discretionary monetary policy relies on period-by-period optimization of the central bank. It is a sequence of unrelated decisions.

Discretionary policy

Sequential optimization; Policy that is optimal in each period without commitment to future actions.

Optimal policy with discretion I

Define $x_t \equiv y_t - y_t^e$, then $\tilde{y}_t \equiv x_t + (y_t^e - y_t^n)$
Each period central bank chooses (x_t, π_t) to *minimize*

$$\alpha_y x_t^2 + \pi_t^2$$

subject to

$$\pi_t = \kappa x_t + v_t,$$

where $v_t \equiv \beta \, \mathrm{E}_t \, \pi_{t+1} + u_t$ is taken as given (since the current action of the central bank does not affect future inflation). We assume that

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \quad \varepsilon_t^u \sim \mathrm{iid}(0, \sigma_u^2).$$

OPTIMAL POLICY WITH DISCRETION II

Optimality condition:

$$x_t = -\frac{\kappa}{\alpha_y}\pi_t.$$

Think this as what the central bank would do if it controlled both inflation and output gap.

Substitute this to the modified expectation-augmented IS curve

$$x_t = -\frac{1}{\sigma}\left(i_t - \mathrm{E}_t\,\pi_{t+1} - r_t^e\right) + \mathrm{E}_t\,x_{t+1},$$

where

$$r_t^e \equiv \rho + \sigma\,\mathrm{E}_t\,\Delta y_{t+1}^e$$

to obtain

$$\pi_t = \frac{\alpha_y\beta}{\alpha_y+\kappa^2}\,\mathrm{E}_t\,\pi_{t+1} + \frac{\alpha_y}{\alpha_y+\kappa^2}u_t.$$

Optimal policy with discretion III

The solution results

$$\pi_t = \alpha_y q u_t$$
$$x_t = -\kappa q u_t$$
$$i_t = r_t^n + q\left(\kappa\sigma(1-\rho_u) + \alpha_y \rho_u\right) u_t,$$

where

$$q \equiv \frac{1}{\kappa^2 + \alpha_y(1 - \beta\rho_u)}.$$

Implementation:

$$i_t = r_t^n + \left[(1-\rho_u)\frac{\kappa\sigma}{\alpha_y} + \rho_u\right]\pi_t$$

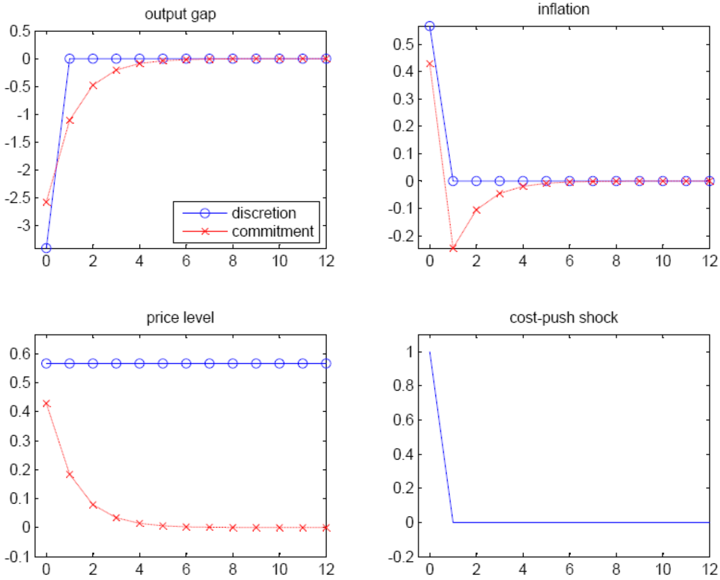
uniqueness condition: $\frac{\kappa\sigma}{\alpha_y} > 1$ (likely if utility-based: $\sigma\epsilon > 1$)

Alternatively,

$$i_t = r_t^n + q\,[\kappa\sigma(1-\rho_u) + \alpha_y\rho_u]\,u_t + \phi_\pi(\pi_t - \alpha_y q\,u_t)$$

uniqueness condition: $\phi_\pi > 1$.

Optimal Monetary Policy: Discretion vs. Commitment



Degree of policy accommodation

- Under discretion, the central bank finds it optimal to accommodate partly the inflationary pressures resulting from the cost-push shock, and thus let inflation increase.
- However, the increase in inflation is smaller than the one that would obtain if the output gap remained unchanged:

$$\pi_t = \frac{1}{1 - \beta\rho_u} u_t.$$

COMMITMENT

Central bank is assumed to be able to commit, with full credibility, to a *policy plan*

In the case of our model, the plan consists of a specification of the desired levels of inflation and output gap at

1. all possible dates, and
2. states of nature,
3. current and future.

Optimal Policy with Commitment

State-contingent policy $\{\widetilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_y \, \widetilde{y}_t^2 + \pi_t^2)$$

subject to the sequence of constraints:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t + u_t$$

Lagrangean:

$$\mathcal{L} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \; [\alpha_y \; \widetilde{y}_t^2 + \pi_t^2 + 2\gamma_t \; (\pi_t - \kappa \; \widetilde{y}_t - \beta \; \pi_{t+1})]$$

First order conditions:

$$\alpha_y \; \widetilde{y}_t - \kappa \; \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

for $t = 0, 1, 2, ...$and where $\gamma_{-1} = 0$.

Eliminating multipliers:

$$\widetilde{y}_0 = -\frac{\kappa}{\alpha_y}\,\pi_0 \tag{7}$$

$$\widetilde{y}_t = \widetilde{y}_{t-1} - \frac{\kappa}{\alpha_y}\,\pi_t \tag{8}$$

for $t = 1, 2, 3, \ldots.$

Alternative representation:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha_y}\,\widehat{p}_t \tag{9}$$

for $t = 0, 1, 2, \ldots$where $\widehat{p}_t \equiv p_t - p_{-1}$.

Equilibrium

$$\widehat{p}_t = a\ \widehat{p}_{t-1} + a\beta\ E_t\{\widehat{p}_{t+1}\} + a\ u_t$$

for $t = 0, 1, 2, \ldots$where $a \equiv \frac{\alpha_y}{\alpha_y(1+\beta)+\kappa^2}$

Stationary solution:

$$\widehat{p}_t = \delta\ \widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}\ u_t \tag{10}$$

for $t = 0, 1, 2, \ldots$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$.

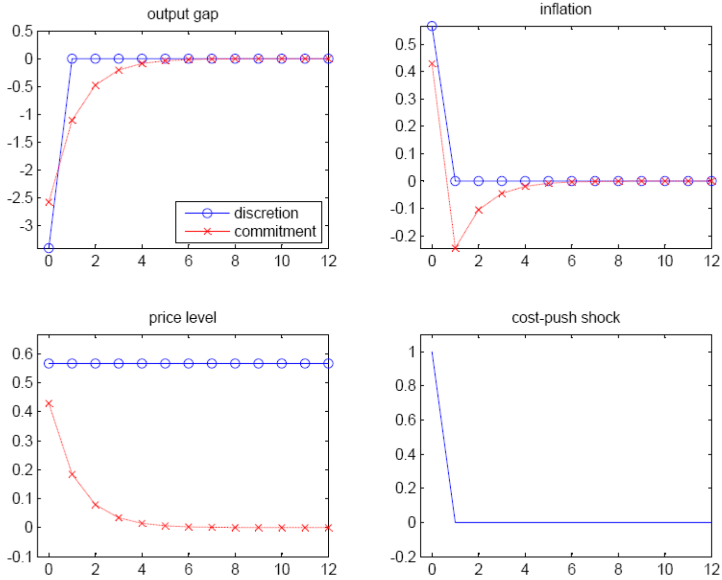
$\rightarrow$ *price level targeting !*

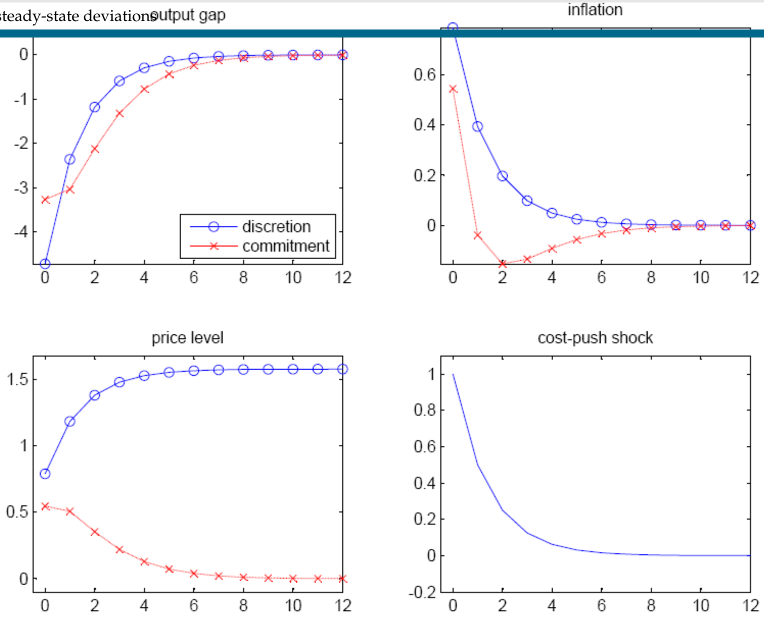
$$\widetilde{y}_t = \delta\ \widetilde{y}_{t-1} - \frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_t \tag{11}$$

for $t = 1, 2, 3, \ldots$as well as

$$\widetilde{y}_0 = -\frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_0$$

Optimal Monetary Policy: Discretion vs. Commitment




output gap
inflation
discretion
commitment
price level
cost-push shock

Comparing discretion and commitment I

- Note, that the loss function is quadratic! Large deviations results relatively higher losses than small deviations.
- In both policies, both output gap and inflation return to zero. In discretionary policy this is reached immediately after the first period and in the case of commitment gradually.

Commitment: why persistently negative output gap and inflation?

By committing to such a response, the central bank manages to improve the output gap/inflation tradeoff in the period when shock occur.

COMPARING DISCRETION AND COMMITMENT II

- A credible central bank is able tie his hands and smooth the losses over time. Discretionary central bank reoptimizes every period (that is known by the agents) and does not have this luxury.

Summary

Given the convexity of the loss function inflation and output gap deviations, the dampening of those dviations in the period of shock brings about an improvement in overall welfare relative to the case of discretion, because the implied benefits are not offset by the (relatively small) losses generated by the deviations in the subsequent periods (and which are absent in the discretionary case).

Stabilization bias

Discretionary policy attempts to stabilize output gap in the medium term more than the optimal policy under commitment calls for, without *internalizing* the benefits in terms of short term stability that results from allowing larger deviations of the output gap at future horizons.

Outline

Distorted steady-state

- Assume that there exist (unmodeled) real distortions/imperfections that generate permanent gap between the natural and the efficient levels of output, which is reflected in an inefficient steady-state

$$-\frac{U_n}{U_c} = MPN(1 - \Phi)$$

- Since $\Phi > 0$, the steady state levels of output and employment are below their respective efficient levels.
- Example: A non-zero steady-state markup (that is NOT corrected by a subsidy), $\Phi \equiv 1 - 1/\mathcal{M}$.

- Losses are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \hat{x}_t^2 \right) - \Lambda \hat{x}_t \right],$$

where $\hat{x}_t$ is the deviation of a welfare relevant output gap from its (negative) steady-state ($x < 0$). $\Lambda \equiv \Phi\lambda/\epsilon$.

DISTORTED STEADY-STATE...

- New-Keynesian Phillips curve is as before, except that

$$u_t \equiv \kappa(\hat{y}_t^e - \hat{y}_t^n).$$

- Furthermore, the steady-state distortion has the same order magnitude as fluctuation in the output gap and inflation, ie "small". We need to be able analyze behavior in the neighborhood of the zero inflation steady-state.

Discretion I

Optimality condition

$$\hat{x}_t = \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\pi_t$$

and solutions

$$\pi_t = \frac{\Lambda\kappa}{\kappa^2 + \alpha_x(1-\beta)} + \alpha_x \Psi u_t$$

$$\hat{x}_t = \frac{\Lambda(1-\beta)}{\kappa^2 + \alpha_x(1-\beta)} - \kappa\Psi u_t$$

The response to cost-push shock is not affected (ie previous impulse responses are valid). Stabilization bias remains. It has effect to the steady-state around which the economy fluctuates.

Discretion II

Inflation bias: Due to inefficiently low level of output, central bank has a desire to increase output by inflating economy. This is known by the agents, and it results positive inflation. (See the constant term in the optimality conditions.)

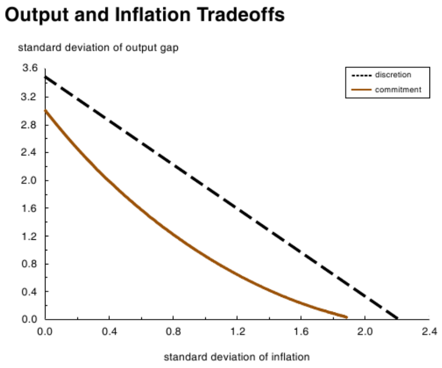
Commitment I

$$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1 - \delta\beta\rho_u} u_t + \frac{\delta\kappa\Lambda}{1 - \delta\beta}$$

$$\hat{x}_t = \delta \hat{x}_{t-1} - \frac{\kappa\delta}{\alpha_x (1 - \delta\beta\rho_u)} u_t + \Lambda \left[1 - \delta \left(1 + \frac{\kappa^2}{\alpha_x (1 - \delta\beta)} \right) \right]$$

Having a committing "technology" (ability) central bank may (asymptotically) get rid of the *inflation bias*. This is a result of additional channel where price level converges to a constant (defined by the preference parameters).

TRADE-OFF



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OUTLINE

OUTLINE

Motivation

- The basic new Keynesian model for the closed economy

  - equilibrium dynamics: simple three-equation representation
  - ability to match much of the evidence on the effects of monetary policy and technology shocks
  - monetary policy: optimality of inflation targeting

- How does the introduction of open economy elements affect that analysis and prescriptions?

- Can a model with nominal rigidities account for the volatility of nominal and real exchange rates?

- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?

Some References

- Kollmann (JIE 01): nominal and real exchange rates, SOE version of EHL, pricing to market, many shocks
- Chari et al. (RES 02): two country model, Taylor type contracts, MP shocks
- Benigno and Benigno (RES 03): one-period contracts, two country, conditions for optimality of price stability
- Svensson (JIE 00): not-fully-optimizing model, strict vs. flexible CPI inflation targeting
- Benigno (JIE 04): staggered, currency union, heterogeneity
- Galí and Monacelli (RES 05): staggered, small open economy, equivalence result, optimal policy.
- Monacelli (JMCB 05): staggered, GM with limited pass-through
- Benigno and Benigno (JME 06): staggered, two countries, optimal policy
- de Paoli (LSE dissertation): generalization of GM

Modeling choices

Open economy aspect brings new concepts: exchange rate, the terms of trade, exports, imports, international financial markets.

Choose, among others, from

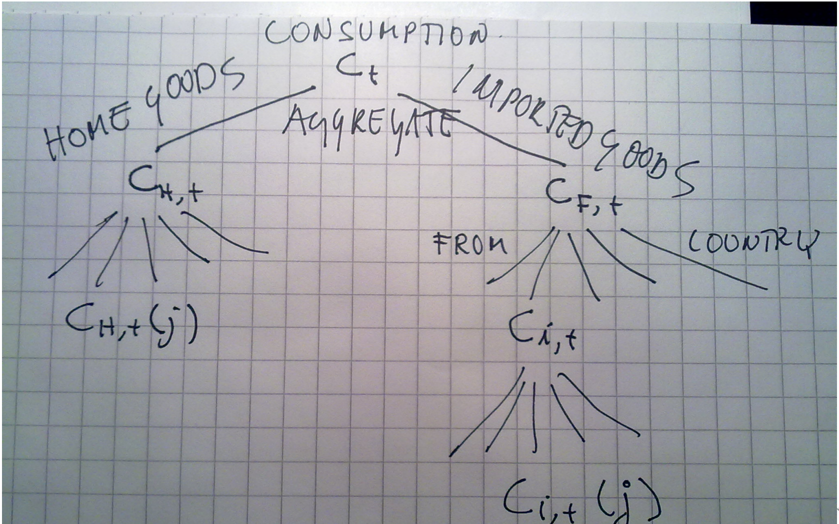
1. Large or small economy
2. Nature of international asset markets: autarky or complete markets.
3. Discrimination between domestic and foreign markets.
4. Pricing behaviour: PCP, LCP
5. Tradeables vs. nontradeables,
6. Trading costs,
7. International policy coordination
8. Exchange rate regimes

Outline

Small open economy

- World economy is a continuum of small open economies represented by the unit interval. $\longrightarrow$ performance of a single economy does not have any impact on the rest of the world.
- Productivity shocks are imperfectly correlated across the economies.
- Identical preference, technology and market structure.
- Notation: no i-index refers to domestic (home) economy, $i \in [0, 1]$ subscript refers to economy i, one in the continuum. Superscript * correspond the world economy as a whole. Superscript i denotes economy i from its own perspective.

Goods structure



Households

Representative household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \tag{9.1}$$

where (as before) N_t denotes hours of labour, and C_t is the CES composite

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{9.2}$$

where α is a measure of *openness* and $1-\alpha$ the *degree of home bias* (Gali 2nd ed. has ν). $\eta > 0$ is elasticity of substitution between domestic and foreign goods.

HOUSEHOLDS...

$C_{H,t}$ is index of consumption domestic goods (Home goods) give by the CES aggregator

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $j \in [0,1]$ denotes the good variety.

HOUSEHOLDS. . . . . .

$C_{F,t}$ is an index of imported goods given aggregegate from countries i by

$$C_{F,t} = \left(\int_0^1 C_{i,t}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

with elasticity of substitution $\gamma > 0$ between importing countries. Imports from each country i is a bundle varieties j

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} .$$

Hence, the varieties in each country are produced by similar technology given by elasticity of substitution $\varepsilon > 0$.

Budget constraint

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi \\ + \mathrm{E}_t\, Q_{t,t+1}D_{t+1} \leq D_t + W_tN_t + T_t, \quad t = 0,1,2,\ldots \quad (9.3)$$

where

- $P_{H,t}(j)$ is the price of domestic variety j,
- $P_{i,t}(j)$ is price of variety j imported from country i.
- D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t.
- W_t is nominal wage, and
- T_t denotes lump-sum transfers/taxes
- $Q_{t,t+1}$ is stochastic discount factor for one period payoff of the household's portfolio.

Complete set of contingent claims traded internationally!

PRICES AND DEMAND FUNCTIONS

Aggregate price index of home goods, $P_{H,t}$, and imported goods from country $i \in [0,1]$ (in domestic currency)

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$$

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$$

Demand for domestic good (home good) of variety j and imported good from country i of variety j

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} C_{H,t} \tag{9.4}$$

$$C_{i,t}(j) = \left[\frac{P_{i,t}(j)}{P_{i,t}} \right]^{-\varepsilon} C_{i,t}$$

Prices and demand functions...

Aggregate price index of imported goods, $P_{F,t}$, and, finally, the aggrage consumtion price index

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

PRICES AND DEMAND FUNCTIONS......

Demand for imported good from country *i* and demand form domestic and imported aggregate goods respectively

$$C_{i,t} = \left[\frac{P_{i,t}}{P_{F,t}}\right]^{-\gamma} C_{F,t} \tag{9.4}$$

$$C_{H,t} = (1-\alpha)\left[\frac{P_{H,t}}{P_t}\right]^{-\eta} C_t \tag{9.5}$$

$$C_{F,t} = \alpha\left[\frac{P_{F,t}}{P_t}\right]^{-\eta} C_t$$

Household budget constraint revisited

Given the market equilibrium (for all of these aggregators), the total consumption expenditure is as follows

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}.$$

The argument is similar to the beginning of the chapter 3.
The budget constraint (9.3) can be written in the following form

$$P_t C_t + \mathrm{E}_t\, Q_{t,t+1} D_{t+1} \leq D_t + W_t N_t + T_t. \tag{9.5}$$

Optimality conditions

Optimality conditions are as before (suppose the familiar utility function, $U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{9.6}$$

and

$$E_t \, \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = E_t \, Q_{t,t+1} \tag{9.8}$$

and loglinearized as

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t \, c_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \, \pi_{t+1} - \rho \right) \end{aligned} \tag{9.8}$$

OUTLINE

Fireworks of definitions!

Terms of trade

Important relative price that deserve a special term! Price of something in terms of price of home good!
Bilateral terms of trade between home and country i:

$$\mathcal{S}_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$$

and (with aggregate) *effective terms of trade*

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}.$$

Terms of trade...

Symmetric steady-state $\mathcal{S}_{i,t} = 1$ for all $i \in [0, 1]$. The loglinearized effective terms of trade is given by

$$s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t} = \int_0^1 s_{i,t} di \tag{9.9}$$

Other useful loglinearizations

- CPI

$$p_t \equiv (1 - \alpha) p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t. \tag{9.10}$$

- CPI inflation π_t and domestic inflation $\pi_{H,t}$

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{9.11}$$

Exchange rates

$\mathcal{E}_{i,t}$ is the *bilateral nominal exchange rate*, ie price of country i's currency in terms of domestic currency, eg how many euros (domestic country) one US (country i) dollar is worth.
$P^i_{i,t}(j)$ is the price of country i's good j expressed in terms of its own currency, eg iPhone (j) in US (i) dollars.

Law of one price

$P_{i,t}(j) = \mathcal{E}_{i,t} P^i_{i,t}(j)$ for all $i, j \in [0, 1]$. Assume it holds at all times for all internationally traded goods. Aggregating results

$P_{i,t} = \mathcal{E}_{i,t} P^i_{i,t}$, where $P^i_{i,t} = \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$

EXCHANGE RATES...

$p^i_{i,t} = \int_0^1 p^i_{i,t}(j)dj$ is the loglinerized domestic price index for country i (in country i's own currency).

$e_t = \int_0^1 e_{i,t} di$ is the log of *effective nominal exchange rate*. Note that it is an index.

$p^*_t = \int_0^1 p^i_{i,t} di$ is the log *world price index*.

Then $P_{F,t}$ may be loglinearized around symmetric steady-state as

$$p_{F,t} = \int_0^1 \left(e_{i,t} + p^i_{i,t} \right) di = e_t + p^*_t,$$

and when combined with the loglinearized terms of trade results

$$s_t = e_t + p^*_t - p_{H,t}. \tag{9.12}$$

EXCHANGE RATES......

Bilateral real exchange rate with country i is defined as

$$\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^i}{P_t},$$

is the ratio of the two countries' CPIs, both expressed in terms of domestic currency.

$q_t = \int_0^1 q_{i,t} di$ is the log *effective real exchange rate*

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t = (1 - \alpha) s_t$$

OUTLINE

International risk-sharing

An analogous optimality condition to (9.8) holds also for the country i. When expressed in the domestic currency, it can be written as

$$\mathrm{E}_t\,\beta\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\left(\frac{\mathcal{E}^i_t P^i_t}{\mathcal{E}^i_{t+1} P^i_{t+1}}\right) = \mathrm{E}_t\, Q_{t,t+1}. \tag{9.13}$$

Combining this with (9.8) (and assuming zero net foreign asset holdings and an *ex ante* identical environment) results

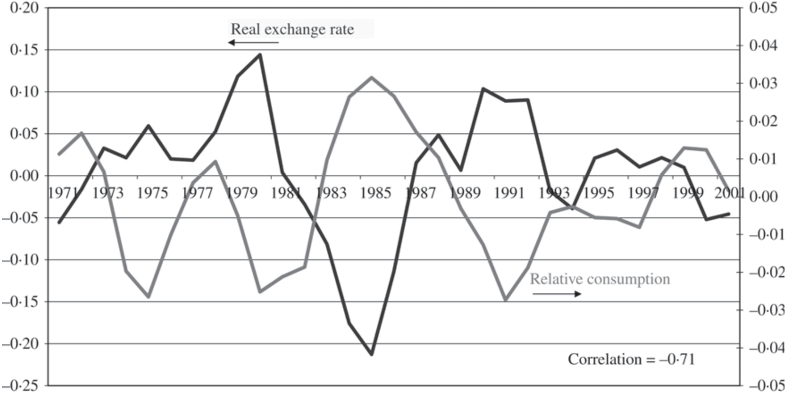
$$C_t = C^i_t \mathcal{Q}^{\frac{1}{\sigma}}_{i,t}. \tag{9.14}$$

Loglinearizing and aggregating ($c^*_t = \int_0^1 c^i_t di$) over i gives

$$c_t = c^*_t + \frac{1}{\sigma} q_t = c^*_t + \left(\frac{1-\alpha}{\sigma}\right) s_t. \tag{9.15}$$

Complete international asset markets equalizes consumption levels between countries up to terms of trade.

INTERNATIONAL RISKSHARING



The real exchange rate and relative consumption are constructed using trade weights as described in the data appendix. Both series are logged and HP-filtered.

Grey line: $c_t - c_t^*$; Black line: q_t. Source: Corsetti et al 2008

UNCOVERED INTEREST PARITY

Allow households to invest both domestic B_t and foreign B_t^* one-period bonds. The budget constraint may be written as

$$P_t C_t + Q_{t,t+1} B_{t+1} + Q^*_{t,t+1} \mathcal{E}_t B^*_{t+1} \leq B_t + \mathcal{E}_t B_t^* + W_t N_t + T_t. \quad (9.16)$$

The optimality conditions wrt to these assets are

$$\beta \, \mathrm{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1},$$

$$\beta \, \mathrm{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) = Q^*_{t,t+1}.$$

Combining these results

$$\mathrm{E}_t\left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) = \frac{Q^*_{t,t+1}}{Q_{t,t+1}},$$

whose loglinear form is the familiar

$$i_t = i^*_t + \mathrm{E}_t\,\Delta e_{t+1}$$

Combining this with the definition of terms of trade, we find that

$$s_t = \mathrm{E}_t \sum_{k=0}^{\infty}\left[\left(i^*_{t+k} - \pi^*_{t+k+1}\right) - \left(i_{t+k} - \pi_{H,t+k+1}\right)\right].$$

That express the terms of trade as the expected sum of real interest rate differential. Note, however, that this is not an equilibrium condition. It simply combines the two from previous slide with the definition of the terms of trade.

OUTLINE

Firms and technologies

The supply side follows the very same structure as in the basic New-Keynesian model. We assume constant-returns-to-scale, $\alpha = 0$ (according to notation of the chapter 3).
Firms price is the domestic price $P_{H,t}(j)$ (price index of domestic production). Therefore, the marginal costs area

$$mc_t = -\nu + w_t - p_{H,t} - a_t,$$

where $\nu \equiv -\log(1-\tau)$ (employment subsidy).
The Calvo-pricing applies here too:

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta)\, \mathrm{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k (mc_{t+k} + p_{H,t+k}), \tag{9.17}$$

where $\bar{p}_{H,t}$ denotes the price of firms allowed to optimize.

OUTLINE

Goods market clearing

Domesticly produced goods

$$\underbrace{Y_t(j)}_{\text{Domestic output}} = \underbrace{C_{H,t}(j)}_{\text{Domestic demand for home goods}} + \underbrace{\int_0^1 C^i_{H,t}(j)di}_{\text{Foreign demand for homegoods, =exports}} \tag{9.18}$$

Notice, that due to nested structure the demand for domestic good j in country i is given by

$$C^i_{H,t}(j) = \alpha \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \overbrace{\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P^i_{F,t}}\right)^{-\gamma} \underbrace{\left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t}_{=C^i_{F,t}}}^{=C^i_{H,t}}$$

EXPRESSING (9.8) IN TERMS OF OUTPUT

Aggregate (9.18) and express in terms of terms of trade

$$\begin{aligned}
Y_t &\equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}^{\eta}_{i,t} C^i_t di \right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 (\mathcal{S}^i_t \mathcal{S}_{i,t})^{\gamma-\eta} \mathcal{Q}^{\eta-\frac{1}{\sigma}}_{i,t} di \right] \qquad (9.19)
\end{aligned}$$

The last step comes from (9.14).

In the Cobb-Douglas case, ie $\sigma = \eta = \gamma = 1$ this aggregates as

$$Y_t = \mathcal{S}_t^{\alpha} C_t. \tag{9.20}$$

Note that at the world level, the terms of trade is unity, ie $\int_0^1 s_t^i di = 0$. The loglinear approximation of (9.19) around the symmetric steady state is the following

$$y_t = c_t + \alpha\gamma s_t + \alpha \left(\eta - \frac{1}{\sigma}\right) q_t = c_t + \frac{\alpha\omega}{\sigma} s_t, \tag{9.21}$$

where $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$. Aggregating the country i counterparts of (9.21) over all countries results

$$y_t^* \equiv \int_0^1 y_t^i di = \int_0^1 c_t^i + \frac{\alpha\omega}{\sigma} s_t^i di = \int_0^1 c_t^i di \equiv c_t^*. \tag{9.22}$$

Combine (9.21), (9.15) and (9.22) to express output y_t in terms of world demand and terms of trade as

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t, \tag{9.23}$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.
Finally, combining (9.8) with (9.21) gives

$$\begin{aligned} y_t &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{t+1} - \rho\right) - \frac{\alpha\omega}{\sigma}\, \mathrm{E}_t\, \Delta s_{t+1} \\ &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathrm{E}_t\, \pi_{H,t+1} - \rho\right) - \frac{\alpha\Theta}{\sigma}\, \mathrm{E}_t\, \Delta s_{t+1} \\ &= \mathrm{E}_t\, y_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - \mathrm{E}_t\, \pi_{H,t+1} - \rho\right) + \alpha\Theta\, \mathrm{E}_t\, \Delta y_{t+1}^* \end{aligned} \tag{9.24}$$

Coefficients in (9.24)

- Real rate sensitivity: $\sigma_\alpha < \sigma$ in the case $\omega > 1$, ie when η and γ are high.
- Foreign output growth sensitivity:

$$\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$$

is positive if η and γ are high (relative to σ).

Intuition: direct negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced ral appreaciation (and the resulting switch toward foreign goods). It is dampened by expected real depreciation (CPI inflation is higher than domestic inflation) which dampens the change in the consumption based real rate $i_t - \mathrm{E}_t\, \pi_{t+1}$ (relative to $i_t - \mathrm{E}_t\, \pi_{H,t+1}$).

THE TRADE BALANCE

In the pure Cobb-Douglas (eg $\omega = 0$) case (9.19) results

$$P_{H,t}Y_t = P_tC_t, \quad t > 0.$$

which means that the *trade balance*

$$nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right)$$

is zero all the time. Loglinearize to obtain

$$nx_t = y_t - c_t - \alpha s_t = \alpha\left(\frac{\omega}{\sigma} - 1\right)s_t. \tag{9.25}$$

Hence, the sign of net exports is ambiguous and depends on (as usual) the elasticities of substitution!

OUTLINE

Output and employment

Labour market clearing condition is

$$N_t \equiv \int_0^1 N_t(j)dj = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj.$$

Loglinearizing

$$y_t = a_t + n_t \tag{9.26}$$

Calvo-pricing implies

$$\pi_{H,t} = \beta \operatorname{E}_t \pi_{H,t+1} + \underbrace{\frac{(1-\beta\theta)(1-\theta)}{\theta}}_{\equiv\lambda} \hat{mc}_t. \tag{9.27}$$

Marginal cost is given by

$$
\begin{aligned}
mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\
&= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
&= -\nu + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
&= -\nu + \sigma y_t^* + \varphi y_t + s - (1+\varphi) a_t \qquad (9.28) \\
&= -\nu + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1+\varphi) a_t \qquad (9.29)
\end{aligned}
$$

Channels

φ: output via employment

σ_α: output via terms of trade

σ: world output via consumption (real wage)

σ_α: world output via terms of trade

Marginal costs and output gap

Under flexible prices

$$mc_t = -\mu.$$

and it is the same form as (9.29)

$$(\sigma_\alpha + \varphi)y_t^n = \nu - \mu + (1 + \varphi)a_t - (\sigma - \sigma_\alpha)y_t^*.$$

Solve a_t, substitute into (9.29) and note that the foreign output y_t^* is independent of domestic factors to obtain

$$\hat{mc} = (\sigma_\alpha + \varphi)\tilde{y}_t$$

OUTLINE

Equilibrium Dynamics in the SOE: A Canonical Representation

$$\pi_{H,t} = \beta \ E_t\{\pi_{H,t+1}\} + \kappa_\alpha \ \widetilde{y}_t$$

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha} \left(i_t - E_t\{\pi_{H,t+1}\} - r_t^n\right)$$

where

$$\begin{aligned} \widetilde{y}_t &= y_t - y_t^n \\ y_t^n &= \Omega + \Gamma \ a_t + \alpha\Psi \ y_t^* \\ r_t^n &\equiv \rho - \sigma_\alpha \Gamma(1-\rho_a) \ a_t + \alpha\sigma_\alpha(\Theta + \Psi) \ E_t\{\Delta y_{t+1}^*\} \end{aligned}$$

$$\kappa_\alpha \equiv \lambda \left(\sigma_\alpha + \varphi\right) \quad ; \quad \sigma_\alpha \equiv \frac{\sigma}{(1-\alpha) + \alpha\omega} \quad ; \quad \omega \equiv \sigma\gamma + (1-\alpha)\left(\sigma\eta - 1\right)$$

$$\Gamma \equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} \quad ; \quad \Psi \equiv - \frac{\Theta \ \sigma_\alpha}{\sigma_\alpha + \varphi}$$

Role of openness: assuming high substitutability (high η, γ)

$$\frac{\partial \sigma_\alpha}{\partial \alpha} < 0 \quad ; \quad \frac{\partial \kappa_\alpha}{\partial \alpha} < 0$$

OUTLINE

OPEN-ECONOMY DISTORTIONS

First of all: this is model dependent!

1. Imperfect competition (monopoly power): corrected with employment subsidy $1 - \tau$.
2. Staggered (Calvo) price setting, resulting fluctuations in the markup: corrected by the inflation stabilization.
3. Terms of trade distortion arising from imperfect substitutability of domestic and foreign goods: increase employment subsidy.

Policy that stabilizes domestic inflation maximizes welfare. Nominal exchange rate, CPI inflation are adjusting to replicate the response of the terms of trade that would obtained under flexible prices.

Optimal Monetary Policy

Background and Strategy

A Special Case

$$\sigma = \eta = \gamma = 1$$

Optimality of Flexible Price Equilibrium:

$$(1-\tau)(1-\alpha) = 1 - \frac{1}{\epsilon}$$

Implied Monetary Policy Objectives

$$y_t = y_t^n$$

$$\pi_{H,t} = 0$$

for all t.

Implementation

$$i_t = r_t^n + \phi_\pi \, \pi_{H,t} + \phi_y \, \widetilde{y}_t$$

Evaluation of Alternative Monetary Policy Regimes

Welfare Losses (special case)

$$\mathbb{W} = -\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t \left[\frac{\epsilon}{\lambda}\,\pi_{H,t}^2 + (1+\varphi)\,\widetilde{y}_t^2\right]$$

Average period losses

$$\mathbb{V} = -\frac{(1-\alpha)}{2}\left[\frac{\epsilon}{\lambda}\,var(\pi_{H,t}) + (1+\varphi)\,var(\widetilde{y}_t)\right]$$

Three Simple Rules

Domestic inflation-based Taylor rule (DITR)

$$i_t = \rho + \phi_\pi \ \pi_{H,t}$$

CPI inflation-based Taylor rule (CITR):

$$i_t = \rho + \phi_\pi \ \pi_t$$

Exchange rate peg (PEG)

$$e_t = 0$$

Impulse Responses and Welfare Evaluation

Outline

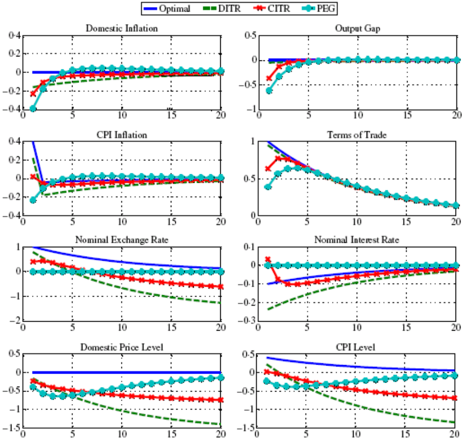
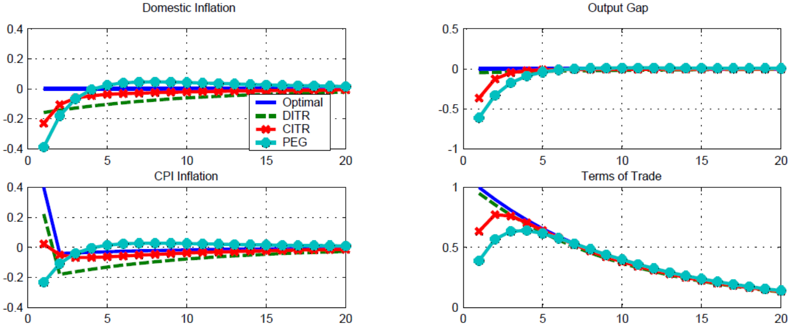
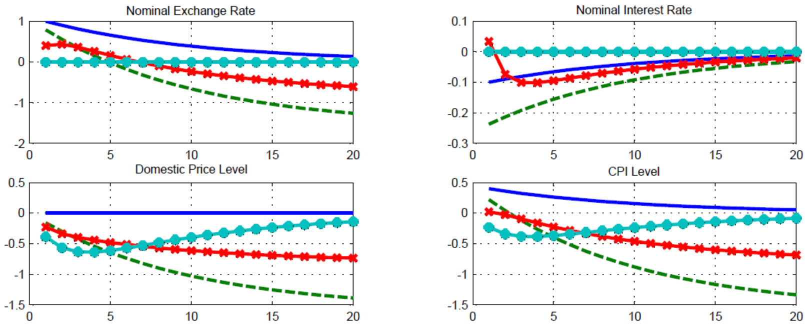


FIGURE 1
Impulse responses to a domestic productivity shock under alternative policy rules

IMPULSE RESPONSES TO POSITIVE TECHNOLOGY SHOCK



IMPULSE RESPONSES TO POSITIVE TECHNOLOGY SHOCK



OUTLINE

TABLE 1

Cyclical properties of alternative policy regimes

	Optimal sd%	DI Taylor sd%	CPI Taylor sd%	Peg sd%
Output	0·95	0·68	0·72	0·86
Domestic inflation	0·00	0·27	0·27	0·36
CPI inflation	0·38	0·41	0·27	0·21
Nominal I. rate	0·32	0·41	0·41	0·21
Terms of trade	1·60	1·53	1·43	1·17
Nominal depr. rate	0·95	0·86	0·53	0·00

Note: Sd denotes standard deviation in %.

TABLE 2

Contribution to welfare losses

	DI Taylor	CPI Taylor	Peg
Benchmark $\mu = 1{\cdot}2, \varphi = 3$			
Var(domestic infl)	0·0157	0·0151	0·0268
Var(output gap)	0·0009	0·0019	0·0053
Total	0·0166	0·0170	0·0321
Low steady state mark-up $\mu = 1{\cdot}1, \varphi = 3$			
Var(Domestic infl)	0·0287	0·0277	0·0491
Var(Output gap)	0·0009	0·0019	0·0053
Total	0·0297	0·0296	0·0544
Low elasticity of labour supply $\mu = 1{\cdot}2, \varphi = 10$			
Var(Domestic infl)	0·0235	0·0240	0·0565
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0240	0·0261	0·0630
Low mark-up and elasticity of labour supply $\mu = 1{\cdot}1, \varphi = 10$			
Var(Domestic infl)	0·0431	0·0441	0·1036
Var(Output gap)	0·0005	0·0020	0·0064
Total	0·0436	0·0461	0·1101

Note: Entries are percentage units of steady state consumption.

An Extension with Imperfect Pass-Through (Monacelli JIE 05)

Setup as in GM, with rest of the world modelled as a single economy.

Key Assumption:

- imports sold through retail firms
- price at the dock: $e_t + p^*_{F,t}(j)$
- staggered price setting by retailers $\Longrightarrow$ in general, $p_{F,t}(j) \neq e_t + p^*_{F,t}(j)$

Law of One Price Gap:

$$\psi_{F,t} \equiv e_t + p^*_t - p_{F,t}$$

Consistent with the evidence (Campa and Goldberg (REStat 05):

- partial pass-through in the short run
- full pass through in the long run (for most industries).

Imported Goods Inflation:

$$\pi_{F,t} = \beta E_t\{\pi_{F,t+1}\} + \lambda_F \, \psi_{F,t}$$

Domestic Goods Inflation

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \lambda_H \, \widehat{mc}_t$$

$\Longrightarrow$ impossibility of replicating flexible price allocation

$\Longrightarrow$ emergence of a policy trade-off

$\Longrightarrow$ gains from commitment

OUTLINE

INTRODUCTION

The fiscal theory of price level (FTPL) tries to fill some gaps in the previous analysis by addressing

- what determines the price level under passive interest rate rules (or fixed interest rate)
- interpreting the public sector present value of the budget constraint
- implicit assumptions about fiscal policy in the previously set monetary theory
- Sargent and Wallace (1981) game of chicken

The theory remains controversial but has generated interest in the current crisis of euro area.

It emphasizes the role of nominal public sector liabilities and their valuation.

See the new book by Cochrane.

(The slides of this section borrow extensive from Eric Leeper's mini-course.)

BIG PICTURE I

Canonical models, like the basic New-Keynesian model, assume:

1. MP can and does control inflation
2. FP can and does ensure solvency
3. MP optimal or obeys Taylor-type rule: unconstrained or "active"
4. FP takes MP and private behavior as given and stabilizes debt: constrained or "passive"

Makes sense in normal times. MP is omnipotent and FP trivial.
Modeling convention a stretch since 2008
What have policies actually been doing?

1. MP at or near zero lower bound: Aggressive MP has pursued growht: thrown Taylor principle out the window

BIG PICTURE II

2. FP bouncing between stimulus and austerity: IMF advice
   - 2008–2009: urgent need to stimulate
   - 2010–2011: urgent need to consolidate
   - 2012: urgent need for stimulative consolidation ("growsterity")

How can such policies anchor monetary expectations on inflation target?
How can such policies anchor fiscal expectations on debt stabilization?
Need to understand implications of policy interactions that deviate from convention
Short-run reasons:

- Europe enters second recession, emerging economies slowing down, U.S. on brink of new recession, Japan still stuck

Big picture III

- Ubiquitous tradeoff between stabilization and sustainability
- What are effect of fiscal policy when MP pegs rate?

Long-run reasons:

- Ageing populations and unfunded old-age benefits
- Huge uncertainty about future fiscal policies
- What are impacts of unresolved long-run fiscal stress?

Conventional modeling cannot address these issues: assumes away the problems

THE MODEL I

Endowment economy at the cashless limit, complete financial markets, one-period nominal debt.
Representative household maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} U(C_t),$$

subject to the following budget constraint

$$P_t C_t + P_t \tau_t + \mathrm{E}_t \left(Q_{t,t+1} B_t \right) = P_t Y_t + P_t G_t + B_{t-1}, \quad B_{-1} > 0$$

where

τ_t is the tax (and other) revenues of the government

G_t is government's expenditures (or purchases more concretely)

The model II

$s_t \equiv \tau_t - G_t$ government's primary surplus.

Define the real stochastic discount factor

$$Q_{t,t+j} = \prod_{k=0}^{j} Q_{t,t+k}, \quad Q_{t,t} = 1.$$

The intertemporal budget constraint can be written as

$$\mathrm{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} C_{t+j} = \frac{B_{t-1}}{P_t} + \mathrm{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} Y_{t+j} - \mathrm{E}_t \sum_{j=0}^{\infty} Q_{t,t+j} s_{t+j}.$$

and the transversality condition

$$\lim_{T \to \infty} \mathrm{E}_t \left(Q_{t,t+T} \frac{B_{T-1}}{P_T} \right) = 0.$$

The model III

(no infinite asset accumulation)
Impose $Y_t = C$ (endowment economy), denote $\frac{1}{1+i_t} = Q_{t,t+1}$ and the above transversality condition to get the following optimality conditions

$$\frac{1}{1+i_t} = \beta \, \mathrm{E}_t \, \frac{P_t}{P_{t+1}} \equiv \beta \, \mathrm{E}_t \, \frac{1}{\Pi_{t+1}}$$

$$\frac{B_{t-1}}{P_t} = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

Price sequence $\{P_t\}$ must satisfy these to be an equilibrium. Without any additional restrictions from policy behaviour, there are many possible equilibrium $\{P_t\}$ sequences.

POLICY RULES I

Specify the policy rules and the government budget constraint

$$\frac{1}{1+i_t} = \frac{1}{1+i^*} + \alpha\left(\frac{1}{\Pi_t} - \frac{1}{\Pi^*}\right)$$
$$s_t = s^* + \gamma\left(\frac{B_{t-1}}{P_t} - b^\star\right)$$
$$\frac{\mathrm{E}_t\left(Q_{t,t+1}B_t\right)}{P_t} + s_t = \frac{B_{t-1}}{P_t}.$$

Steady state

$$\frac{B_{t-1}}{P_t} = b^\star, \quad s^\star = (1-\beta)b^\star, \quad 1+i^\star = \frac{\Pi^\star}{\beta}, \quad m^\star = \beta.$$

Policy rules II

Combine policy rules with equilibrium conditions to obtain

$$\mathrm{E}_t\left(\frac{1}{\Pi_{t+1}} - \frac{1}{\Pi^*}\right) = \frac{\alpha}{\beta}\left(\frac{1}{\Pi_t} - \frac{1}{\Pi^*}\right)$$

$$\frac{B_t}{P_{t+1}} - b^\star = \frac{1-\gamma}{\beta}\left(\frac{B_{t-1}}{P_t} - b^\star\right).$$

Policy tasks

1. Control inflation
2. Stabilize public debt

POLICY RULES III

Two different policy mixes can accomplish these tasks

Active MP/passive FP

Active monetary policy, and passive fiscal policy:
⟶MP targets inflation; FP targets real debt.
Normal state of affairs

Passive MP/active FP

Passive monetary policy, active fiscal policy:
⟶MP maintains the value of debt, and FP controls inflation.
Can arise in an era of fiscal stress.

Regime of active monetary policy I

Familiar:

- monetary policy targets inflation by adjusting nominal interest rates aggressively:

$$\frac{\alpha}{\beta} > 1$$

- fiscal policy adjusts future surpluses to cover interest plus principal of the debt

$$\frac{1-\gamma}{\beta} < 1, \quad \iff \quad \gamma > 1-\beta.$$

Equilibrium

Regime of active monetary policy II

- Unique bounded equilibrium

$$\Pi_t = \Pi^\star$$

- evolution of the government debt

$$\mathrm{E}_t \frac{B_t}{P_{t+1}} - b^\star = \frac{1-\gamma}{\beta}\left(\frac{B_{t-1}}{P_t} - b^\star\right)$$

which results $\mathrm{E}_t\, b_T \to b^\star$ when $T \to \infty$.

Keeping the *phase-diagram of the ZLB section* in mind, we cannot rule out

$$\lim_{T\to\infty} \Pi_T = \infty.$$

by monetary policy and private behaviour.
Can be resolved only by fiscal policy!

Regime of active monetary policy III

- any shock that changes debt must create expectations that future surpluses will adjust to stabilize debt value.
- people must believe that those adjustments will occur
- this eliminates the wealth effect from government debt $\longrightarrow$Ricardian fiscal policy!
- Fiscal expectations must be achored!

In other words: MP delivers the equilibrium inflation process, FB stabilizes the debt such that the equilibrium condition

$$\frac{B_{t-1}}{P_t} = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

is satisfied.

Regime of passive monetary policy I

Most of the governments issue nominal bonds: 90 % US, 80 % UK, etc.

In passive monetary policy, active fiscal policy regime

- Fiscal policy sets primary surpluses independently of the debt.
- Monetary policy prevents interest payments on debt from destabilizing debt.

Nominal debt is revalued to align its value with expected surpluses: price level is defined by the equilibrium condition

$$\frac{B_{t-1}}{P_t} = \mathrm{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

In other words:

REGIME OF PASSIVE MONETARY POLICY II

- Fiscal policy responds weakly (or not at all) to state of government indebtedness (B_{t-1}):

$$\gamma < 1 - \beta$$

- Monetary policy prevents nominal interest rate from reacting strongly to inflation:

$$0 < \frac{\alpha}{\beta} < 1$$

Consider a special case where $\alpha = \gamma = 0$:

- MP sets $1 + i_t$ exogenously
- FB sets $\{s_t\}$ exogenously

Regime of passive monetary policy III

In equilibrium

$$E_t \frac{1}{\Pi_{t+1}} = \frac{1}{\beta(1+i^\star)} = \frac{1}{\Pi^\star}$$

and price level is solved from

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

note that at period t, B_{t-1} and $E_t\, s_{t+j}$ are given
$\longrightarrow P_t$ becomes endogenous.
To summarize:

- FP delivers unique equilibrium price process

Regime of passive monetary policy IV

- taking this inflation process as given, MP must choose compatible interest rate policy: stabilizes debt
- this imposes restrictions on P_t and on monetary policy if price level remains stable.

DISCUSSION I

The equilibrium condition

- Holds any model, and any regime
  $\longrightarrow$no empirical tests of regimes possible
- It is not an "intertemporal government budget constraint" since we have imposed market clearing, optimality conditions, transversality condition
- debt valuation equation (like stock valuation)

Other issues

- **?** is about segniorage where as FTPL is not
- Debt maturity structure: MP is not impotent but it cannot control both actual and expected inflation
  $\longrightarrow$Consol: MP determines only the timing, whereas FP determines the present value of inflation.

Discussion II

- Fiscal union is needed to support monetary union

See Cochrane's panel discussion.

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