Evolution of the Universe by the Principle of Least Action

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Evolution of the Universe is described by the principle of least action. This tenet is motivated because the general law of nature accounts for ubiquitous patterns, most notably power laws that display themselves also on cosmic scale, for example, in distributions of galaxies and voids. The least-time consumption of free energy will invariably process the Universe toward isotropy and homogeneity on its largest scale without superluminal expansion. Notably, the least-time evolution results in history. The path-dependence of early evolution manifests itself as correlated multipoles in angular decomposition of cosmic microwave background. These characteristics of the universal evolution accounted by the least-time principle distinguish from those resulting from the standard cosmological model.

Keywords: anomaly, cosmic microwave background, free energy, horizon problem, inflation model, the principle of least action

1. Introduction

The Universe comprises everything, by definition. In view of that, evolution of the Universe amasses from evolution of all subsystems. Also by definition no subsystem can single out to a universe of its own, and hence all subsystems must comply with universal laws of nature. Conversely, evolution of the whole Universe must follow the same evolutionary principle as any of its subsystem.

Evolution by a natural law, known as the principle of least action [1,2,3], yields ubiquitous scale-invariant patterns [4,5,6]. Namely, skewed distributions are found throughout nature. They accumulate along sigmoid curves that span log-log plots mostly as straight lines. These ubiquitous patterns manifest themselves also on cosmic scale [7]. Galaxies populate space pairwise in a power-law manner [8,9,10,11,12]. Intergalactic voids distribute in line with a broken power law that can be deduced from dispersion of hydrogen emission lines, known as Lyman alpha-forest [13,14]. Type 1a supernovae magnitude vs. redshift data that reports from expansion of the Universe, also complies with a broken power law [15,16]. Similarly, flux of cosmic rays that probes the structure of space over 12 orders of magnitude in energy, follows a broken power [17,18,19], just as the number of faintest radio sources vs. flux density [20]. Also the number of quasars vs. redshift is a skewed distribution [21,22]. Temperature spectrum of free space has the familiar skew form of Planck's radiation law. Moreover, cosmic microwave background (CMB) radiation is scale-invariant and its power of distributes nearly log-normally about the maximum at multipole 200 that corresponds to an angle slightly less than 1° in the sky [23]. The power at higher multipoles decreases in oscillatory manner which is also a characteristic of natural processes that follow the principle of least action by consuming free energy in least time [6].

The universal patterns pervade similarly on shorter scales. On the galactic scale, for example, characteristic bands of star luminosity vs. color are basically straight lines on the renowned log-log plot of Hertzsprung-Russell (H-R) diagram [24]. Also the initial mass function for a population of stars follows mostly a power law [25]. Likewise, the Faber-Jackson relation for luminosity vs. central stellar velocity dispersion of elliptical galaxies [26,27] as well as Tully-Fisher relation for mass vs. intrinsic luminosity of spiral galaxies [28,29] comply with power laws. The logarithmic spirals themselves are skewed distributions in polar coordinates [5,6,30]. Moreover, orbital velocities vs. distance in spiral galaxies display damping oscillations which are characteristic of natural processes that are subject to abrupt changes or perturbations in energy density [31,32].

The least-time free energy consumption does not only account for the universal patterns, but also for history [3]. In contrast, the time- and path-dependent trajectories do not result from many models of physics whose constant-energy equations of motions are determined by an initial phase, and hence can, at least in principle, be transformed to a time-independent frame [33]. Thus, when using time-symmetric formulas, traces of irreversibility, that is, dissipation are often dealt approximately or eventually deemed as anomalous.

On the cosmic scale, angular decomposition of CMB radiation reveals that low multipoles are somewhat anomalously correlated [34,35,36]. Also the coldest spot on the map is regarded somewhat anomalously large and cold [37]. Moreover, contemporary interpretation of the type 1a supernovae data that extends over eons, calls for extraordinary dark energy to be consistent with the lambda cold dark matter (ΛCDM) model [16,38].

In general, the arrow of time couples to the leveling off energy gradients [3]. Specifically, the universal energy gradient across time of 13.8 billion years manifest itself most evidently on the galactic scale. However, when it is ignored, the galaxy rotation is attributed to unobservable dark matter [31,39,40]. On the solar system's scale, a small but distinct fraction of the advancing Mercury perihelion, albeit accounted for by general relativity, is referred to as anomalous [41,42]. On the planetary scale an additional velocity gain of a spacecraft during its flyby of Earth is perceived as anomalous [43] when the energy density gradient due to Earth's rotation is omitted from analysis.

Nature is not abnormal, by definition, and hence anomalies imply imperfect understanding. In contrast, the principle of least action, in its original dissipative form [1,2,3,6,44], accounts for anomalies, not as oddities, but as natural manifestations of the least-time free energy consumption so that changes in kinetic energy

$$d_t 2K = -\mathbf{v}\cdot\nabla U + i d_t Q \tag{1}$$

from one state to another proceed along the steepest gradients in potential energy U with velocity $\mathbf{v}$ concurrent with dissipation d_tQ. We employ this basic law to obtain perspective on evolution of the nascent Universe. We emphasize that only the least-time principle, not the factual process, will be analyzed and demonstrated, since very little is known about primordial mechanisms of free energy consumption.

2. Inferences from Universality

The universal patterns, by displaying themselves as far as can be observed [22, 45], imply to us that evolution of the early Universe was by principle no different from evolution of the present day Universe, only the primordial mechanisms of free energy consumption were different from the present ones. Today stars of various kind are the most effective mechanisms that transform matter-bound quanta to freely propagating photons that embody vacuum.

It is worth reminding that the easily observable radiation density of free space is only a small fraction of the vacuum's total energy density [46]. The vacuum devoid of electromagnetic fields is hard to detect, hence various embodiments have been suggested [47], including pairs of photons that co-propagate exactly out-of-phase [19,48]. These paired quanta emerge for free propagation from matter, *e.g.*, when electrons and positron annihilate, but also in general when elementary constituents of matter, most notably quarks of opposite charge annihilate each other [49,50].

When the physical vacuum originates from matter, it is inescapable that the vacuum's energy density is in balance with the average energy density of matter in the Universe [51]. In other words, according to the least-time principle the Universe is inevitably flat. No geodesic can be shorter than that taken by the photons [52]. The universal balance between the vacuum energy and matter-bound energy is stated by integrating Eq. 1 to the stationary form familiar from Kepler's 3rd law [16,31]

$$c^2 R = GM \tag{2}$$

over the huge radius $R = ct$ of the Universe at its current age t housing mass M. As usual, c is the speed of light and G is the gravitational constant. The energy density can be given equivalently to Eq. 1 by $\rho = c^2/4\pi G t^2$ [42]. In view of that c and G are not constants, but functions of the decreasing universal energy density [53,54]. The time-dependence is also contained in the unitary condition $c^2\varepsilon_o\mu_o = 1$ where permittivity ε_o and permeability μ_o are decreasing concurrently with increasing c.

According to the least-time principle a star or any other contemporary dissipative mechanism, just as any mechanism in the past, processes its bound quanta to free quanta or *vice versa* to attain thermodynamic balance with its surrounding energy density as soon as possible. This imperative is, of course, nothing but a restatement of Newton's 2nd law which says that the change in momentum $\mathbf{p}$ from one state to another directs along the steepest gradient in energy. In other words, evolution follows the net force, $\mathbf{F} = d\mathbf{p}/dt$ [44]. This means that on both the local and universal scales the densest regions, that is, the hottest loci are processed first and foremost. For example, today the biggest stars are the most luminous and short-lived. By the same token we reason that the nascent Universe, regardless of its mechanisms, were processing bound quanta to free quanta as soon as possible. Thus, via Eq. 2, the force of expansion is

$$F = \frac{Mc}{t} = \frac{c^4}{G} \tag{3}$$

and power $P = Fc = c^5/G$. In terms of Hubble constant $H = 1/t$, the rate of change $dH/dt = -1/t^2 = -4\pi\rho G/c^2$ decreases with decreasing energy density ρ, but since c and G are not constants, it is not so obvious how the universal acceleration $a_R = cH$, known also as the scale factor, changes over the eons. The scale-invariant imperative suggests that also the universal change follows mostly a power law. The universal acceleration due to the energy density gradient from the sparse present to the dense past, manifests itself, for instance, in the rotation and velocity dispersion of galaxies [31].

The least-time free energy consumption means that a gradient will level off at a rate that is proportional to the gradient itself. Put differently, among diverse mechanisms those that consume free energy in least time will be naturally selected by flows of energy themselves. By this percept matter aggregates to dissipative structures, such as stars and galaxies, to consume free energy. By the same token, even when not knowing the dissipative mechanisms that operated in the beginning, we expect the nascent Universe to have cooled quickest where it was hottest. Therefore, the Universe did process and continues to process, regardless of its mechanisms, toward high homogeneity on its largest scale. This least-time outcome does not necessitate thermal contact across the Universe or superluminal expansion. Moreover, there is no need fine-tune parameters or to specify initial-state characteristics.

3. Least-Time Decimation

Commonly the cosmic microwave background is regarded as a relic radiation of the earliest events in our sight [55]. Its temperature on average 2.725 K is highly uniform when galactic sources are masked and the dipolar gradient across the sky, inferred to display our motion along with the Milky Way, is subtracted from the map. Then only tiny variations with the root mean square variation of 18 μK are apparent [56]. These have been ascribed to quantum fluctuations in the nascent Universe that have by now blown up to patches that span on average angles just below 1°. The initial anisotropy is thought to have seeded galaxies. This current consensus about how CMB reports from the early evolution of the Universe is worth comparing with the perspective provided by the least-time principle. We exemplify this by a series of simulated temperature maps that mimic cooling from the initial state to the present state.

The initial state of the Universe we assume to have been isotropic, in a sense a condensate. If not, we would have to postulate some forces to account for non-uniformity. Either way being isotropic or anisotropic, as it will become apparent, the Universe will be subject to a long series of steps that will level off energy differences in least time.

Our initial map has a uniform temperature T_o across all indiscernible loci. At the first step our algorithm will chose one locus on the map among the identical loci where matter-bound quanta are pictured to break loose for free quanta that form space (void) for the first time. We model this first step of expansion by lowering temperature at the chosen locus to T_1, being a fraction of T_o, and set a linear temperature gradient radially across the whole map to the opposite pole that remains at the intact temperature T_o (Fig. 1). We model the second step of cooling down to T_2 to take place at the opposite pole, since it is then the hottest locus, and set the temperature gradient within a cone that opens up to an angle $\theta(T_2/T_o)$ as a function of the fraction T_2/T_o. Thereafter the hottest loci are at the equatorial zone where we model the third step of expansion to take place by lowering the temperature down to T_3 and setting the temperature gradient within the cone angle $\theta(T_3/T_o)$. Accordingly at each step i our algorithm will choose at random one among the hottest loci on the map and lower its temperature down to T_i and set the temperature gradient within the cone of radius $r_i \propto (T_i/T_o)^{1/3}$ to comply with the common equation of state for the energy balance $pV = nk_BT$ where n number of quanta with average energy k_BT generate a pressure p in a volume $V \propto r^3$ of radius r. In this manner the simulated series of steps mimics the least-time cooling by expansion that disperses from the first sequence of steps to parallel processes in numerous loci.

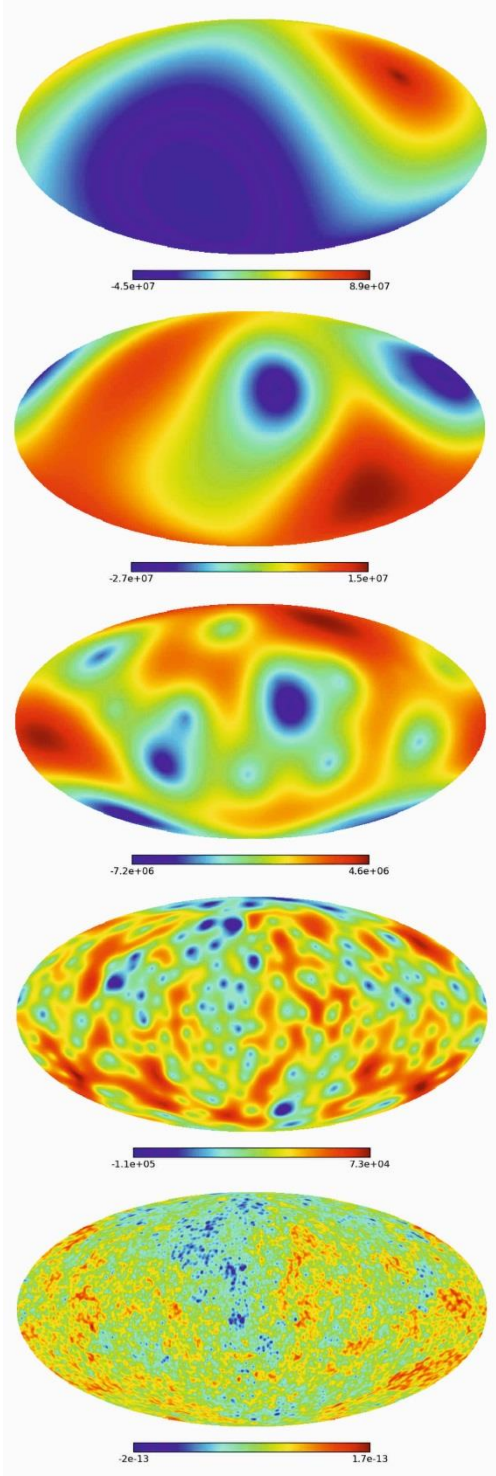

FIGURE 1. Full-sky maps of a simulated least-time expansion from top to down at the cooling step indexed with an increasing number n = {1, 4, 16, 256, 65536}. Temperature variation about the average, *i.e.*, anisotropy is shown on an arbitrary color scale. The least-time cooling yields maps that depend on the earlier maps, *i.e.*, show path-dependence, most notably the overall distribution at n = 256 is still discernable at n = 65536.

Our algorithm, essentially a for-loop of cooling steps, was written in Java to benefit from HEALPix software-package (version 3.20). The software for hierarchical equal area isolatitude pixelation of the sphere was specifically made to construct full-sky maps of the microwave sky for data analysis, simulations and visualization [57]. The simulated maps contained $12*2048^2$ pixels.

Parametrization of the least-time simulation, just as that of an inflatory model, dictates the overall dispersion of temperatures. However, when the rate of cooling is proportional to temperature, that is, at each step the hottest loci are cooled at first, then each step depends on all previous steps. Thus, the least-time cooling by expansion will result in history where low multipoles are correlated. Due to this path-dependence the observed alignments of low multipoles inferred from the WMAP and Planck satellite data [34,35,36] are no anomalies, but normal outcomes of the least-time decimation. In other words, when the "cake" is to be cut in pieces as soon as possible, lines of divisions are obviously not arbitrary but correlated. Also the quadrupole's low amplitude in the decomposed measured map appears to us a natural outcome because the step-by-step least-time cooling does not produce perfect multipoles. By the same token, the observed asymmetry in the average temperatures on the opposite hemispheres appears to us as a natural relic of the early steps. These primordial gradients across the Universe underlie subsequent steps.

The observed cold spot as deep as 70 μK and as large as 5° does seem like an anomaly or at least an unlikely feature when one is expecting normally distributed data with standard deviation σ = 18 μK on the angular scale of about 1° [37]. The cold spot is not an oddity according to the least-time decimation which invariably results in skewed distributions with fat tails. Our simulated maps of evolution down the finest details tail beyond 3σ. However, we emphasize that while this conclusion about the variance is consistent with the least-time characteristics [6], it remains only qualitative, because the simulation's parametrization does not match quantitatively the nascent expansion where the sequence of distinct initial steps soon disperses to numerous parallel processes throughout the Universe. Eventually, our serial CPU-algorithm could be converted to a parallel GPU-program to mimic the actual evolution of energy dispersal more faithfully and more effectively.

The peak power of CMB at the multipole 200, corresponding to the angle slightly below 1°, we take as a signature of a specific state along the evolutionary path where the early Universe had consumed all sources of free energy available to its nascent mechanisms. Then there were no more loci in the whole Universe that were dense enough for the primordial mechanisms to process for blazing radiation. In general this type of a stasis is common to evolutionary processes [6]. For example, when a galaxy has consumed all of its hydrogen in gas clouds, then at that stasis no more new stars will form. Thereafter, the galaxy still continues to evolve by other mechanisms, most notably by its black holes that devour matter, *i.e.*, free energy, from the dying stars. Likewise, we reason that the primordial mechanisms capable of operating at high densities were taken over by other mechanisms in the diluted conditions. Presumably this transition from the primordial to modern mechanisms happened when the early forms of matter had been broken down to quarks and gluons so that hadrons and leptons could form [49,50]. In general, a transition from one mechanism to another presents itself as a change in the power-law index [6,19].

We could not afford long enough simulations to demonstrate the least-time processing down to fine details well below the 1° degree cut-off. Instead, we had to terminate the least-time decimation at 4° (Fig. 1), and hence also our simulated power spectrum does not extend up to fine angular details (Fig. 2). Eventually, after copious processes, the whole map will be patterned by the cut-off spots. When the least-time decimation terminates at the sharp cut-off, even those unprocessed smaller regions will not be arbitrary in size, but display in excess characteristic sizes due to the closely packed cut-off spots. Similarly, circles that are packed in triangles, squares and pentagons leave smaller regions of certain sizes in between [58]. By the same token, the distribution of galaxies and the distribution of voids are necessarily in relation to each other.

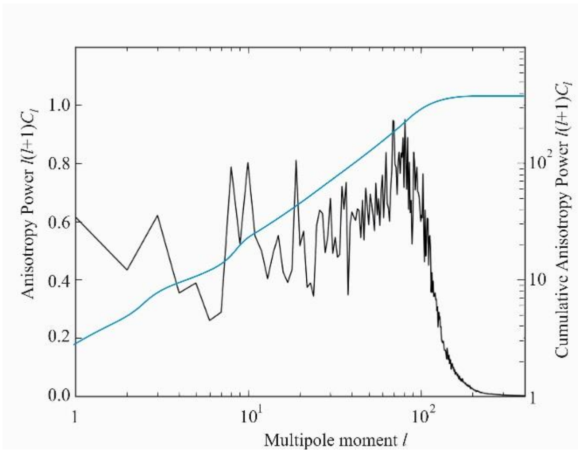

FIGURE 2. Angular power spectrum (black) of the simulated least-time expansion at the cooling step n = 65536. Due to our limited computational resources longer simulations below 4° resolution could not be executed. Thus, in our immature maps details at high multipoles are absent and low multipoles remain more pronounced than in the measured power spectra. The cumulative power spectrum (blue) on the log-log scale follows mostly a straight line until leveling off at the cut-off which is characteristic of the least-time free energy consumption.

4. Discussion

Physics is an empirical science, yet no observation means anything without some interpretation. At best an observation may exclude a theory, but in many cases measurements are too few and unambiguous to rule out one tenet or another. Consequently, a timely model that matches data tends to trump belated alternatives. Since data of the early Universe is scant, there is on one hand plenty of room for theorizing and on the other hand pressing need of a general theory that encompasses all there is. To this end we have introduced the principle of least-action in its original dissipative form. It accounts for the arrow of time, ubiquitous patterns and path-dependence that characterize the universal evolution [3,6]. Therefore when interpreting astronomical observations [16,19,32,42], the least-time principle serves as an alternative to the standard cosmological model that seems so sovereign today. Similarly, the steady-state model of the Universe contrasting the standard cosmological model pressed for more critical analysis of observations. Put differently, alternatives expose the character of a convention.

Clearly our interpretation of the CMB power spectrum by the least-time imperative is at variance with cosmic concordance that finds the first three peaks to report from geometry of the universe, baryon density and from the total density of matter as well implying a dark matter proportion [59]. According to the least-time principle there is no alternative for the flat geometry because the space constitutes from the quanta that break free from matter. In established parlance, the density cannot but match the critical density. By the same token, there is no room for dark energy and dark matter when using the least-time imperative to interpret CMB, galaxy rotation as well as bending and frequency shifts of light [16,32].

When summarizing differences to the standard cosmology, the high homogeneity of matter distribution on the largest scale and the uniformity of microwave cosmic background are found as inexorable outcomes of the least-time free energy consumption over eons rather than being innate and prevalent characteristics of the nascent Universe that should somehow survive and lens up as modelled by cosmic inflation [60]. Moreover, it is not perplexing that there are almost exclusively baryons and hardly any antibaryons, because the asymmetric matter-antimatter ratio is regarded merely as a handedness convention that facilitates free energy consumption just like any other standard in nature [49,50,61].

The again, not everything is different. Results of nucleosynthesis can be calculated as in the standard cosmology by equating the familiar expressions of density $\rho \propto t^2$ and $\rho \propto T^4$ to an approximate the relationship for time and temperature, i.e., tT^2 = constant. However, our objective is not to model but to explain. The explanation identifies the cause of expansion to the least-time free energy consumption that transforms the matter-bound quanta to the freely propagating quanta that embody the vacuum. This natural process yields the principle characteristics of the Universe, that is, flatness, high-homogeneity, baryon asymmetry, aggregation of matter and the low vacuum energy density, *i.e.*, the cosmological constant. The imperative accounts as well for the observed scale-free patterns, most notably power laws, whose changing slopes of lines on log-log plots relate to changing rates of free energy consumption. Such a change as far back in time as can be detected [45] could mark the end of the super-luminous primordial epoch that outshone all subsequent era.

In addition to the structures exposed by angular decomposition to multipoles, also rings have been spotted on the measured CMB maps, albeit their existence, origin and meaning have been questioned [62,63,64]. Our simulated maps obviously display rings (Fig. 1) because we explicitly modeled the diluting expansion in radial forms. Therefore, we cannot make any specific statement about rings in the measured maps. In general though, the inverse square law is a natural form for the least-time dispersal of energy, and hence various rings and circular forms are found all over in nature.

Our simulation, by reproducing the principle characteristics of CMB, suggests an evolutionary scenario where the early anisotropy resulting from the giant leaps of expansion in the beginning, or eventually an initial anisotropy of an unknown origin, was trodden down to almost undetectable low-multipole imprints by numerous smaller steps over the eons. Evolution of this kind can be modelled by considering the Universe as an expanding resonator whose energy density is decreasing step-by-step to lower and lower harmonics [65]. Then the flattening landscape of kinetic energy and dissipation can be indexed with phasors that span an algebraic number field and sum up as Riemann zeta function $\Sigma_1^\infty n^{-s}$, familiar from the spectral density integral. Its zeros correspond to the states of fixed energy. This unitary condition for a solution to exist dictates that the complex exponent's $s = \sigma + i\tau$ real part $\sigma = ½$.

Logically the least-time evolution, just as the standard cosmological model, implies that initially all quanta in the Universe were bound together. We do not know why the quanta in total amount to a staggering number $Mc^2t/h = c^5t^2/Gh = Pt^2/h \approx 10^{121}$ where Planck's constant is h, as usual. This number though is familiar from the huge ratio between the calculated and observed vacuum density [42]. Nevertheless, we reason that a smaller universe would have been too small to trigger the expansion, for example, recalling that many a reaction requires certain critical energy density to spark. Then again, a much bigger universe would have corresponded to an unlikely overheated initial state. Ultimately, after eons of combustion all quanta will be free to embody the extremely dilute and cold Universe. When all gradients have been consumed, any one locus will be indiscernible from another. In a sense, such a state will be a condensate.

When quanta are understood to embody everything, including space, also gravity is understood as a force, like any other [19,32,49,52]. It is an energy difference between the system of bodies and their universal surroundings. This means that gravity is an attractive force when the surroundings is sparse in energy density. Then a close-by body, *e.g.*, the Andromeda Galaxy, is inbound by emitting quanta, say, the paired photons from the gravitational potential between us and the body to the surrounding vacuum. Conversely, gravity is a repulsive force when the quanta from numerous sources in the Universe enter between us and a distant galaxy which then is outbound. In other words, gravity is also a manifestation of the least-time free energy consumption, not a phenomenon in its own right that would dictate the course of the Universe [32,52].

The principle of least action, albeit being the universal law, cannot exhaust all questions in accordance with Gödel's incompleteness theorems. For instance, the general evolutionary tenet cannot address what preceded the initial state and what will succeed the ultimate state. When there is no surroundings for the universal system, by definition, there cannot be any force either discernable to us as a cause for the birth and death of the Universe.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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