The substance of gravity

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Abstract: It is well established that a body of mass will bend a beam of light, but conversely does the beam of light displace the body? Our experimental result obtained with a torsion balance does not confirm the anticipated effect, but the negative result is in line with an elementary estimate that the effect is expected to be beyond our means of detection. Energy density of a power laser beam is minute relative to the universal reference energy density, known as the vacuum, and hence the gravitational force imposed on the body by light is well below the resolving power of our apparatus. We reason that gravity is mediated by photon pairs that embody the vacuum. Therefore, the vacuum exhibits electromagnetic characteristics, namely, permittivity and permeability. However, the physical vacuum embodied by photons is devoid of electromagnetic fields because the distribution of photon phases is even and random. This leaves the space only with the photon-embodied energy density differences which will manifest themselves as gravity. The energy density of free space maintains balance with the total mass of the Universe; and a local energy density, known as the gravitational potential, tends to be in balance with the body of mass. Accordingly, inertia is understood as a least-time reaction taken by the photon-embodied vacuum to restore the universal balance that has been perturbed by the body's change in momentum. © *2015 Physics Essays Publication*. [http://dx.doi.org/10.4006/0836-1398-28.2.208]

Résumé: Il est bien établi qu'un corps de masse courbeun faisceau de lumière mais inversement, peut-le faisceau de lumière déplacer le corps? Notre résultat expérimental obtenu avec une balance de torsion ne confirme pas l'effet prevu, mais le résultat négatif est en ligne avec une estimation élémentaire que l'effet est au-delà de nos moyens de détection. La densité d'énergie dans un puissant faisceau de laser est minuscule par rapport à la densité d'énergie de référence universelle, connue sous le nom du vide, et donc la force de gravitation imposée sur le corps par la lumière est bien en dessous du pouvoir de résolution de notre dispositif. Nous raisonnons que la gravité est secondée par des paires de photons qui incorporent le vide. Par conséquent, le vide présente des caractéristiques électromagnétiques, c'est-à-dire la permittivité et la perméabilité. Cependant, le vide physique incorporé par des photons est dépourvu de champs électromagnétiques en raison de la distribution de phase des photons, qui est uniforme et aléatoire. Il ne reste que les différences de densité d'énergie des photons qui peuvent se manifester comme gravité. La densité d'énergie de l'espace libre maintient l'équilibre avec la masse totale de l'univers; et une densité d'énergie locale, connue sous le nom de potentiel gravitationnel, a tendance à être en équilibre avec un corps de masse. Par conséquent; l'inertie est décrite comme le temps de réaction minimum pris par le vide, constitué par des photons, pour rétablir l'équilibre universel qui a été perturbé par le changement de la quantité de mouvement du corps.

Key words: Fermat's Last Theorem; Free Energy; Gravitation; Inertia; The Principle of Least Action; Vacuum.

I. INTRODUCTION

It is well known that a ray of light will bend when passing by a body of mass.[1–3] Conversely, will a body displace when a ray of light is passing close by? The latter, logically expected effect was recently sought for so that a high-power laser beam was pointed to pass close by a body suspended by a torsion balance.[4] To observe the body's deflection away from the balance's equilibrium position, when the light beam is switched on, seems like a straight-forward way to demonstrate the light-induced gravitational effect. However, the experiment is delicate. Just like in the original Cavendish experiment that yielded the strength of gravitational interaction between two bodies, the torsion balance is difficult to shield from disturbances, for instance, from thermally induced effects. In fact, as will be described below, we failed to prove the light-induced gravitational effect with our apparatus. Yet, the experiment inspired us to propose how photons interact with matter in gravitational terms. Our reasoning follows that of Newton who maintained that light and matter are fundamentally equivalent by asking, as if knowing:[5] Are not the rays of light very small bodies emitted from shining substances? and Are not gross bodies and light convertible into one another,…?

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Parallels between gravitational and electromagnetic interactions imply that they have a common origin.[6–8] Most notably, both gravitational and Coulomb potentials fall inversely with distance r from a source. Moreover, the squared speed of light relates via $c^2 = 1/\varepsilon_o\mu_o = GM/R$ electromagnetic characteristics permittivity ε_o and permeability μ_o of free space, known also as the physical vacuum, to the total mass M of the Universe[9] within its huge radius $R = cT$ at the age of T by the gravitational constant G. The energy density of free space is on the order 10^{-9} J/m^3. The value matches the average energy density of matter within the Universe.[9] Yet, it remains unclear, what exactly is the substance that associates both with gravity and electromagnetism. The ratio of gravitational and electromagnetic interactions relate to one and other by the ratio of the huge radius $R = cT$ of the Universe at the current age $T = 13.8 \times 10^9$ years and the tiny radius r_e of electron via

$$\frac{R}{r_e} = R\frac{m_e c^2}{\hbar c} = \frac{Gm_e M}{\hbar c} = \frac{4\pi\alpha\varepsilon_o}{e^2} Gm_e \frac{m_e R^2}{r_e^2} \Rightarrow \frac{R}{r_e} = \frac{1}{\alpha}\frac{e^2}{4\pi\varepsilon_o G m_e^2}, \quad (1)$$

where energy $E = \hbar c/r_e = m_e c^2 = e^2/4\pi\alpha\varepsilon_o r_e$ of an electron with wavelength $2\pi r_e$ and mass m_e is expressed in terms of the reduced Planck constant $\hbar = h/2\pi$ and the fine structure constant α. So, the ratio of the electrostatic to gravitational coupling constant is $\alpha/\alpha_G = e^2/4\pi\varepsilon_o G m_e^2 = 4.17 \times 10^{42}$. However, it remains a mystery what does the huge ratio of strengths signify.

The parallels between gravitational and electromagnetic interactions as well as the physical characteristics of vacuum indicate to us that the free space is, after all, embodied by quanta of light, i.e., photons. When space is understood as being embodied by a physical substance in the form of photons, gravity can be comprehended as a force, like any other force, as an energy gradient, i.e., an energy difference per length in the continuum limit, that exists between the energy density that is bound with a system of bodies, known as the local gravitational potential, and the energy density that embodies the free space, known as the universal gravitational potential due all bodies in the Universe. This free energy perspective on gravity allows us to evaluate not only whether, or not, to expect the body of a torsion balance to displace when a high-power laser light is passing close by, but also to address other gravitational phenomena.

II. HYPOTHESIS ON THE PHYSICAL VACUUM

Gravitational force is expected to be mediated between bodies by force carriers in analogy with electromagnetic force that is mediated by the photons, and in analogy with weak force that is mediated by $W^{\pm}$ and Z bosons as well as with strong interaction that is mediated by gluons.[10] The carrier of gravitational force has been coined as the graviton. However, it has not been discovered. The notion of gravitational force carrier implies some substance that embodies a local gravitational potential about a body and the universal gravitational potential, i.e., the free space known also as the physical vacuum, due to all matter in the Universe.

Although the idea of luminiferous aether as a physical medium which supports propagation of light is old and abandoned,[11] we will argue for the hypothesis that the photons do embody the vacuum, however, not in the form of plain light. Specifically, our hypothesis is that the vacuum is embodied by the photons which are on average in pairs of opposite polarization, and hence the photon pair does not display electromagnetic fields. In other words, we propose that the pair of photons with opposite polarization is the graviton.

At first sight, our hypothesis of the photon-embodied vacuum may seem bizarre because the space is not bright but dark. However, a second look at the renowned diffraction experiment, where two coherent rays of light combine with each other to produce a pattern of bright and dark fringes, suggests to us that at a dark fringe electromagnetic fields do indeed cancel each other, but the quanta of light themselves, when in pairs of opposite phases, do not vanish at the destructive interference for nothing but continue propagating. This revelation of interference implies that quanta are conserved.[12] It parallels the old atomistic tenet by regarding the photons as the basic building blocks of the Universe.[13] We motivate this tenet by a counterargument: if the photons were to vanish for nothing when interfering destructively, then the envelope curve of diffraction pattern should change with increasing distance from the slits to the canvas, because more and more quanta would have vanished in destructive interferes over an increasing distance to the canvas. Eventually, the pattern should disappear altogether because no photons from numerous destructive inferences would be left over in propagation to impinge on the canvas. However, no such decrease in the integrated intensity, as a function of increasing distance to the detector, has been observed. So, we conclude: the quanta of actions are conserved. This means that the total number of the quantized actions in the Universe is invariant. We obtain an estimate on the order of 10^{121} for the total number of quanta from the ratio Mc^2T/h of the total action $Mc^2T = c^5T^2/G$ within the Universe to the elementary action h using the renowned balance equation, i.e., the virial theorem, $2K + U = 0 \Leftrightarrow Mc^2 - GM^2/R = 0 \Leftrightarrow GM = c^2R = c^3T$ for the entire Universe. Just as Newton perceived,[5] the quanta of actions as the basic building blocks of everything are either bound in material bodies or free in propagation.[13,14]

Our reasoning that the photons embody the vacuum's energy density, so that the photons propagate on average in pairs of opposite polarization, is, to our knowledge, compatible with observations. The free space is dark and without net polarization just as the diffraction fringe is dark because the photons in pairs are on average with opposite phases. The free space, i.e., the space where no matter resides, when embodied by the photons in pairs with opposite polarizations, is devoid of net electromagnetic fields. Therefore, the vacuum has electromagnetic characteristics ε_o, μ_o and impedance $Z = (\varepsilon_o/\mu_o)$.

The photon pairs without net polarization do not manifest themselves as light, and hence this form of energy density in propagation is not as apparent and observable as the plain light, i.e., electromagnetic radiation in general. Yet, the photon-embodied vacuum manifests itself in many ways, most notably in gravitational terms. The photon pairs will by propagating carry energy and thereby will move readily to attain and maintain energetic balance within the Universe. In particular, the freely propagating energy density that embodies the physical vacuum will maintain balance with the universal energy density that is bound in all bodies. Therefore, the energy density of the free space equals the energy density of all matter in the Universe. By the same token, the energy density in the vicinity of a body, which houses bound energy, is higher than the energy density of the vacuum. This local increase above the background energy density of the vacuum is commonly referred to as the local gravitational potential.

The notion of photon-embodied physical vacuum appears to resolve many quandaries. For example, it allows us to perceive the double-slit experiment without conceptual conundrums of quantum mechanics. The interference pattern emerges because a photon or an electron or any other projectile in propagation induces perturbations in the photon-embodied vacuum density[15] and subsequently interferes with them. Likewise, the photon-embodied vacuum makes it easy for us to conceive that an applied vector potential imposed by a solenoid in Aharonov–Bohm experiment[16] will, in addition to the ubiquitous vacuum density, affect the interference pattern by increasing the energy density along the optical path. By the same token Casimir effect[17] can be understood so that a force between the walls of a tiny cavity arises from difference between the energy density in the cavity and the energy density in its surroundings. The cavity-confined density is in balance with energy density that is bound within the materialization of the cavity. Therefore, the energy density in the cavity differs from that of the surrounding free space, and that energy difference displays itself as a force. Furthermore, the Unruh effect[18] is understood so that a body accelerating with $a = dv/dt$ will experience counteraction by the vacuum. This will show up as an increase in the average energy $k_BT = \hbar a/2\pi c$. For example, an orbiter with integrated velocity $v = \int a dt$ over the period t of acceleration a will sense an increase in energy relative to the average energy of the vacuum $\langle E \rangle$ proportional to the velocity ratio $v/c = 4\pi^2 k_BT/\langle E \rangle$.

III. THE SUBSTANCE OF GRAVITY

According to our hypothesis, a local gravitational potential is a tangible physical substance embodied in the form of photon pairs without net polarization. Likewise, the universal gravitational potential is a tangible physical substance in the form of photon pairs that propagate throughout the Universe. In this way, gravity is understood as a force, like any other force, that is, an energy difference per length in the continuum limit between the local potential and the universal potential. Thus, these photon pairs as gravitons make the substance of gravity.

One body will be attracted to the other body, when the local energy density, i.e., the local gravitational potential which resides between the bodies, is higher than that of the surrounding space, i.e., the universal gravitational potential, that houses all bodies in the Universe. When the bodies are moving toward each other, the energy density that resides within the system of bodies, i.e., in the form of local gravitational potential, will escape as in the form of photon pairs to the sparser surrounding space. According to the principle of least action, the free energy in this form of energy density difference per length in the continuum limit will be consumed as soon as possible. Therefore bodies, such as falling objects, move along least-time trajectories. In other words, the bodies, just as rays of light, move along geodesics. The gravitational force when perceived as an energy density gradient will increase with decreasing distance r according to the familiar form $1/r^2$. This can be understood as follows. The photon-embodied energy density in the gravitational potential between the two bodies confines photon-embodied waves of various lengths. When the two bodies move closer and closer to each other, these waves bound within the system of bodies will, on average, become shorter and shorter, and hence their energy density will increase.[14] So, the energy density difference per length in the continuum limit between the local gravitational potential due to the system of bodies and the universal gravitational potential due to all bodies will increase, when the bodies approach each other. The photons pairs of shorter and shorter wavelength have higher and higher energy.

Conversely, when the surrounding energy density of the universal vacuum is higher than that local gravitational potential within a system comprising, e.g., two distant galaxies, gravity will manifest itself as a repulsive force.[14] The distant galaxies will move further apart from each other, because the energy density carried by the pairs of photons resulting from combustion of matter to freely propagating photons by stars, black holes, and other mechanisms in the vast Universe that surrounds the two galaxies exceeds the energy density within the system of two distant galaxies. For instance, the further away a distant galaxy is from the Milky Way, the faster it will recede away from us, because more and more photon pairs will enter from the increasingly younger, and hence, denser universal surroundings into the contemporary sparser space. Thus, the expansion of the Universe is merely a manifestation of the least-time quest of attaining the universal balance between bound and free forms of energy.

Likewise, when a body is lifted from the ground, the action is ultimately powered by photons that have been acquired from the energy-dense Sun. Therefore, this process, when the body is being lifted up, can be logically considered as a reverse reaction, where photons are acquired from the energy-dense surrounding source, to the reverse process when the body is falling down, where photons escape to the energy-sparse cold space. In other words, when the energy density of the local gravitational potential about a body as well as the energy density of the universal gravitational potential about all bodies is acknowledged as the photon-embodied substance, then gravity is not erroneously seen simply as an attractive force, but a force, like any other, whose direction depends on the sign of the energy gradient between the system and its surroundings.

This understanding of gravity yields the usual form of gravitational force $\mathbf{F} = -\nabla V$ when derived from the general principle of least action as follows. The variational principle in its original form[19–21] states that a change in kinetic energy $d_t 2K$ balances changes in the scalar $\partial_t U$ and vector $\partial_t Q$ potentials. The principle means for a system of body with mass m orbiting a central mass M_o with velocity v that[21]

$$d_t 2K = -\partial_t U + \partial_t Q \Rightarrow d_t mv^2 = -\partial_t \frac{GmM_o}{r} + \partial_t mc^2$$
$$= -\mathbf{v} \cdot \nabla \frac{GmM_o}{r} + \partial_t mc^2 \qquad (2)$$

since $\partial_t = \mathbf{v} \cdot \nabla$. The constant of gravitation $G = c^2 R/M$ relates the squared speed of light c^2, the current radius R, and the total mass $M = \int \rho_m 4\pi r^2 dr$ integrated over the radius R of the Universe, where $\rho_m = 1/2\pi G t^2$ is the average density of matter.[7,22,23] When Eq. (2) is re-expressed at any moment of time t using the mass–energy equivalence $mc^2 = hf$, the energy density around the local body M_o can be given relative to the universal surroundings in terms of the refractive index $n = c/v$

$$n^2 = \left(1 - \frac{GM_o}{c^2 r}\right)^{-1} \approx 1 + \frac{GM_o}{c^2 r}$$
$$\Rightarrow \left(n^2 - 1\right) mc^2 = \frac{GmM_o}{r}. \qquad (3)$$

Equation (3) shows that the energy density difference per length in the continuum limit between the local gravitational potential and the universal gravitational potential, i.e., the density of free space defined with refractive index of unity, is $U = (n^2 - 1)mc^2 = GmM_o/r$. The energy gradient per length in the continuum limit, i.e., the force, of this familiar form of gravitational potential will be used when the light-induced gravitational effect is analyzed in Sec. IV.

The photon-embodied vacuum energy density does not manifest itself only as gravity, but it will materialize itself in pair production of a particle and its antiparticle. Conversely, an annihilation process will contribute to the vacuum density by releasing pairs of co-propagating quanta without net polarization, in addition to the readily observable pair of photons that will depart in opposite directions to balance the momentum of a decay process.[13] Difficulties in detecting the pairs of photons without net polarization have apparently eluded us to perceive that space would be without any substance and also deluded us to think that the number of quanta would not be conserved. This error has caused, for example, misunderstanding about the nature of antimatter. However, when everything is described in terms of quantum of actions, the ordinary matter, where the proton carries a unit positive charge and the electron a unit negative charge, is understood to be merely a chirality consensus of quantized actions that constitute all particles.[13] Accordingly, antimatter merely stands for the opposite chirality consensus of quantized actions that constitute all antiparticles. In other words, antimatter did not disappear next to nothing in some fierce nascent annihilation, but in the proton and neutron, for instance, both left and right handed constituents, i.e., up and down quarks coexist. When the right- and left-handed constituents are equal in numbers the particle, for instance, the neutron is neutral. Accordingly, the particle which is its own antiparticle, such as the photon or Z-boson, has no net chirality, since one of its chiral constituents mirrors the other chiral form. Also a particle, such as neutrino, which is achiral altogether, is neutral.[13]

The notion of photon-embodied vacuum allows us to reconsider also the origin of electromagnetic potentials. In the vicinity of a charged particle, the photons that embody the vacuum will adapt both their density to the particle's energy density and their phases to the chirality of bound quanta that gives rise to the charge of the body. Therefore, the vacuum's distribution of photon phases next to the charged body deviates from the otherwise random, i.e., uniform and universal distribution which is without net polarization. Accordingly, the photon phase distribution adjacent to a net neutral particle does not deviate in total from the uniformity, but still displays itself in the form of anisotropy, e.g., as dipole moment of the neutron. The charge-induced uneven distribution of photon phases in the vacuum is customarily known as the electromagnetic potential.

We emphasize that the notion of photon-embodied vacuum does not mean virtual photons, but real photons. For example, when an atom is ionized, the electromagnetic potential does not materialize from nothing, instead the completely random and even distribution of photon phases will change to display net polarization in the vicinity of the charge. Since the vacuum is embodied by actual photons, it is only natural that the electromagnetic properties of free space relate to the speed of light. Since both the electromagnetic and gravitational potentials are manifestations of the photon-embodied vacuum, their functional $1/r$-form is necessarily the same, and their ratio is given by the electrostatic and gravitational coupling constants α and α_G.

IV. PROBING GRAVITATIONAL INTERACTION OF LIGHT WITH MATTER

Gravity, when understood as the energy density difference per length in the continuum limit between the energy density associated with a system of bodies in the form of local gravitational potential and its surrounding energy density in the form of photon-embodied vacuum, allows us to analyze also light-induced gravitational effects. For example, a system which comprises a beam of light and a body of mass should display light-induced gravitational effect on a body. The effect should be possible to observe in an experiment where the energy density in the beam of light supersedes the energy density of the surrounding vacuum at least by the amount that is needed for an unambiguous detection. The light-induced displacement of a body could be seen, for instance, as rotation of a torsion balance's lever arm in analogy of the Cavendish experiment where an introduced body with mass causes rotation of lever arm. While we do not question the light-induced gravitational effect itself, we suspect, based on an elementary estimate as elaborated below, that the corresponding force is way too weak to be detected, at least by the torsion balance at our disposal. It is capable of resolving a force only down to about 1 pN.

We used the PASCO scientific AP-8215A gravitational torsion balance.[24] It is designed to reprise the measurement of the gravitational constant, first performed in 1798 by Cavendish.[25] In this standard setup, two small tungsten balls, each of 38.3 g mass, are at the ends of a pendulum arm of 100 mm length which is suspended from a highly sensitive torsion ribbon. The torsion ribbon, made of beryllium copper, is approximately 260 mm long and its cross section is 0.017×0.150 mm. When two large masses, each 1.5 kg, are placed near the smaller masses opposed to each other, the gravitational force between the large and small masses is measured by observing the twist of the torsion ribbon. The centers of small and large masses can be placed no closer than 46.5 mm from each other. The ribbon's tiny twist caused by the gravitational attraction is accurately recorded by an optical lever arm produced so that a laser pointer's light is shone on to a mirror affixed to the torsion pendulum and reflected on a canvas few meters away. In this way, any torsion of the ribbon will be tracked and greatly amplified on the canvas as a motion of the laser pointer's spot of reflected light. The torsion balance provides, on one hand, an ingenious means of negating the otherwise overwhelming Earth's gravitational attraction, and on the other hand, a delicate counter force that will unwind a twist in the torsion ribbon back toward its free energy minimum state.

To begin with, we reproduced the standard measurement of gravitational constant as instructed by the manufacturer of the apparatus.[24] Initially, the balance was allowed to settle for an equilibrium position over night. Thereafter, the pendulum was brought into disequilibrium by revising the position of the large masses. The induced oscillatory rotation of the lever arm toward a new balance position was monitored for few hours from the movement of the laser pointer's beam that was deflected by the ribbon-affixed mirror on the canvas (Fig. 1). We attained the nominal 5% precision of the apparatus when it was set up on an optical table to reduce coupling to spurious vibrations in the environment. A video camera was used to record the position of laser pointer's bright spot on the canvas. Subsequently, to determine displacements, the spot's position was automatically tracked[26] from the series of video frames.

Second, we prepared for the actual experiment by removing the large masses altogether. Instead a beam of 10 W laser light with wavelength $\lambda = 532$ nm, generated by a light source (Coherent Verdi V10), placed on the optical table about 1.5 m away from the torsion pendulum was aligned by a mirror to go vertically, i.e., orthogonally respect to the lever arm, in front of one of the small masses. Thereafter, the beam was directed by another mirror to terminate at a beam stopper which was placed about 0.2 m away from the torsion balance to reduce convection of air caused by heating. The beam of a nominal area 1 mm^2 could not be aligned any closer than 15.0 mm from the center of the small mass.

Initially, the power laser was off and the pendulum was allowed to settle for an equilibrium position over night.

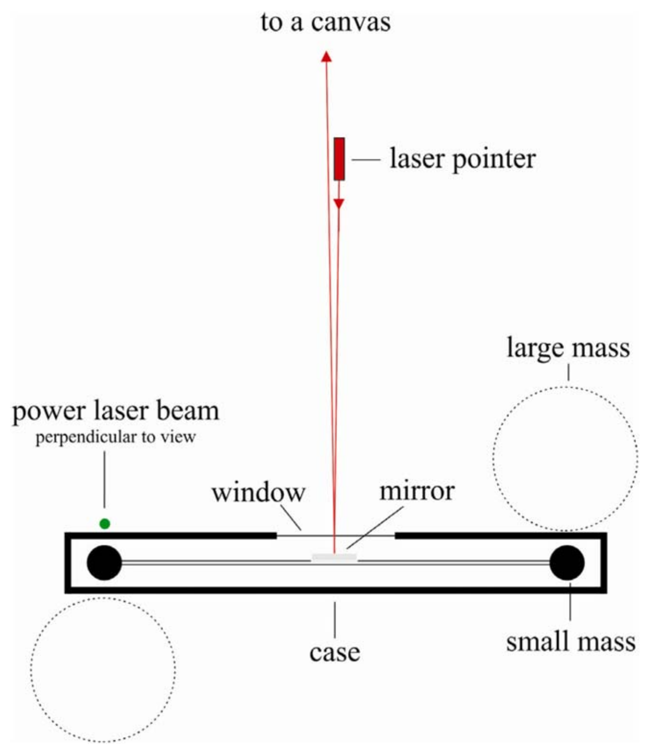

FIG. 1. (Color online) Top view of the reproduced Cavendish experiment. Two small bodies of masses at ends of a lever arm are suspended from the arm's middle point by a fine ribbon (along the viewing direction). The lever arm is closed in a case to shield the measurement from spurious ambient disturbances. When two large masses (dashed spheres) are brought next to the small masses on the opposite sides of the lever arm, the gravitational attraction results in a small torsion of the fine ribbon. The ribbon's twist away from the equilibrium position is seen as a deflection of a laser pointer's beam in a vertical direction (red) that is reflected from the mirror, that is affixed to the ribbon, onto a canvas (not shown) standing few meters away. When attempting to observe the light-induced gravitational effect on the small masses, suspended by the fine ribbon, the large masses were removed altogether, and instead a beam of power laser was pointed vertically, i.e., upward next to one of the small masses. The drawing has been adapted from Ref. 24.

Thereafter, the power laser was switched on. Indeed we began to observe the lever arm to oscillate, but the amplitude of swinging was about a factor of ten smaller than that we had recorded earlier in the course of the standard Cavendish experiment. Hence, the acquired data (Fig. 2) fell below the nominal precession of the instrument. The light-induced oscillations tended toward a new equilibrium position. However, that position indicated repulsion rather than attraction between the beam of light and the body of mass. Consequently, we revised the setup so that the beam of light was directed to go vertically behind of one of the small masses. The revised measurement displayed also light-induced oscillations that tended toward a new equilibrium position, but that position indicated attraction between the ray of light and the body of mass. Since the two experiments differed only in terms of whether the beam passed in the front or behind the small mass, yet yielded opposite results, we reason that the observed oscillations could not stem from light-induced gravitational interaction but were due to some other force that was imposed on the delicate apparatus when the power laser was turned on. Moreover, we noticed that it took a longer time for the oscillations to commence when the power

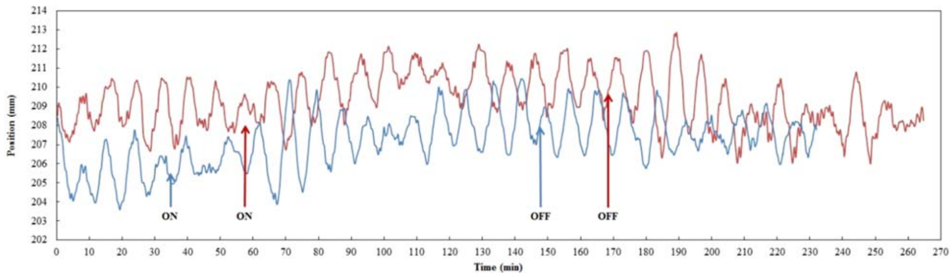

FIG. 2. (Color online) Laser pointer's beam position (mm) on a canvas recorded as a function of time during two experiments, upper and lower (red and blue) traces. The laser pointer's beam was reflected on the canvas from a mirror that was affixed to the ribbon of a torsion balance which stood on an optical table four meters away from the canvas. When the power laser beam was turned on (ON) to pass close by a small mass suspended by the torsion balance, the average position of the pointer's beam began to shift slowly, however, the change was small, about one order of magnitude less than the nominal precision (5%) specified by the manufacturer of the torsion balance. Conversely, when the beam was turned off (OFF), the average position of the pointer's beam began gradually to move toward the initial balance position. The data were extracted from video camera sequence of images, each having resolution of approximately 2 pixels/mm. The lever arm's oscillations were made more apparent by applying a five-point moving average over the low resolution data.

laser was turned on or off than it took when the positions of large masses were changed. This difference in the rate of recovery toward a stationary state implied to us that the observed effects in the swinging of the pendulum were not caused by gravity but induced by something else that we failed to exclude from affecting the measurement. In conclusion, we failed with our modified Cavendish apparatus to prove the gravitational interaction between light and matter.

An elementary estimate of the expected light-induced gravitational effect in our experiment is obtained from the energy density difference per length in the continuum limit, i.e., the force between the system and its surroundings. This force given by Eq. (3) yields the familiar Newton's form of the strength of gravity $F = GmM/r^2$ for a system consisting of a mass m and M that are separated by a distance r. This form asserts the negative outcome of our experiment when the energy E equivalent to mass $M = E/c^2$ is used for the energy in the ray of light when propagating near to the small mass as follows. The strength of force F between the beam of light and the body of mass can be estimated from the energy $E = Pt$ in the beam of power $P = 10\,\text{W}$ that passes by the body of mass $m = 38.3\,\text{g}$ at a distance $r = 15\,\text{mm}$ during the period of time $t = l/c$ when the photons are propagating over a distance l in the vicinity of the body

$$F = G\frac{mM}{r^2} = G\frac{mhf/c^2}{r^2} = G\frac{mPt/c^2}{r^2} = G\frac{mPl/c^3}{r^2}, \quad (4)$$

where G is the gravitational constant and c is the speed of light. The energy $E = hf$ in the beam of light with frequency f has been expressed using the mass–energy equivalence $M = E/c^2$. Since $F \propto 1/r^2$, we reason that the ray of light will exert most of its gravitational effect on the body, when propagating in the vicinity of the body. Therefore, we consider the effect only over a distance $l \approx r = 15\,\text{mm}$ which is, for the sake of simplicity, comparable to the distance r between the beam and the small body. We reason that this range of l is generous enough, since the gravitational force falls off as $1/r^2$. Then Eq. (4) gives an elementary estimate of about 10^{-35} N. Thus, the expected force is some 23 orders of magnitude less than that we could measure with our apparatus. So, we conclude that our apparatus is far from being sensitive enough to detect the light-induced gravitational displacement of a body.

It is worth noting that our estimate for the light-induced gravitational force is minute compared with the change in momentum $d\mathbf{p}/dt$, i.e., compared with the force that the beam of light with momentum $p = E/c$ can possibly bring about, e.g., by absorption. Specifically, the magnitude $F = dp/dt = d(E/c)/dt = P/c$ for light with power $P = 10\,\text{W}$ is on the order of 10^{-8} N. This value of force, in turn, is some four orders of magnitude larger the nominal sensitivity of our torsion balance. Therefore, it does not appear to us inconceivable that the torsion balance could, despite our efforts, couple to, e.g., thermally induced perturbations when the power laser is switched on and off.

V. THE NOTIONS OF INERTIA AND MASS

It may seem somewhat perplexing that gravity as a very feeble force is nevertheless so evident to us yet. In other words, what is so big that scales such a small force to become so apparent to us? By the same token, what is so universal that the gravitational and inertial masses are equal?

We argue that the photon-embodied vacuum as a tangible substance makes it easy to understand that changes in the body's state of motion will be met with resistance, known as inertia. When the body experiences a change in momentum, the vacuum density must move to restore balance with the energy density that is bound in all other bodies of the Universe. We reason that the vacuum density in the form of photon pairs, like energy density in any other form, will move down along energy density gradients according to the ubiquitous quest for attaining a free energy minimum state in least time.[19,20] Although it may appear at first sight that inertia would be an intrinsic property of a body, it is actually the vacuum density that endeavors to maintain and regain balance with the body's energy density and all other energy densities in the Universe.

When inertia is understood as the measure of vacuum's resistance to the change in the state of body, it follows that every body will endeavor to preserve its present state, whether it be at rest or moving uniformly along a straight line.[27] So, only an infinite homogenous medium would exert no effect on a moving body because in such a fictitious medium any position along the body's trajectory would be indistinguishable from any other position. Along such a steady-state trajectory, there would obviously be nothing to rebalanced and no inertial effects. But the Universe is heterogeneous. Therefore, any one body is ultimately moving relative to some other bodies, and hence inertial effects manifest themselves readily.

The photon-embodied vacuum density will maintain and retain balance between all bodies by moving at the speed light. So, a body will reside in a complete rest only in the center of the Universe where forces due to all other bodies cancel each other exactly, i.e., the resultant force is zero. Conversely, when the body is displaced away from the central position of balance, it will be subject to a density gradient due to its new position that is in an imbalance relative to other bodies. The ensuing energy density gradients will force the body to move toward a new balance position. This force manifests itself, for instance, when the body is rotating with velocity v at a distance r from the axis of rotation. It will experience a centrifugal force proportional to $(v^2/r)(R/c^2)$ due to all other bodies in the Universe of radius R, which are at any moment invariably distributed in an heterogeneous manner about the curved circular trajectory. The orbit at radius r is stable and stationary over a period of rotation t only when the centrifugal force due to all bodies totaling mass M in the Universe[28] is exactly balanced by a central force due to a body. This is, of course, the familiar case of planetary motion about a central body of mass M_o given by Kepler's third law, yet a form of the well-known virial theorem. This balance can be expressed [23]as the ratio $v^2r/c^2R = M_o/M$ of the local acceleration $a_o = v^2/r = GM_o/r^2$ and the universal acceleration $a = c^2/R = GM_o/R^2$.

Accordingly, the moment of inertia I is the measure of vacuum's resistance to the acceleration of a body with mass m about an axis of rotation. When the angular momentum of a constant magnitude $L = I\omega = mr^2\omega^2\tau = (1/2\pi)\int 2Kd\tau = n\hbar$, i.e., kinetic energy K integrated over an orbital period τ, is constant, there is no net flux of quanta from the system comprising an orbiter with mass m and angular velocity ω and a central body to the rest of Universe. As is expressed by Noether's theorem, this means that the number n of quantized actions, each having the measure of reduced Planck's constant ħ, is constant.[12] Yet, the state of an orbiter keeps changing along its closed trajectory relative to all other bodies in the Universe, and hence the vacuum density keeps moving to retain the balance. Therefore, the orbital motion displays the moment of inertia.

Since gravity and inertia are both manifestations of the vacuum's energy density, the gravitational mass and inertial mass are equal. Yet, to fully comprehend the nature of inertia, the notion of mass deserves to be explained. The starting point is the quantum of action. It parallels the old atomistic tenet where everything is ultimately composed of the same basic building blocks. Accordingly, also a particle totals an integral number n of quanta in an action,[12] $A = 2K\tau = \mathbf{p}\cdot\mathbf{x} = n\hbar$, where kinetic energy $2K$ is confined on the least-time period τ or equivalently momentum $\mathbf{p}$ is on the least-action path $\mathbf{x}$. The geometric notion of a quantized path

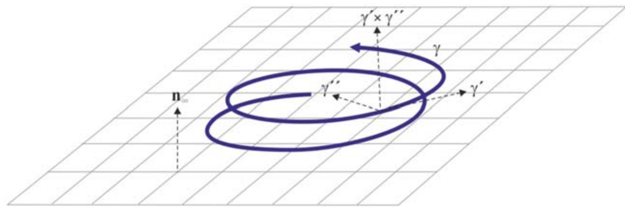

FIG. 3. (Color online) A curve γ, exemplified as a spiral (blue) can be compared at any given point to straight reference lines that span a plane with normal $\mathbf{n}$, in terms of a projection, known as geodesic curvature k_g, which is computed from the velocity γ' and acceleration γ'' of the curve. The sum of geodesic curvature over the curved path that is composed of quantized actions, corresponding to a particle, yields a characteristic quantity χthat is proportional to the mass of particle.

is familiar, e.g., from Taniyama-Shimura conjecture which says that every rational elliptic curve is a modular.[29] The action's curved geodesic can be characterized, according to Euler, by a quantity $\chi = \int k_g dg$ which sums up the geodesic curvature[30] $k_g = \mathbf{n}_\infty\cdot(\gamma' \times \gamma'')/|\gamma'|^3$ along the curved path γ characterized by the cross product $\times$ of velocity γ' and acceleration γ'', as a projection on a reference plane whose unit normal is $\mathbf{n}_\infty$ (Fig. 3). This connection between curvature and its characteristic quantity χ is also familiar from the theorem of Gauss and Bonnet.[31] The Euler characteristic χ essentially measures the effective curvature $1/r = \gamma''/|\gamma'|^2$ where γ'' is acceleration and γ' is velocity of the curved action that constitutes the particle. Physically speaking, the common reference frame with unit normal $\mathbf{n}_\infty$ is the universal vacuum embodied by the freely propagating photons. In other words, the integrated geodesic curvature, i.e., χ, quantifies how much the photons that embody the Universe within its huge yet a finite radius R will deviate from straight lines when adapting to the energy density bound in the body. The actual numeric value of mass m is obtained from r using $m/M = r^2/R^2$ which relates the mass of the particle with the mass of the Universe $M = \sum m_i$ due to all bodies m_i with the universal radius of curvature R defined by free photon paths. For example, the known mass of electron $m_e = 9.11 \times 10^{-31}$ kg and the known ratio $R/r_e = 6.34 \times 10^{41}$ give an estimate of the total mass $M = 3.66 \times 10^{53}$ kg of the Universe. It is consistent with the mass $M = 2c^2R/G = 3.50 \times 10^{53}$ kg and comparable to other estimates, approximately 10^{53} kg for the total mass of (ordinary) matter.[13,21,22,32–34] The ratio of a particle mass to the mass of the Universe can be understood so that inertial effects are frame independent, and hence equal.

Many modern theories of gravity involve high-dimensional spaces, and hence the norm of space, i.e., the universal reference substance is worth analyzing. Our analysis, as shown below, implies that the least-time paths of consuming free energy, i.e., geodesics, are natural trajectories for bodies as well as for rays of light. In other words, any trajectory in a putative high-dimensional space will be inferior to the least-time trajectories in our common conception of space and time.

The norm of photon-embodied free space is defined by the speed of light in the vacuum $\|c\| = (c^*c)^{1/2} = (c^2)^{1/2}$. The L^2-norm $\|x\|_2 = (\sum_i^n |x_i|^2)^{1/2}$ of the flat Euclidean space of axis x_i is, e.g., familiar from Pythagorean theorem $a^2+b^2=c^2$. This least norm compares with higher, $p>2$, norms $\|x\|_p = \left(\sum_i^n |x_i|^p\right)^{1/p}$ of curved L^p spaces according to[35]

$$\|x\|_p \le \|x\|_2 \le n^{1/2-1/p}\|x\|_p, \tag{5}$$

where the integer n denotes the number of base vector dimensions of space. The relation [Eq. (5)] means, e.g., that the familiar unit circle in a plane of two dimensions relates to the two-norm whereas a superellipse, as an $\mathbf{R}^2$-projection of a curve in the L^p space, relates to a higher p-norm ($p>2$). Therefore, the perimeter of the unit circle, in accordance with the least-time path, is always shorter than that of a superellipse which limits to a bounding square when $p\to\infty$. Thus mathematically speaking, when the mass of a particle, being proportional to χ, is measured, the corresponding curved path of the particle's action is projected from the L^p space onto the complex reference plane in the L^2 space that signifies the free space having the least norm. The projection of the action's path is an algebraic curve that is defined in homogeneous coordinates (a,b,c) by the Fermat equation $a^p+b^p=c^p$. When the equation is normalized by c^p, i.e., after homogenization, the polynomial equation of degree p in the affine plane is $(a/c)^p+(b/c)^p=1^p$. Since for all one-dimensional L^p spaces, the unit norm equals 1, i.e., is unitary, the relation between norms [Eq. (5)] for $n=2$ proves by

$$\|1\|_p = \|1\|_2 = \sqrt{c^*c/|c|^2} \le 2^{1/2-1/p}\left(\left|\frac{a}{c}\right|^p + \left|\frac{b}{c}\right|^p\right)^{1/p} \tag{6}$$

that there are no nontrivial integer solutions when $p>2$ for Fermat's last theorem

$$a^p + b^p = c^p. \tag{7}$$

Only when $p=2$, the factor $2^{1/2-1/p}$ in Eq. (6) equals 1, and hence corresponds to the Pythagorean theorem. Geometrically speaking, no curve in L^p, when $p>2$, will project onto L^2 a shorter or an equal circumference than that of the familiar unit circle. Physically speaking, the action of no particle will project exactly onto the action of photon in the free space. Not even an electron neutrino, that is described as a single quantum planar loop,[13] will map exactly one-to-one on the photon propagating in the universal surroundings which is characterized by the huge, yet finite radius of curvature $R=cT$ at its present age T and the speed of light c. Since mass characterizes the geometry of a particle relative to the geometry of the free space, it follows that the mass of a particle is not an invariant during the evolution of the Universe because the universal curvature of free space is flattening. Likewise particles, such as ultrahigh energy cosmic rays, originating from high-energy reactions will gain mass, i.e., energy on their way from nascent curvilinear ambiences of an active galactic nucleus to prevailing low-energy density flat surroundings of our observatories.[36] The same dependence of particle's properties on the surrounding energy density is witnessed in flavor changes of solar neutrinos as they traverse through Earth.[37]

VI. DISCUSSION

Our study aimed at observing a gravitational effect induced by light on a body, but we failed to prove it within precision of our apparatus. However, in need of understanding the negative outcome of our experiment we reached the old, yet meaningful conclusion by Newton, that gravity is a force like any other force, i.e., an energy density difference per length in the continuum limit between the system of bodies and its surroundings. So, the principle result of this study is the free energy perspective of gravity. The energy density both in a local gravitational potential and the universal gravitational potential, known also as the physical vacuum or free space, is embodied by pairs of photons in opposite polarizations. These pairs are the force carriers of gravity that move to even out the energy density differences in space.

Space is a pivotal notion in gravitational theories. The free space, i.e., the vacuum was perceived as a physical, fluidlike medium already by Newton[27,38] and later by Einstein[39] as well as by others.[40,41] The importance of understanding the free space as a tangible substance, rather than being accounted for by abstract notions such as curved space-time of general relativity or as fluctuations in quantum theory, has been emphasized rightfully.[42] In concord, neither we regard the vacuum as an abstract mathematical construct, instead we maintain that the photons embody the vacuum. Yet, the vacuum energy density does not manifest itself as light or as electromagnetic potential because the distribution of photon phases in the free space is even and random. In other words, the free space is dark because the photons on average pair with opposite polarizations, and hence no net electromagnetic fields and no net polarization. Nevertheless, the photon-embodied space manifests itself in gravitational and inertial effects. The vacuum density moves to maintain balance with all bodies in the Universe. Conversely, any one body, when displaced from balance, such as a bucket of spinning water,[27,28,33] will be subject to forces, i.e., to the energy density differences due to all other bodies in the Universe. These inertial forces are communicated with high, yet finite speed of the photon pairs in propagation. In other words, the space does not exist independently of matter but is embodied by the photons that were processed free from matter by stars and other mechanisms.[33] Thus, during the evolution of the Universe, the vacuum energy density maintains by diluting the balance with diminishing material forms of energy densities.

According to this physical portrayal of space, the substance of gravity is the photon-embodied energy density in the form of local and universal gravitational potentials. Hence, gravity can be understood simply as a phenomenon, as any other, where bodies move to diminish energy gradients. Since a local gravitational potential is in a local balance with a body and the total density of the photon-embodied free space is, in turn, in universal balance with all bodies in the Universe, any perturbation away from the balance will cause flows of energy, i.e., net propagation of photon pairs from a locus to another. In accordance with Fermat's principle, the free energy will be consumed in the least time. The equation of motion for the flows of energy embodied by the photon pairs is the principle of least action given in its original form [Eq. (2)].

The old flow equation [Eq. (2)] can be shown to be equivalent to Newton's 2nd law of motion in its original complete form $\mathbf{F} = d_t\mathbf{p} = m\mathbf{a} + \mathbf{v}d_t m$ by multiplication with velocity $\mathbf{v}$ and identification of change $d_t m$ in mass as dissipation to the free space characterized by the squared speed of light c^2 via $v^2 d_t m = (v^2/c^2) d_t E = d_t Q$. Since the dissipative evolution from one state to another consumes its driving forces, the equation of evolution [Eq. (2)] cannot be solved, e.g., by separation of variables. When there are two or more degrees of freedom for the flows of energy, the problem becomes intractable. This characteristic is familiar from the three-body problem.[43]

The least-time imperative accounts also for homogeneity of the Universe at the largest scale as well as for observations that are customarily addressed by general relativity including those that seem to require notions of dark energy and dark matter.[33,44] Moreover, the photon-embodied space resolves difficulties in comprehending coherence, e.g., between optical paths of interferometer, which is customarily ascribed as a peculiarity of quantum mechanics.[15] Also the notion of time as the component of an evolving space should not be regarded as an abstract construct but embodied by the quantized actions that are either emitted to or absorbed from its surroundings by the system. So, a flow of energy from the system to its surroundings or vice versa is invariably coupled with a flow of time since both energy E and time t are attributes of the quantum of action in its invariant measure $h = Et$ known as Planck's constant.[45–47]

The photon-mediated gravity is by no means a new idea, but it has for long been thought as being inconsistent with observations. For example, already Newton rejected light as waves propagating in some universal medium because such a medium would disturb and retard the motions of those great bodies. Yet, the planets have seemingly stationary orbits. However, here we argue that indeed there is no medium that would support photon propagation, instead the photon pairs without net polarization in propagation themselves constitute the medium known as the vacuum. So, if there was a net loss of energy over an orbital period, it would indeed indicate that the body was not at a stationary-state balance with its surrounding energy density, i.e., the density due all other bodies in the Universe. Accordingly, any perturbation of a system away from the energy density balance with its surroundings would inflict a reactive force, i.e., a density difference that would result in opposing flows of energy when restoring the balance. In other words, the system at a stationary state, e.g., the planet on its orbit, just as a chemical reaction equilibrium, is thermodynamically stable state according to Lyapunov criteria.[48]

Our proposition of the photon-embodied space is falsifiable. As far as we can judge the tenet is in concord with experiments that have already been carried out. Since co-propagating photons, when in pairs of opposite polarization do not couple to charges, the finite energy density ought to manifest itself in interference phenomena and its perturbations ought to manifest themselves as gravitational waves. In general, interference phenomena are familiar from slit experiments that give rise to diffraction patterns as well as from interferometry that produces fringes. In these contexts, it has customarily been seen particularly puzzling how the pattern of bright and dark bands can possibly emerge even when photons or other projectiles, such as electrons, are shot so infrequently that at any time there is only one in propagation. Here, we argue that the projectile is not propagating in emptiness but in the photon-embodied vacuum, and hence the photons that embody the vacuum will also contribute to the interference pattern. So, when the vacuum is understood as a tangible substance, there is nothing mysterious about interference that would have to be described by quantum mechanics.

Perturbations in the vacuum density emerge, for instance, from annihilation processes where pairs of photons in opposite polarization are released along with readily observable photons. Therefore, we reason that the amplitude of a density wave originating from such a process is proportional to the concurrent luminosity. So, we expect that difficulties in detecting a temporal increase in the vacuum density due to an incoming gravitational wave are comparable to those we faced in our torsion balance experiment. Although current means of interferometry[49] enable detection of changes in the optical path down to 10^{-22}, it is not obvious to us, which events would be of both sufficient amplitude and brief enough to cause such a measurable path difference along the two arms of an interferometer. For example, although quasars are the most luminous, powerful, and energetic objects known in the Universe, they are billons of light years away from us. So, any perturbation originating from a very distant process has spread, dispersed and shifted down in frequency, and hence would be well below current detection limits for energy density waves. Moreover, no quasar is known to have a periodicity less than an hour which is well beyond the time resolution provided by the two arms of any ground based observatory.

Much closer, in the center of Milky Way there are massive orbiters that spiral toward a central mass and emit light as well as are expected to produce density waves. Nonetheless, we think that the amplitude of such a density wave and its slow temporal characteristics are insufficient for detection. Also, it does not seem apparent to us that gravitational shielding[50,51] would provide superior means of detection. Finally, considering the dynamic Casimir effect,[52] there are perhaps some possibilities to convert a density wave to a readily observable light wave.

Surely, one may argue that gravitational waves have already been detected. Namely, the Hulse–Taylor binary pulsar's orbital period is decaying in agreement with the predicted loss of energy due to gravitational waves.[53] Also the BICEP2 telescope has revealed B-mode polarization in the cosmic microwave background that was hastily ascribed to primordial gravitational waves.[54] We are not in position to question these observations were waves of light bring us signals from distant gravitational phenomena in the past. On the contrary, we reason that in many changes of state, in addition to the pairs of quanta with opposite polarization, also readily observable single quanta of light are emitted. Yet, to capture the energy density waves of paired quanta, the actual carriers of gravity, would first meet our criterion for detecting a gravitational wave.

Since we argue that gravity is mediated by the pairs of photons in opposite polarization, it is of interest to examine whether to expect gravitational analogies to optical phenomena of reflection, refraction, polarization, and eventually shielding. In general, the pairs of photons in opposite polarization, and hence without net electromagnetic force and net polarization, do not couple to charges. Therefore, it is not easy for us to conceive interfaces to reflect, refract, polarize or shield single photons. The gravitational force carriers as the photon pairs follow the geodesics, i.e., the least-time paths that are governed exclusive by energy density gradients. These energy gradients display themselves primarily as gravitational lensing.

Our doubtful views about the feasibility to observe directly gravitational waves undoubtedly reflect also our insufficient knowledge of both detection techniques and potential sources of gravitational waves. Yet, our understanding of the substance of gravity as the photon-embodied energy density in the form of local gravitational potential about a local system of bodies as well as in the form of universal gravitational potential, known as the vacuum energy density, is a falsifiable conjecture. It has already been assessed by reanalyzing and reinterpreting old experiments and observations, namely, the double-slit experiment and action at a distance[15] as well as bending of light, redshift, time dilatation,[44] perihelion precession,[23] and the gravity probe B data.[21]

Finally, it is worth emphasizing that the outcome of any experiment is always subject to some theoretical interpretation, and hence no single observation is an independent proof of a theory. Yet, a single observation can prove a theory wrong. In Einstein's words: Whether you can observe a thing or not depends on the theory which you use. It is the theory which decides what can be observed. No amount of experimentation can ever prove me right; a single experiment can prove me wrong.

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