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Least-action perihelion precession

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ABSTRACT

The perihelion precession of Mercury is reinspected using the original form of the principle of least action á la Maupertuis. The anomalous advancement of the apside line that is customarily accounted by the theory of general relativity is ascribed here to the gravitational effect due to the entire Universe. When the least action is written in the reference frame of the Sun, the residual precession in the planet's orbital motion can be related to all bodies that are dispersed throughout the Universe. The universal potential energy, just as a local potential, contributes to the precession in accordance with the virial theorem.

Key words: gravitation – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The theory of general relativity triumphed by providing a numerical value for the anomalous advancement of Mercury perihelion in excellent agreement with observations (Einstein 1916; Chow 2008). The residual rotation of the line of apsides was ascribed to the space–time that curves about the Sun (Berry 1974; Narlikar 1993). However, for a layman the successful calculation, as such, does not disclose what exactly the curved space–time means.

Of course, conceptual conundrums of general relativity do not only trouble the perihelion precession, but also involve other phenomena too, e.g. bending of light that is passing by a massive body (Narlikar 1993; Will 2006), ticking of a clock that is subject to acceleration (Resnick 1968) and increasing angular size of a galaxy that is ever further away from us (Narlikar 1993; Raine & Thomas 2001). Yet, the perihelion precession as a seemingly simple process prompts us to ask what will *cause* the planet to lengthen its path past the exact closure of an elliptic orbit.

Surely, the anomalous part of the precession can be calculated from the Schwarzschild solution of the Einstein field equations (Weinberg 1972), but then our naïve request to understand the physical reason of perihelion precession will not be met. For us a causal connection would entail a flow time (Tuisku, Pernu & Annila 2009). To look for some irreversible process in the context of a seemingly stationary motion may not appear well motivated. Yet our call for comprehension is perhaps not entirely obsolete because at least the expanding universal setting is not at the equilibrium. In other words, the universal energy density is diluting, hence not zero, and ensuing effects are worth considering.

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2 CONTRIBUTIONS TO THE LEAST-ACTION ORBIT

Mercury's perihelion rotates 5600 arcsec per century in reference to the Earth's equinox line (Brown 1999). The rotation of our reference frame is 5025 arcsec and the gravitational tug of the other planets contributes 532 arcsec per century so that their sum 5557 arcsec falls short by 43 arcsec from the measured value (Clemence 1947). This anomaly is attributed by general relativity to space–time that curves due to the presence of mass, here most notably due to that of the Sun. Since the other planets also contribute, it seems logical to us that the rest of the Universe must contribute to the precession too. Thus, we regard the curved space–time as a mathematical explanation that effectively sums up gravitational effects due to all bodies in the Universe (Fig. 1). This logic of reasoning is in a sense sound but it would be an unorthodox resolution since, according to the general relativity, the curved space–time, i.e. the metric, gives rise to gravity (Wheeler 1990). Yet, is our direct way of perceiving the curved space–time as physical entirely wrong?

At first it may seem like a formidable task to account accurately for the total inertia due to some 100 billion stars in the Milky Way (ESA 2011) as well as all those other stars in some 170 billion other galaxies that the Universe is estimated to house (Gott et al. 2005). But formally the inertia $I = \Sigma m_i r_i^2$ due to all masses m_i at center-of-mass distances r_i,

$$I = \sum_i m_i r_i^2 = \frac{\sum_{i<j} m_i m_j r_{ij}^2}{\sum_i m_i} = \frac{M_{\text{o}} m r^2}{M} + \cdots, \tag{1}$$

can be written in terms of distances r_{ij} between any two bodies m_i and m_j in the Universe that total the mass $M = \Sigma m_i$. In particular, the term involving the inertia $I = mr^2$ of the planet of mass m orbiting at a distance r from the Sun of mass M_{o} can be singled out from the

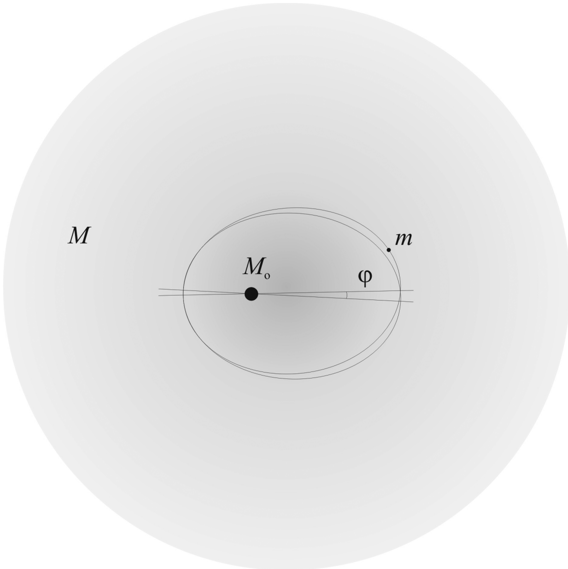

Figure 1. Mercury (m) is on an elliptic orbit about the Sun (M_o). The line of apsides advances about the Sun at a gradual rate $d_t\varphi$. The gravitational tug of the other planets contributes to the precession 532 arcsec per century, whereas a residual of $\varphi = 43$ arcsec is described as a relativistic effect due to the curved space–time. However, it is reasoned here that, besides the Sun and its planets, the residual precession stems from the gravitational effect of all other bodies that totals the mass (M) of the Universe.

huge series to see that it, just as any other term involving a pair of bodies, is in relation by M to everything else in the Universe.

The residual precession φ can be identified by the principle of least action á la Maupertuis to stem from all other bodies in the Universe (De Maupertuis 1744; Kaila & Annila 2008). The original and general form of the least-action principle applies to both evolutionary and stationary systems. In contrast, the standard Lagrangian implies by its conserved form a stationary setting. However, the Universe is not at the equilibrium but expanding. Its diluting energy density is small but still finite, and therefore the Schwarzschild solution of the Einstein field equations in the zero-density vacuum is not warranted.

When the general form of the action principle is used, kinetic energy $2K = I\omega^2$, given in terms of inertia I and angular velocity ω, will be integrated over time t. Thus, according to the variational principle the orbit will be optimal when $2K$ is a constant integrand of the action $\int 2K\,dt$. This least-time criterion means that the action of the planet is stationary, so that its orbit advances a full precession period in $\tau = 1/\omega = 1/2\pi f = t/2\pi$:

$$I\omega^2 = mr^2\omega^2 = (2\pi)^2 m\frac{v^2}{c^2}c^2 = \frac{2\pi^2 GM_o}{c^2 r}mc^2$$
$$= (2\pi)^2\frac{M_o}{r}\frac{R}{M}mc^2 = \varphi mc^2, \quad (2)$$

where the kinematic equation $r = \frac{1}{2}a_o t^2$ for the local acceleration $a_o = GM_o/r^2$ (Breithaupt 2008) is used to relate velocity $v = r/t$ of the planet to the mass M_o of the Sun via the constant of gravitation G. Likewise, $R = \frac{1}{2}aT^2$ for the universal acceleration $a = GM/R^2$ is used to relate the speed of light $c = R/T$ to the mass M of the Universe that has expanded during time T up to its present radius R. As usual, the semimajor axis r of a circular or elliptical orbit is related via Kepler's third law $t^2 = (2\pi)^2 r^3/GM_o$ to the orbital period t of a small body with mass m that revolves about the central body with mass M_o.

The obtained formula $\varphi = (2\pi)^2 M_o R/Mr$ for the anomalous advancement of apsides places the local energy density due to the Sun in relation to the energy density due to all bodies in the Universe. The result can also be stated so that the advancing arc $r\varphi$ of the perihelion precession about the Sun makes the fraction M_o/M of the universal arc $R\Phi$. In this way it becomes explicit that the average energy density of the Universe due to all its bodies, just as any local potential, contributes to the kinetic energy of the planet in accordance with the virial theorem.

The obtained value $\varphi = 5.035 \times 10^{-7}$ rad, equivalent to 43.09 arcsec per century, is in excellent agreement with observations 43.1 ± 0.5 arcsec (Clemence 1947). Numerical values of Mercury's semimajor axis $r = 5.79 \times 10^{10}$ m and the Sun's mass $M_o = 1.989 \times 10^{30}$ kg (Jet Propulsion Laboratory 2011) were used in equation (2).

3 THE MASS OF THE UNIVERSE

The residual part of the perihelion precession (equation 2) can also be written as the energy ratio

$$\varphi = \frac{d_t L}{Q} = \frac{(2\pi)^2}{2}\frac{GmM_o}{r}\frac{1}{mc^2} = -\frac{(2\pi)^2}{2}\frac{U}{Q}$$
$$= (2\pi)^2\frac{K}{Q} = (2\pi)^2\frac{\frac{1}{2}mv^2}{mc^2} \quad (3)$$

of the Sun's gravitational potential $U = GmM_o/r$ and the planet's energy $Q = mc^2$ given in energy equivalents that characterize the universal surroundings, i.e. the free space. Alternatively, when using the virial theorem $2K + U = 0, \varphi$ can be given by the ratio of kinetic energy $K = \frac{1}{2}mv^2$ in the orbital motion and the dissipation $Q = mc^2$ to the ultimate sink of the free space. In terms of thermodynamics the energy ratio relates a planet's revolution to the energy in the universal surroundings in the same way as the maximum revs of an engine relate to temperature of the surroundings into which the engine exhausts its waste heat (Kittel 1969). Likewise, a temperature gradient between a warm ocean and cold atmosphere is a non-equilibrium setting that powers a whirling hurricane (Emanuel 1994). When the residual perihelion precession is ascribed to result from the all around hovering energy density, so-called vacuum density, the measured value of φ together with M_o and R can be inserted in equation (2) to calculate the mass of the Universe $M = 3.488 \times 10^{53}$ kg.

The energy density of free space displays itself in the cosmic background radiation photon temperature, at the moment $T = 2.725$ K, as well as in the squared speed of light $c^2 = (\varepsilon_o\mu_o)^{-1}$ and the invariant squared impedance $Z^2 = \mu_o/\varepsilon_o$ characterized by permittivity ε_o and permeability μ_o. The non-zero energy density manifests itself also in the Casimir effect (Casimir & Polder 1948) and Aharanov–Bohm experiment (Aharanov & Bohm 1959).

Moreover, the tiny but non-negligible universal acceleration $a = GM/R^2 = 1/\varepsilon_o\mu_o R = c^2/R = c/T$ results from the energy density difference between the energy density that is still bound in the total mass M of the Universe and 'the zero-density surroundings'. This energy gradient is diminishing at the rate $H = 1/t$. Today, after some $T = 13.7$ billion years of diluting combustion, the radius $R(t)$ is huge (1.295×10^{26} m). So the corresponding curvature is small but, when ignored, its effects, such as the perihelion precession, will be regarded as anomalous. The rate $H = d_t a/a$ is changing as the scalefactor $a(t) = \sqrt{c^2\varepsilon\mu}$ (Weinberg 1972) is stretching from the reference index of refraction $n = 1/a(T) = 1$ defined by the present-time permittivity and permeability. The universal curvature tends towards complete flatness as the combustion of mass to free photons will exhaust all repositories of bound quanta.

The contribution of the universal energy density to the perihelion precession can be calculated by integrating the mass density ρ from the present moment $t = 0$ back to $t = T$ that marks the nascent Universe. The density falls with t so that the more distant an object is, the less it will contribute to Mercury's motion. Ultimately, the nascent Universe that is receding from us at the speed of light will cause no effect at all. The calculation enclosing the entire Universe is facilitated by the cosmological principle, i.e. ρ is homogenous at the largest scale. However, here the uniformity is not taken as granted but understood as a consequence of the principle of least action. The least-time energy dispersal entails that diverse sources of radiation, such as black holes, stars, planets and gaseous clouds as well as entire galaxies, are dispersing from each other further and further apart so that the rate

$$\mathrm{d_t L} = \mathrm{d}_t(2Kt) = \mathrm{d}_t(Mc^2 t) = \mathrm{d}_t(Mr^2\omega) \\ = r^2\omega \mathrm{d}_t M + M\mathrm{d}_t(r^2\omega) \quad (4)$$

is maximal. The non-conserved first term in equation (4) denotes by $\mathrm{d}_t M$ the combustion of energy that is bound in the form of mass to freely propagating photons. Thus, the mass of the Universe is not a constant, but the combustion powers the expansion of the Universe. The conserved second term is familiar from Kepler's second law where the specific relative angular momentum $r^2\mathrm{d}_t\theta$ is a constant of motion, e.g. so that the line from a planet to the Sun sweeps out equal areas during equal intervals of time of rotation $\omega = \mathrm{d}_t\theta$. The term $\mathrm{d}_t(r^2\mathrm{d}_t\theta)$ vanishes for a constant angular acceleration (Alonso & Finn 1983).

In the quest of consuming free energy in the least time, the flows of energy will naturally select the most effective means and mechanisms (Sharma & Annila 2007; Annila 2009). Thus, also accumulation of matter to stars, galaxies and group of galaxies is regarded merely as a way to progress towards the equilibrium because this process generates mechanisms that will transform energy bound in mass to freely propagating photons. The irrevocable processing of bound quanta that embody the loci of space also implies that time is physical. In other words, the irreversible combustion of bound to free photons $\mathrm{d}_t M/M = \mathrm{d}_t r/r = H = 1/t$ is seen to generate the universal arrow of time (Tuisku et al. 2009).

At any moment of time and in any place of space, the combustion $r^2\omega\mathrm{d}_t M$ will be maximal when on the average the mass density $\rho = 1/2\pi G t^2$ falls from any point inversely proportional to the square of the distance r. Thus, the least-time expansion results in the uniform distribution of energy at the largest scale. Anisotropy in the cosmic background radiation is minute (Wright 2004). This is to say that kinematics succeeds in positioning each and every locus of space very close to a center of the Universe. For example, our Local Group of galaxies is moving at a moderate speed relative to the universal reference frame of the cosmic microwave background (Kogut et al. 1993).

When ρ is taken homogenous at all scales, the total mass of the expanding Universe will be obtained by integration of the mass density from $r = 0$ to the radius $R = cT$:

$$M = \int_0^R \rho 4\pi r^2\,\mathrm{d}r = \int_0^R \frac{1}{2\pi G t^2} 4\pi r^2\,\mathrm{d}r = \frac{2c^2 R}{G}. \quad (5)$$

The result is consistent with equation (2). The geometry of the expanding Universe can also be written so that ρ relates to the area spanned by the angle of observation θ that contains the Universe at a past moment (Fig. 2). Then the integration is over a spherical cap of height $h = r[1 - \cos(\theta/2)] = 2r\sin^2(\theta/4) = 2r/t^2$ up to the most distant past $T = R/c$:

$$M = \int_0^R \frac{1}{2\pi G} 2\pi r h\,\mathrm{d}r = \int_0^R \frac{1}{2\pi G} 4\pi r^2 \sin^2(\theta/4)\,\mathrm{d}r \\ = \int_0^R \frac{4\pi r^2}{2\pi G t^2}\,\mathrm{d}r = \int_0^T \frac{2c^3}{G}\,\mathrm{d}t = \frac{2c^3 T}{G}, \quad (6)$$

where $r\sin(\theta/4) = r'$ can be regarded as the radius of the Universe at a moment t back in time. A view of sky captures a connected 3-manifold (e.g. a spherical cap) that is homeomorphic to the 3-sphere at a moment back in time. The opening angle θ is decreasing with increasing t and, ultimately, when $t \to \infty, \theta \to 0$. For example, a distant object such as a galaxy, when viewed from the Earth, spans an arc $r\theta$ that is decreasing monotonically with $r = ct$. This is in contrast to Lambda cosmology, where objects with increasing redshift beyond $z \approx 1.5$ would appear larger and larger (Mattig 1958; Raine & Thomas 2001).

The light that was emitted at a frequency f_e to energy-dense surroundings at a moment t_e back in time distributes its energy on a longer and longer period, i.e. shifts red during its passage to the contemporary sparse-energy density. Thus, while the energy is conserved ($2K = mv^2 = hf_\mathrm{o}v^2/c^2 = hf_\mathrm{e}$), the redshift $z = f_\mathrm{e}/f_\mathrm{o} - 1 = n^2 - 1$ results from the decreasing squared index of refraction $n^2 = c^2/v^2 = \varepsilon\mu/\varepsilon_\mathrm{o}\mu_\mathrm{o}$ from the increasing radius of curvature due to the dilution of the energy density of free space. This understanding of the Doppler shift is in agreement with Planck's law which says that on the average at any moment and at any place the energy bound in matter and the energy contained in freely propagating photons are in balance with each other.

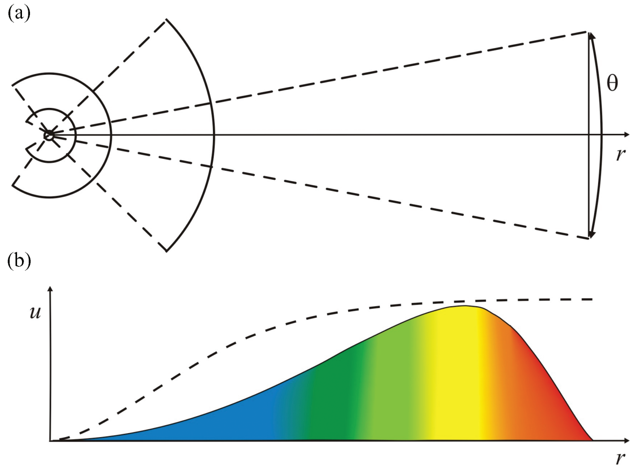

Figure 2. (a) Geometry of the expanding Universe is such that the area spanned by the angle of observation θ, when viewed from any locus of space, contains the Universe at the moment t back in time at the radius $r = ct$. The present moment $t = 0$ and $r = 0$ corresponds to $\theta = 2\pi$, whereas the nascent Universe at $T = 13.7$ billion years back of radius $R = cT$ corresponds to a tiny angle $\theta = 2.3 \times 10^{-18}$ rad. Actually, the same spot of the nascent Universe is seen in every direction. However, nothing can be observed from this ultradeep field spot that is receding from us at the speed of light. Since the huge Universe is to a good approximation flat, a small arc $r\theta$ can be approximated by a chord of Euclidean geometry according to the Pythagorean theorem $r(1 - v^2/c^2)^{1/2}$. (b) On the average, the energy density $u(r)$ (solid line) accumulates (dashed line) with increasing r about any locus in the Universe, so the few brightest (blue) bodies are nearby, whereas numerous faint (red) bodies are further away. Ultimately, the spectral density vanishes altogether at R. The skewed distribution displays the balance between spectral energy density and energy density of mass according to Planck's law.

The geometry of the expanding Universe relates acceleration a, as usual, to the radius of curvature $r = \frac{1}{2}at^2$ at time t. The universal acceleration a in the distant past relates to the height $h = 2r/t^2 = 2c^2r/r^2 = GM/r^2 = a$ of the spherical cap contained in the view angle. In other words, when the Universe was young, r was smaller than today and accordingly a was stronger. The force of expansion today, after $t = 13.7$ billion years of dilution, is very weak because the average mass density $\rho = 1/2\pi Gt^2$ has become very low (12.78×10^{-27} kg m^{-3}). This calculated value though is higher than the estimate 9.9×10^{-27} kg m^{-3} obtained from the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) measurements (Bennett et al. 2003). However, the *WMAP* data have been interpreted on the basis of Friedmann–Lemaître–Robertson–Walker metric. Therefore, the critical density $\rho_c = 3H^2/8\pi G$ of the Friedman equation is lower by the geometrical factor $\frac{3}{4}$ than that provided by the formula $\rho = H^2/2\pi G$ (Lemaître 1927; Hubble 1929; Unsöld & Baschek 2002).

4 DISCUSSION

Our calculation $\varphi = 43.09$ arcsec per century for Mercury's perihelion precession and the result 42.94 arcsec of relativistic calculation using the values from NASA (The Solar System Exploration Web Team 2011) are both in excellent agreement with observations (43.1 ± 0.5 arcsec; Clemence 1947). Moreover, the functional form $\varphi = 2\pi^2 GM_o/c^2r$ that is based on the principle of least action á la Maupertuis and the form $\varphi_{GR} = 6\pi GM_o/c^2r(1 - e^2)$ derived from general relativity both relate the local characteristics (i.e. the mass of Sun M_o, the semimajor axis r and eccentricity e of Mercury's orbit) to the characteristics of the Universe (i.e. the constant of gravitation G and the speed of light c), but φ differs from φ_{GR} as much as 2π differs from $6/(1 - e^2)$. Kepler's third law $GM_o = (2\pi)^2r^3/t^2$ delivers $(2\pi)^2$ to the form of φ and leaves it independent of e, whereas the factor 6 in φ_{GR} and its dependence on e follow from Schwarzschild's solution in the zero-density vacuum.

On the basis of observations, however, it is not so easy to judge between φ and φ_{GR}. When e is small, it will be difficult to measure the apsidal precession of a nearly circular orbit with small error. This is apparent in the case of Venus (Clemence 1947). Conversely when e is big, the semimajor axis tends to be large and the overall effect will be small and thus subject to a large measurement error as well. In any case, our elementary calculation will deliver values for other planets in the Solar system, the asteroid Icarus (Shapiro, Ash & Smith 1968) and binary pulsar systems (Kramer et al. 2006) in agreement with observations.

Moreover, the elementary calculation gives a meaning to the relativistic calculation by relating the Schwarzschild radius r_s of the Sun, about 3 km, with the radius R of the Universe. In other words, it is not the mass M_o of an object that defines the radius of a horizon $r_h = GM_o/2c^2 = RM_o/M$ but it is the ratio of the local energy density to the surrounding energy density contained in the mass M of the Universe. Consequently, when the Universe was young and dense, r_h of a given mass M_o was smaller than it is in today's sparse surroundings.

In general, calculations that are based directly on the concept of action, like the one presented here, are not troubled by a singularity $r \to 0$, unlike those based on energy, because the absolutely least action, the quantum of action h, is finite. This is to say that the symmetry group U(1) that characterizes electromagnetic radiation cannot be broken down. According to the principle of least action á la Maupertuis, the evolving Universe is a natural process that breaks irreversibly from one stationary state of symmetry (Noether 1918) to another of different symmetry (Annila 2010, 2011). Since the Poincaré group is the full symmetry group of any relativistic field theory (Weinberg 1995), also general relativity cannot but describe a stationary state (Birkhoff 1924). Therefore, it fails to account accurately for the series of state changes that the expanding Universe is undergoing.

Evolution of space is not a continuous process but proceeds from one state of symmetry to another in steps of quantized actions. When one confined circulation transforms to another by excising quanta from the bound state to the surroundings, time will step forward. Thus, it follows from the quantized character of nature that a loop of space cannot be continuously tightened to a point as was conjectured by Poincaré (Milnor 2011), but ultimately the spontaneous symmetry-breaking processes will attain the absolutely least closed circulation. When it breaks open, the resulting open action will belong to the absolutely lowest group of symmetry U(1) which is the symmetry group of electromagnetic radiation (Griffiths 1999; Annila 2010).

The holistic view of gravity as the manifestation of energy density differences obtained by means of the original form of the principle of least action parallels Mach's thinking of inertia. Gravity couples everything to everything else via flows of energy densities so that nothing can be done without affecting everything else (Einstein 1923; Bondi & Samuel 1997; Von Bayer 2001). The space is connected, i.e. affine via these flows of energy and finite but without boundary. Since there are numerous pathways for the force carriers to propagate in the quest of diminishing the energy density differences among all bound forms of energy, the Universe will expand uniformly in the least time. When gravity is understood as a force due to energy density differences, then the residual in the perihelion precession is also understood to stem from the ultimate energy density difference between the Universe and its 'zero-density surroundings'. This universal force, often described as the curvature of space–time, is the requested reason of the anomalous perihelion precession.

The holistic portrayal of the perihelion precession by the principle of least action is self-consistent. Yet the original form of least action by Maupertuis may appear outdated to some, and perhaps even fortuitous when giving the numerical value for the apsidal precession in excellent agreement with observations. However, the explanation is not an ad hoc resolution since it is based on the same natural principle that has been used to clarify diverse phenomena ranging from quarks to galaxies and from biology to economics (Feynman 1948; Salthe 1985; Beeson 1992; Bak 1996; Georgiev & Georgiev 2002; Lineweaver & Egan 2008; Annila & Salthe 2009, 2010).

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