

# On the Origin of Universal Patterns



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**Abstract** The most comprehensive result of scientific inquiry across disciplines is that data, irrespective of origin, display skewed distributions, sigmoid curves, and power laws as well as oscillations and, at times, chaos. While mathematical models and computer simulations can be made to reproduce these ubiquitous patterns of nature, science is not only about modeling and mimicking the data but making sense of it. We argue that the ubiquitous patterns follow from the least-time consumption of free energy. These natural processes can be described by the many-body theory of open systems, i.e., nonequilibrium statistical physics for quantized systems. This theory, also known as the second law of thermodynamics, explains the arrow of time in terms of flows of quanta as well as non-determinate and path-dependent evolution that yields the scale-free patterns.

**Keywords** Atomism · Complexity · Free energy · The principle of least action · Scale-free patterns · The second law of thermodynamics

## 1 Introduction

Today, the spectrum of scientific knowledge extends from tiny elementary particles to gigantic galaxies and from the richness of genes to the abundance of species. As startling as it is, the data are highly similar, regardless of what we look at. The universal characteristics are evident in immense masses of information called “big data” (Albert and Barabási 2002; Clauset et al. 2009; Newman 2005; Bak 1997; Sornette 2006; Buchanan 2002). The mathematical models of lognormal distributions, S-curves, and power laws match data irrespective of the field. Complex

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systems science is the new discipline that models this great regularity (Waldrop 1993).

Unless headers and units are labeled in the descriptors of different datasets, we could not say from where the data originates. For example, the length distribution of genes looks much like the length distribution of words. The distributions of animal and plant populations are just as skewed as the distributions of genes and words. Distributions of wages and wealth are also skewed. The size distribution of earthquakes looks similar to the size distribution of the activated cortical areas in the brain (Bak 1997). Moreover, natural spirals, such as clamshells, heads of flowers, hurricanes, and galaxies, all whirl in the same way (Hargittai and Pickover 1992; McNally 2010). Many growth curves follow the characteristic form of the letter “S”; that is, they are sigmoidal. For example, a bacterial population grows thus. Chemical reactions progress and economies develop likewise. The world is clearly not random but regular. Could it be consistent with just one single rule?

At first, it may seem all crazy to compare any data to any other data without any unit of measure. But, in this way, we are free from barriers to realizing that the world is everywhere amazingly similar. Only the units and scales that we have set ourselves vary from one dataset to another. When we cannot infer the origin of data from the data itself, we must accept that the data are similar, although not identical. The regularity stands for that which we cannot distinguish one dataset from another. What does this point to?

The similarity of the data is inconceivably broad. It is expressly puzzling unless we can see a common cause. The more general the explanation we must look for, the more different things share the same shape. Also, Newton was after the same reason for similar natural phenomena in his rules of scientific reasoning (Rossi 2001). The great regularity has been noticed. It has been modeled but not explained. For example, the lognormal distribution is a good model of skewed distributions. However, it does not say why the data distributes nearly lognormally. Likewise, the logistic curve matches many datasets of growth. However, a good fit does not say why growth is sigmoidal.

Statistical mechanics, as a theory of many-body systems, has the potential to explain the origin of the universal patterns. That promise is realized with statistical physics of open systems. According to that theory, flows of quanta between the system and its surroundings drive toward the mutual thermodynamic balance in the least time. The analysis reveals that evolution results in skewed distributions, sigmoid growth curves, spirals, and power laws that are found throughout nature.

## 2 Statistical Physics of Open Systems

At one time, Ludwig Boltzmann understood that gas attains thermodynamic balance by way of the gas molecules colliding on each other and on walls of the tank. Albert Einstein understood conversely that Planck’s law of radiation accounts for light as a gas of photons. Willard Gibbs, in turn, comprehended that chemical

compounds attain thermodynamic balance via reactions. Now, we can understand likewise the quanta, e.g., photons, as the fundamental elements of everything, redistribute energetically ever more favorably in all kinds of events. This process toward thermodynamic balance can be described by the equation of evolution, which, in turn, can be derived from the equation of state.

## 2.1 The Equation of State

The equation of state describes any system that comprises the basic building blocks, the quanta. In events, the system moves from one state to another either by gaining quanta from the surroundings or by giving away quanta to the surroundings. When all systems consist of quanta, the description is inherently universal. Thus, we can start by examining any constituent of any system. A constituent, indexed with  $j$ , exists with the probability  ${}^1P_j = \phi_1\phi_2\phi_3 \dots = \Pi_k\phi_k$ , which is the product,  $\Pi_k$ , of its ingredients, indexed with  $k$ . Thus, if any one entity  $k$  is missing altogether, i.e.,  $\phi_k = 0$ , then also  ${}^1P_j = 0$ . For example, an enzyme could not exist if any one of its ingredients was missing.

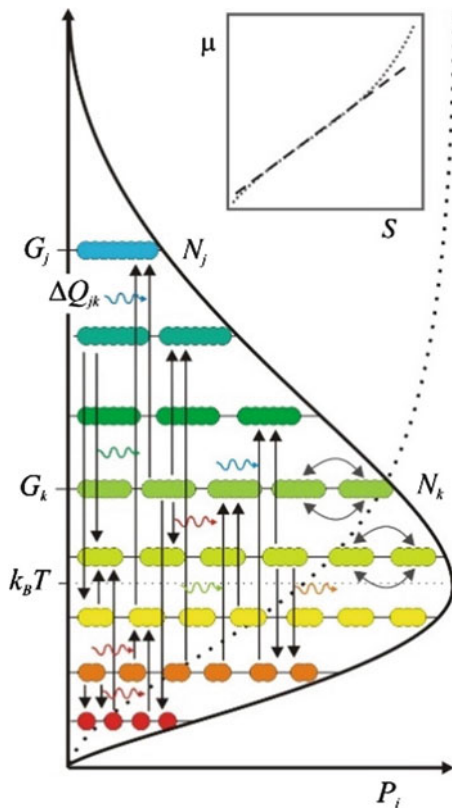
The ingenuity of statistical physics is that we can express the probability  ${}^1P_j$ , even when we do not know what entities  $\phi_k$  are in the product  $\Pi_k$  because ultimately all entities comprise the quanta. Therefore, the equation of state includes all the details with the formal precision of the quantum.

When the system houses several entities of equal energy, for example, a cell having multiple copies of the same enzyme, the probability of the population  $P_j = [{}^1P_j][{}^1P_j][{}^1P_j] \dots /N_j! = [{}^1P_j]^{N_j}/N_j!$  is the product of the partial probabilities, where  $N_j$  is the size of the population. The product form ensures that if any one entity is missing, i.e.,  ${}^1P_j = 0$ , then also  $P_j = 0$ . When the entities are identical, their mutual order makes no difference. Hence, the expression of  $P_j$  is divided by the number of ways  $N_j!$  that the entities, in total  $N_j$ , can be arranged into a sequence (Fig. 1).

The total probability  $P$  of the system is the product  $\Pi_j$  of the partial probabilities  $P_j$ :

$$P = \prod_{j=1} P_j = \prod_{j=1} \left[ \prod_{k=1} \phi_k \right]^{N_j} /N_j! \quad (1)$$

where each factor  $\phi_k = N_k \exp[(-\Delta G_{jk} + i\Delta Q_{jk})/k_B T]$  denotes the number of starting materials  $N_k$  and the energy difference  $\Delta G_{jk}$  between the starting material, indexed with  $k$ , and the product, indexed with  $j$ . The higher the energy of the starting materials, the less energy  $i\Delta Q_{jk}$  is needed from the surroundings to bridge the energy gaps from the starting materials into the product. The label  $i$  in front of the energy term means that the system is open for the flows of quanta. For example, the



**Fig. 1** When everything comprises quanta, any system can be pictured in terms of an energy-level diagram. The entities of a system, in numbers  $N_k$ , which have the same energy  $G_k$ , are on the same level. The bow arrows indicate their mutual exchange, which changes nothing and hence causes no change in the average energy of the system  $k_B T$  either. By contrast, the vertical arrows indicate events, in which the entities move from one level to another. For example, in a chemical reaction, starting materials  $N_k$  transform into products  $N_j$ . The horizontal wave arrows denote the quanta of light that either come from the environment to the system or go away from the system to the environment. Since the quanta carry energy  $\Delta Q_{jk}$ , the events as flows of quanta move the system and its surroundings toward the thermodynamic balance. When the energy of the surroundings is higher than that of the system, the system will evolve toward higher average energy and the surrounding systems toward lower average energy, and vice versa. The cumulative probability distribution curve (dotted line) is a sigmoid. When the logarithm of the total probability, i.e., entropy  $S$ , is plotted as a function of [chemical] potential energy  $\mu$ , the S-shaped curve follows on the logarithm-logarithm scale mainly a straight line (inset), that is, it follows a power law

photons from the Sun make photosynthesis happen. Conversely, the heat generated by our body goes away into the surroundings.

Energy differences between the products and starting materials are relative to the average energy of the system,  $k_B T$ . Since the concept of temperature  $T$  was adopted long before the concept of energy,  $T$  is multiplied by the Boltzmann constant  $k_B$  to

make it commensurate with the other terms of energy. Naturally, the average energy changes in every event. However, when a single event perturbs the average energy only a little bit, the system evolves smoothly.

The state equation (Eq. 1) is the theory in essence. Thus, it is relevant to summarize the assumptions that have already been made since conclusions just follow from a straightforward mathematical analysis. (1) The state equation applies when the same elements make up everything. This atomistic axiom underlies statistical mechanics in general. (2) The system is statistical when there are numerous entities. Then the average energy  $k_B T$  is a meaningful concept, and the energy differences relative to it can be expressed as exponential functions (exp) (Gibbs 1948; Phillies 2017). When these assumptions hold, the statistical theory explains why data of various kinds have the same form.

The state of the system is customarily given by an additive measure, which is obtained as the logarithm (ln) of the state equation (Eq. 1). For historical reasons, the logarithm of probability, when multiplied by the Boltzmann constant  $k_B$ , is known as the entropy:

$$S = k_B \ln P = k_B \sum_{j=1} \ln P_j \approx \frac{1}{T} \sum_{j=1} N_j \left( \sum_{k=1} -\Delta\mu_{jk} + i\Delta Q_{jk} + k_B T \right). \quad (2)$$

In the equation,  $\Delta\mu_{jk}$  denotes the potential energy difference between the populations  $N_k$  and  $N_j$ . The energy that is bound to the  $k$ -entity population  $\mu_k = k_B T \ln \phi_k$  is called [chemical] potential. Similarly,  $\mu_j$  denotes the potential energy of the  $j$ -entities. In the equation for entropy (Eq. 2), the entry  $\approx$  stands for the statistical approximation  $\ln N_j! \approx N_j \ln N_j - N_j$ , which is excellent when  $N_j > 10$ . It is worth emphasizing that the entropy expression (Eq. 2) is just the logarithm of probability (Eq. 1). In other words, mathematics does not change anything. It just keeps conclusions within the atomistic axiom of the theory.

## 2.2 The Equation of Evolution

The total energy of the system  $TS$  equals temperature  $T$  times entropy  $S$ . It comprises the system-bound energy  $\sum N_j k_B T$  and the free energy  $\sum N_j (-\Delta\mu_{jk} + i\Delta Q_{jk})$ . When free energy  $-\Delta\mu_{jk} + i\Delta Q_{jk}$  is decreasing, the populations  $N_j$  are changing, and hence, also the total energy of the system  $TS$  (Eq. 1) is changing with time  $t$ :

$$T \frac{dS}{dt} = \sum_{j=1} \frac{dS}{dN_j} \frac{dN_j}{dt} = \sum_{j=1} \frac{dN_j}{dt} \left( \sum_{k=1} -\Delta\mu_{jk} + i\Delta Q_{jk} \right). \quad (3)$$

It is convenient to denote the change as continuous, i.e., as a differential  $dN_j$  because for a statistical system, the change, quantum by quantum, appears as if it were continuous.

We cannot solve the equation of motion (Eq. 3) because the change in population is proportional to free energy, i.e., force:

$$\frac{dN_j}{dt} = \frac{1}{k_B T} \sum_{k=1} \sigma_{jk} (-\Delta\mu_{jk} + i\Delta Q_{jk}), \quad (4)$$

where  $\sigma_{jk} > 0$  represents a mechanism that facilitates the flow of quanta. For example, an enzyme catalyzes the conversion of starting materials  $N_k$  into the products  $N_j$  or vice versa. The flows of quanta naturally select efficient mechanisms because then the thermodynamic balance is attained in the least time.

While the course of events cannot be predicted because forces and flows cannot be separated, the process can be simulated step by step, according to Eq. (4). In this way, the emergence of standards, skewed divisions, growth curves, oscillations, and chaotic courses can be demonstrated and modeled (Annala and Annala 2008; Jaakkola et al. 2008a, 2008b; Karnani and Annala 2009; Annala and Salthe 2009).

Equations (3) and (4) describe the flows of quanta so that the imbalance between the system and the environment decreases in the least time. When we substitute in Eq. (3) the change in the number  $dN_j/dt$  with Eq. (4), we see that the entropy cannot decrease. This is known as the second law of thermodynamics  $dS \geq 0$ . There is no exception since the quanta, as conserved entities, cannot come out of nothingness or vanish into nothingness. The quantum that leaves the system will end up in the environment or vice versa.

According to Eqs. (3) and (4), there are no energy barriers for the evolution from one state to another. If such barriers existed, thermodynamics and kinetics would be in conflict with each other. This is not the case. Free energy can only decrease. For example, the flow does not open until the water level rises over the spillway crest. Likewise, the chemical reaction does not proceed from the starting material to the product until the energy of the starting materials, including absorbed photons, exceeds the energy of the products. The catalyst does not change the energy level diagram or landscape. It is a mechanism that only speeds up the conversion of the starting materials into the products or vice versa. According to the theory of time, the flows direct so that energy differences are diminishing as soon as possible. Thus, entropy does not just increase, but it increases as fast as possible.

### 3 The Universal Patterns

The equation of evolution (Eq. 4) reproduces the S-shape of growth piecewise. At the beginning of the growth, there is a wealth of resources, i.e., free energy. Then,

we can assume that mechanisms  $\Sigma_k \sigma_{jk}$  of the system consume free energy  $-\Delta\mu_{jk} + i\Delta Q_{jk}$  almost steadily, and hence the population  $N_j$  changes with time:

$$\begin{aligned} \frac{d}{dt} \frac{1}{k_B T} \sum_{k=1} (-\Delta\mu_{jk} + i\Delta Q_{jk}) &= \frac{dN_j}{dt} \frac{d}{dN_j} \frac{1}{k_B T} \sum_{k=1} (-\Delta\mu_{jk} + i\Delta Q_{jk}) \approx \sum_{k=1} \sigma_{jk} \\ &\Rightarrow \frac{dN_j}{N_j} = \sum_{k=1} \sigma_{jk} dt. \end{aligned} \quad (5)$$

Here  $d\mu_j/dN_j = k_B T/N_j$ . The growth by Eq. (5) is approximately exponential because initially, the amount of free energy seems as if it were infinite, and only the mechanisms are limiting the growth. Likewise, the growth is decreasing almost exponentially when the free energy is dwindling down while the balance is approached.

The growth between the initial and final phases follows a power law closely. We see this by expressing the population  $N_j$  as the product of the elements  $N_1$  using the atomistic axiom  $N_j = \Pi_k \phi_k = \alpha_j N_1^j$ . The factor  $\alpha_j = \Pi_{mn} \exp[\Sigma N_j (-\Delta\mu_{mn} + i\Delta Q_{mn})/k_B T]$  contains the free energy terms that force the assembly of  $N_j$  from the elements  $N_1$ . So, the change

$$\frac{dN_j}{dt} = j\alpha_j N_1^{j-1} \frac{dN_1}{dt} = j \frac{N_j}{N_1} \frac{dN_1}{dt} \Rightarrow \frac{dN_j}{N_j} = j \frac{dN_1}{N_1} \quad (6)$$

when integrated follows the power law  $\ln N_j = j \ln N_1 + \text{constant}$ .

When the assumption of a nearly constant change in free energy does not hold, we can *model* the change by adding the term  $-\beta N_j$  to the equation of the population change (Eq. 5):

$$\frac{dN_j}{N_j} \approx \left( \sum_{k=1} \sigma_{jk} - \beta N_j \right) dt \Rightarrow N_j(t) = N_j(t_0) \left( \sum_{k=1} \sigma_{jk} - \beta N_j(t_0) \right). \quad (7)$$

In this model (Mäkelä and Annala 2010; May 1976), the population  $N_j(t_0)$  at present  $t_0$  determines the population  $N_j(t)$  at a later time  $t$ . According to the model, evolution is almost predictable when the change in free energy is small compared with average energy, i.e.,  $|(-\Delta\mu_{jk} + i\Delta Q_{jk})/k_B T| \ll 1$ . In contrast, oscillations and chaos occur when the condition is not fulfilled. This is the case, for example, when a solid-state laser is turned on or when the animal population proliferates and exceeds the carrying capacity of the environment or when the banks need more money than is available.

Logarithmic, exponential, or truncated distributions and their power-law-like cumulative distribution functions are mathematical models of the physical processes

given by Eqs. (3) and (4). The models allow us to describe and categorize various data, but they do not explain how the data came about, that is, causality. It is worth emphasizing that Eqs. (3) and (4) cannot be solved, except at balance, since the variables cannot be separated. This means that a chain of events is fundamentally unpredictable due to mutual dependencies rather than due to complexity or uncertainty in the initial conditions.

When the system evolves gradually, the change in energy is small compared with the average energy, i.e.,  $|(-\Delta\mu_{jk} + i\Delta Q_{jk})/k_B T| \ll 1$ . Therefore, the variation  $n$  is small  $n \ll j$  around a typical or an average factor, indexed with  $j$ . When the factors  $\phi_j$  are given in logarithmic terms  $\ln\phi_j = j\ln\phi_1$  of the elemental factor  $\phi_1$ , we see that the natural distribution

$$\ln\phi_{j-n, j+n, j, n} = \ln\phi_j + \sum_n n \ln\phi_1 \quad (8)$$

is approximately logarithmic. The distribution of Eq. (8) shows that the typical form  $j$  can be recognized in each member within the distribution  $j \pm n$ . For example, all-sized Northern pike looks like pike and not bream. On the other hand, if weights of pikes and cars were presented in the same figure, we would see two distributions: one about a typical pike and the other about an ordinary car. Moreover, spiral forms of nature, such as shells, cyclones, and galaxies, are also approximately lognormal distributions but in polar coordinates (Mäkelä and Annila 2010). Logarithmic spirals are thus energetically optimal shapes.

## 4 Discussion

Statistical physics of open systems accounts for processes as flows of quanta in accordance with observations and measurements. The correspondence between the theory and data implies further that everything comprises the indivisible elements, quanta, and every process seeks thermodynamic balance in the least time. The natural law is contained in the quantum itself. Planck's constant  $h = Et$ , as the complementary product of energy and time, determines the change in energy over time  $dE/dt = -E/t = -\mathbf{F} \cdot \mathbf{v}$ . Thus, the power  $dE/dt$  decreases due to motion with velocity  $\mathbf{v}$  in the direction of force  $\mathbf{F}$ .

From this perspective, statistical physics of open quantized systems could be falsified (i) if a phenomenon were found where the system moves away from the thermodynamic balance; (ii) if a quantum, say, a photon, was found to split into pieces; or (iii) if something were found that is not quanta. Earlier, we have argued that elementary particles, as well as the void, comprise quanta (Annila 2012; Grahn et al. 2018; Annila 2010; Annila and Kolehmainen 2016).



## 5 Conclusions

Traditional statistical mechanics is limited to closed or equilibrium systems. When no net fluxes are included, the system is stationary, and hence calculations are precise. In contrast, the statistical physics of open systems, i.e., nonequilibrium statistical mechanics, includes net fluxes, and accordingly, the system is described in evolution from one state to another. However, the evolution is nondeterministic because the boundary conditions, the surroundings, which are the sources and sinks of the fluxes, are changing too. The future is genuinely unpredictable.

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