The 2nd Law of Thermodynamics Delineates Dispersal of Energy

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Abstract – *The 2nd law of thermodynamics as an equation of motion is derived from statistical physics of open systems. It describes, by the principle of increasing entropy, systems in evolution from one state to another, more probable one when energy flows from high to low densities along the paths of least action, i.e., geodesics toward the stationary state of free energy minimum in respective surroundings. The universal law equates changes in kinetic energy with changes in scalar and vector potentials, as is given by equations of basic, continuum and quantum mechanics as well as by those of fluid, electro- and thermodynamics. The Law in its scale-independent form gives holistic understanding of nature in motion with no distinction between animate and inanimate.*

***Keywords**: Entropy, Evolution, Free energy, Probability, Statistical Physics*

k_BT	average energy per particle
ϕ	energy density
$2K$	kinetic energy
U	scalar potential
Q	vector potential
A	free energy
S	entropy

I. Introduction

The 2nd law of thermodynamics is often praised as the supreme among the laws of nature. Yet, the general principle is seldom pronounced explicitly in diverse disciplines of physics, i.e., in basic, continuum and quantum mechanics or in fluid and electrodynamics or in optics. Likewise, the Law is rarely related directly to other universal imperatives, e.g., to the principle of increasing entropy, minimum energy, minimum time, least action or to the maximum power principle. It may well be that by today physics has submerged to specialties that recognize no reason to call for the common ground. Nonetheless, the unifying principle is valuable in providing holistic view on how nature works.

In the old days, when natural philosophy was diversifying into sciences, the aim was to maintain broad oneness, and nowadays too, when sciences are branching further, the struggle is to attain unity in each field. In the quest for the universal understanding few names stand out from the history of science. Louis Moreau de Maupertuis struck that both Newton's laws of motion and Fermat's principle of least time were articulated in the principle of least action [1]. His heirs, though, thought that it was ambiguous to minimize an adjunct momentum-coordinate (geometric) product [2], and revised integrand for Lagrangian. Ludwig Boltzmann, in turn, in admiration of Darwin came up with the astounding idea that nature is in motion toward increasingly more probable states. His successors hailed the simple theory of complex systems [3] but apparently Boltzmann himself remained distressed. The statistical theory did not fully comply with the 2nd law but limited to closed systems whereas living systems are unmistakably open to energy flows from surroundings.

We will begin this study by re-examining the probability notion that Boltzmann placed as the corner stone of his statistical mechanics when trying to bridge from reversible microscopic phenomena to irreversible macroscopic processes following the 2nd law. Boltzmann adopted the probability concept from Decartes, Fermat, Pascal and others who had computed combinatorial possibilities in context of gambling but Boltzmann could have also resorted to the posthumous paper [4] of the Reverend Thomas Bayes who had considered circumstantial possibilities in context of collecting information [5]. It turns out that new insights to the supreme law of nature are available from these old thoughts. Although we fail to present any novel results, hopefully it will be found gratifying to recognize that some central concepts in diverse disciplines are in fact re-expressions of the 2nd law of thermodynamics.

II. Physical Probability

Boltzmann enumerated, just as counting pips on dice, isoenergetic configurations that are commonly referred to as microstates. This invariant, Cartesian probability notion is constant in energy and thereby it identifies to conserved systems. Hence the statistical theory, by founding solely on it, limits to changes in phase of stationary systems. In contrast, the Bayesian probability varies in energy and thereby it relates to non-conserved systems. Hence the statistical theory, by including it, will extend to changes in state of evolutionary systems.

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The distinction between the Cartesian and Bayesian probability concepts can be exemplified by examining a chemical system. Chemical reactions, like other natural processes, direct toward the most probable state, i.e., the entropy maximum. That dynamic steady state resides in the free energy minimum which depends on surrounding conditions as stated by Le Châtelier's principle [6].

A chemical system, for example, a reaction mixture in a vessel at an organic chemistry laboratory or a metabolic network in a living cell may house a myriad of molecules. In statistical physics the system is depicted as a level diagram where distinguishable molecules occupy distinct energy levels (Fig. 1). Reactions bring about changes in populations. These transformations from one molecular state to another are either absorptive, i.e., endergonic or emissive, i.e., exergonic transitions. Conversely, configurational swapping of entities at any level is isergonic exchange without net influx or efflux.

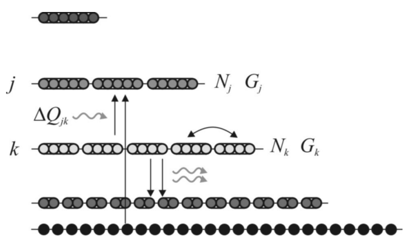

Fig. 1. A system is represented by a diagram where distinct j-entities in numbers N_j populate levels of energy G_j relative to k_BT. The system evolves from one state to more probable one by diminishing energy differences, i.e., consuming free energy in endergonic ΔQ_{jk} or exergonic $-\Delta Q_{jk}$ transitions until the steady state is attained in its surroundings.

The probability P_j of a particular molecule, indexed by j, depends on its substrates as well as on energy in the surroundings that couples to the transformation from substrates to the product. When any one vital k-substrate is missing entirely ($N_k = 0$), no jk-synthesis will yield the j-product. No j-product will be obtained either when the surroundings do not supply any quanta for the endergonic reaction or cannot accept any quanta from the exergonic reaction. When substrates and energy are available, the yield, i.e., P_j will depend on the difference $\Delta G_{jk} = G_j - G_k$ between energy G_j of the j-product and G_k of its k-substrate. That difference, given in the exponential form, can be bridged by the energy influx ΔQ_{jk} from the surroundings that couples orthogonal to the jk-transition. These circumstantial considerations define the conditional probability as [7,8]

$$P_j = \prod_k N_k e^{-(\Delta G_{jk} - i\Delta Q_{jk})/k_BT} \qquad (1)$$

where the energy difference $-\Delta G_{jk} + i\Delta Q_{jk}$ is relative to k_BT. The ingredients in Eq. 1 are densities-in-energy $\phi_k = N_k \exp(G_k/k_BT)$ as defined by Gibbs [9].

During the natural process the energy difference between the j and k-repositories of energy (fermions) that form space by excluding each other, is bridged orthogonally, hence denoted by i, by the flux of quanta ΔQ_{jk} (bosons) that form the flow of time [8]. In the continuum these two forms of energy are referred to as the scalar and vector potentials. A gradient of the former is the conserved, irrotational part and the latter is the non-conserved, rotational part of the force.

In addition to the circumstantial conditions given by Eq. 1 the probability depends, as usual, on the isergonic configurations. The k-substrates that are incorporated in the j-product as indistinguishable (symmetric) copies are numbered by the (stoichiometric) degeneracy g_{jk}. Likewise, N_j enumerates indistinguishable j-products. These combinations are taken into account by factorials to give P_j for a pool of j-products [7,8]

$$P_j = \left(\left(\prod_k N_k e^{-(\Delta G_{jk} - i\Delta Q_{jk})/k_BT} \right)^{g_{jk}} \Big/ g_{jk}! \right)^{N_j} \Big/ N_j!. \qquad (2)$$

And the probability of the entire system $P = \Pi P_j$ is obtained by considering actions over all levels of hierarchy as statistically independent [7,8]

$$P = \prod_j \left(\left(\prod_k N_k e^{-(\Delta G_{jk} - i\Delta Q_{jk})/k_BT} \right)^{g_{jk}} \Big/ g_{jk}! \right)^{N_j} \Big/ N_j!. \qquad (3)$$

The obtained nested (recursive), self-similar formula means that each k-substrate is considered as a product of some earlier evolutionary processes. For instance, elements, that make molecules, are products of nuclear reactions in stars. In turn, molecules are substrates for cellular assembly, and cells are ingredients for individual development, and so on.

Since thermodynamics pictures everything in terms of energy, the scale-independent form (Eq. 2) that was derived exemplifying chemical reactions specifically, is generally applicable to various natural processes, including transport phenomena, like diffusion where dissipation is in practice negligible but conceptually central. Since it remains impossible to recognize any unambiguous border line between animate and inanimate, abiotic phenomena are viewed as evolutionary processes that are naturally happening as soon as possible. Conversely, biotic systems are looked upon as undergoing merely time-dependent physical processes that are naturally selecting the steepest directional descents toward the free energy minimum. As will be shown in the subsequent section, the probability P (Eq. 3) relates to some familiar forms of physics.

III. Forms of Energy Dispersal

The 2nd law of thermodynamics is conceptually simple. It says, when adopting the words of Carnot [10]:

Wherever there exists a difference of energy density, a flow of energy can appear to diminish that difference. The energy difference is the motive force and the flow of energy is motion that naturally selects the fastest ways, i.e., the most voluminous steepest descents to level the free energy landscape in least time. Thus, when the system is evolving from one state to another toward the free energy minimum, entropy $S = k_B\ln P$, as the logarithmic, hence additive, probability measure [5], will not only be increasing but it will be increasing as soon as possible. This principle of least time [11] for flows of energy is equivalent to the maximum power principle [12] and also, as will become apparent below, equal to the maximum entropy production principle [5]. The differential equation of motion along an extremum path, i.e., geodesic, is known in its integral form as the principle of least action [13]. The 2[nd] law as the equation of motion for flows of energy has been given in a variety of forms in various branches of physics.

III.1. Statistical Mechanics

A system of many bodies allows us to use Stirling's approximation $\ln N_j! \approx N_j\ln N_j - N_j$ valid for large N_j to simplify the equation of P (Eq. 3) to the additive statistical status measure [7,8]

$$S = k_B \ln P = k_B \sum_j \ln P_j \approx k_B \sum_j N_j \left(1 - \sum_k A_{jk} / k_B T \right) \quad (4)$$

where the free energy $A_{jk} = \Delta\mu_{jk} - i\Delta Q_{jk}$, i.e., affinity [14], is the motive force that directs the transforming flow dN_j/dt from N_k to N_j. The logarithmic density difference is usually denoted as the scalar (chemical) potential difference $\Delta\mu_{jk} = \mu_j - \Sigma\mu_k = k_BT(\ln\phi_j - \Sigma g_{jk}\ln\phi_k/g_{jk}!)$. The statistical approximation implies that the j-system is able to absorb or emit quanta without a marked change in the average energy density $A_{jk}/k_BT << 1$. Otherwise, e.g., when a population N_j goes extinct or emerges, the particular $\ln P_j$ is not a sufficient statistic for k_BT [15]. When an ensemble of entities is not sufficiently tied together by mutual interactions to establish common k_BT, it is not a system, rather surroundings of sufficiently statistical systems at a lower level of hierarchy where the self-similar equations apply [16,17].

When the system is transforming from a state to more probable one by consuming A_{jk}, S is increasing [7,8]

$$d_t S = k_B \partial_t \ln P = -\sum_{j,k} d_t N_j \, A_{jk} / T = k_B L \geq 0 \quad (5)$$

Curiously, the flows d_tN_j and forces A_{jk} are inseparable in $L = -\Sigma d_tN_jA_{jk}/k_BT$ when there are alternative paths for energy dispersal [8,18]. In other words, the open system with three or more degrees of freedom is non-Hamiltonian, and the seemingly simple equation of motion $\partial_tP = LP$ cannot be solved. The non-conserved system has no norm hence there is no unitary transformation to yield eigenvalues of the characteristic equation. Likewise the equation cannot be integrated to a closed form hence open evolutionary trajectories are inherently intractable. Finally, when evolution has arrived at a stationary state $\partial_tP = 0$, conserved currents circulate on closed orbits governed by the symmetry of Hamiltonian [19]. These motions can be transformed to a standstill, time-independent frame.

The principle of increasing entropy is equivalent to the conservation in the flows energy [7,8]

$$T d_t S = -\sum_{j,k} d_t N_j A_{jk} = -\sum_{j,k} d_t N_j \left(\Delta\mu_{jk} - i\Delta Q_{jk} \right) \quad (6)$$

found from Eq. 5 by multiplying with T. When $A_{jk} > 0$ (< 0), the j-system is higher (lower) in energy than its surroundings and $d_tN_j < 0$ (> 0) will diminish that difference. Thus, the free energy minimum state is Lyapunov-stable $S(\delta N_j) < 0$, $d_tS(\delta N_j) > 0$ against perturbations δN_j [20]. The influx to the system must match exactly the efflux from its surroundings. Therefore, there is no justification for the common caveat put against the 2[nd] law that entropy of a biotic system would be decreasing at the expense of increasing entropy in its abiotic surroundings. That statement violates the conservation of energy. However, it is possible, though statistically unlikely, that energy would be flowing up against gradients and entropy both of a system and its surroundings would be decreasing.

A population change in a statistical system is proportional to A_{jk} by a conductance σ_{jk} [7,8]

$$d_t N_j = -\sum_k \sigma_{jk} A_{jk} / k_B T \quad (7)$$

to satisfy the conservation of energy across jk-interfaces. However, anyone σ_{jk} is not necessarily invariant because the conducting mechanism is a system of its own that may evolve further to facilitate the flow. Also entirely new transduction mechanisms may emerge when the energy influx incorporates into the system's constituents. Likewise, old mechanisms will face extinction when the flows redirect and abandon them.

III.2. Basic and Continuum Mechanics

In continuum the discrete change $-\Sigma d_tN_j\Delta\mu_{jk}$ in the chemical potential is considered as the continuous directional derivate $-v_x\partial_xU$ of the scalar (internal) potential U and the quantized flux $\Sigma d_tN_j\Delta Q_{jk}$ as the continuous temporal gradient ∂_tQ of the vector (external) potential Q. Likewise, the change Td_tS is recognized as the change d_t2K in the kinetic energy. This equivalence is apparent, e.g., from the maximum entropy partition of gas whose internal energy U matches pressure p in a volume V. At a steady state $TS = \Sigma N_j k_B T = pV = \int \mathbf{F}\cdot d\mathbf{x} = \int \partial_t \mathbf{p}\cdot\mathbf{v} = 2K = -U$. The continuum flow balance (Eq. 6) is

$$d_t 2K = -\sum_{x,y,z} \left(v_x \partial_x U - i\partial_t Q_x \right) \tag{8}$$

so that a change in $2K$ balances changes in U and Q. The evolving system spirals along an open trajectory due to the action over a time-interval dt to satisfy the conservation $2K + U = Q$. Conversely, the stationary system stays on a closed orbit over the period to satisfy $2K + U = \langle Q \rangle = 0$ which is the familiar virial theorem.

The continuum equation (Eq. 8) is also available from the Newton's 2nd law of motion $\mathbf{F} = d\mathbf{p}/dt$ when taking a product with $\mathbf{v}$ and using the famous relation in the differential form $dm = dE/c^2 = dQ/v^2$. The 2nd law as the equation motion in Cartesian components $j,k = \{x,y,z\}$

$$\begin{aligned} \sum_{j,k} v_j F_{jk} &= \sum_{j,k} v_j m_{jk} a_k \; + i \sum_{j,k} v_j \left(\partial_t m_{jk} \right) v_k \\ \Leftrightarrow \sum_{j,k} d_t 2K_{jk} &= -\sum_{j,k} v_j \partial_{x_j} U_{jk} \; + i \sum_{j,k} \partial_t Q_{jk} \end{aligned} \tag{9}$$

says that the change in $2K_{jk} = v_j m_{jk} v_k$ equals the changes in U_{jk} and Q_{jk}. The force is composed of the irrotational gradient $-\partial_{xj} U_{jk} = m_{jk} a_k$ that is defined, as usual, for the Cartesian combinations and of the divergence-free field, *i.e.*, the gradient of the vector potential which dissipates $\partial_t Q_{jk} = v_k(\partial_t m_{jk}) v_j$ (Fig. 2). Dissipation stems from the changes in the mass given by the energy equivalent $dm_{jk}c^2 = dE_{jk} = n^2 dQ_{jk}$ that can be radiated in the respective surroundings defined by the isotropic index of refraction $n = c/v$. The mass change is often ignored but it signifies the changes in interactions when the system evolves from one state to another. Thus, the mass is understood by $E = mc^2$ only as a convenient way to denote a stationary system's the total energy content that can be radiated to the surroundings.

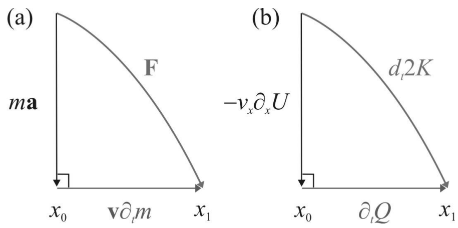

Fig. 2. (a) Force $\mathbf{F}$ as the change in momentum $d_t\mathbf{p} = d_t(m\mathbf{v})$ is a resultant of conserved $m\mathbf{a}$ and non-conserved $\mathbf{v}\partial_t m$ parts that bring about a change in state from x_0 to x_1. (b) The corresponding balance for the flows of energy equates the change $d_t 2K$ with the change $-\partial_t U$ and $\partial_t Q$.

Change in energy density is the hallmark of evolving systems but the lack of norm renders evolutionary trajectories non-integrable. For this reason Eq. 9, known also as Cauchy momentum equation or in fluid dynamics as Navier-Stokes equation does not have solution when there are three or more degrees of freedom [21].

III.3. Quantum Mechanics

A system that is not sufficiently statistical to accept or discard quanta without a marked change in $k_B T$, is referred to as microscopic. It is characterized by $P = \int \psi^* \psi dx = \langle \psi | \psi \rangle$ as a sum over densities-in-energy, just as P is a measure of the macroscopic system (Eq. 3), but since the microscopic system is perturbed by mere observation, dynamic densities are denoted by the wave function $\psi(x,t)$ and its complex conjugate orbiting in the opposite sense. At the most probable state P_{max}, i.e., at the steady state the spatial U and temporal iQ components of A balance over the period of integration so that $e^{-A/k_B T} = e^{-(U-iQ)/k_B T} = 1$. During dt the density moves $\partial_t \psi = \hat{L}\, \psi$ by the operator $\hat{L}$ whose expectation value is the force $L = \langle \psi | \hat{L} | \psi \rangle = -\partial_t A / k_B T$. Likewise, the complex conjugate moves $\partial_t \psi = -\hat{L}^\dagger\, \psi$ by the adjoint operator (Fig. 3). When expanding $\psi\rangle = \Sigma c_k | k \rangle$ in a basis $|k\rangle$ of the unit operator $\hat{P} = \Sigma |k\rangle\langle k|$, the equation of motion [8,22]

$$\begin{aligned} \partial_t P &= \left\langle \psi \,|\, \partial_t \psi \right\rangle + \left\langle \psi \partial_t \,|\, \psi \right\rangle = \left\langle \psi \,|\, \hat{L} \psi \right\rangle - \left\langle \psi\, \hat{L}^\dagger \,|\, \psi \right\rangle \\ &= \left\langle \psi \,|\, \hat{L}\hat{P} \,|\, \psi \right\rangle - \left\langle \psi \,|\, \hat{P}\hat{L}^\dagger \,|\, \psi \right\rangle = 2LP - \partial_t \Delta\varphi \end{aligned} \tag{10}$$

describes transition of state driven by L. The system is stepping from one stationary orbit to another by emission or absorption. Concomitantly the phase of precession relative to the detector is changing $\partial_t \Delta\varphi = \langle \psi \partial_t | \psi \rangle - \langle \psi | \partial_t \psi \rangle$. Free energy is consumed along the directional action denoted by the momentum-coordinate (geometric) product. The non-Abelian characteristic is also embedded in the commutation relation $[\hat{p}, \hat{x}] = -i\hbar$. When net dissipation vanishes $\langle \psi | \hat{Q} | \psi \rangle = 0$, the system arrives at the stationary state $\partial_t P = 0$ where the operator is unitary. In the Hamiltonian system only the phase precession prevails among the energetically equivalent configurations (microstates) at a constant rate, i.e., $\partial_t \Delta\omega t = \Delta\omega$ relative to the frame of detection.

Interference effects arise from phase-coherent motions, well-known from Aharanov-Bohm experiment [23], where a geometric phase difference [24] $\Delta\varphi = \varphi_a - \varphi_b$ develops between the flows of energy via distinct paths a and b through differing densities. A change $\partial_t A_{jk} \neq 0$, will change the path length and so will the diffraction pattern also change $\partial_t \Delta\varphi \neq 0$.

The 2nd law emphasizes that the detection transforms the system via the energy transduction that is inherent in any observation [22]. A macroscopic system does not mind much but P of the microscopic system will jump when the detection forces an abrupt change in its status, and the ensuing evolutionary step may appear as incomprehensible trajectory [25,26].

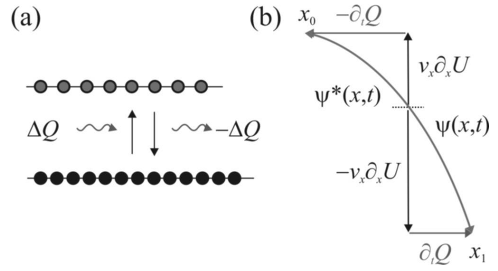

Fig. 3. (a) The energy density of a microscopic system is represented by $\psi(x,t)$ and its counter rotating complex conjugate $\psi^*(x,t)$ that contain the spatial x scalar and temporal t vector potentials. (b) At a stationary state there are no net fluxes between the system and its surroundings, hence $\partial_t P = \partial_t\langle\psi|\psi\rangle = 0$. Conversely, detection induces transduction of energy from the system to the surroundings or vice versa depending on the phase of the system's motions relative to the detector frame.

III.4. Electrodynamics

The 2nd law as the equation of motion for flows of energy is in electrodynamics known as Poynting's theorem. It is customarily derived from Maxwell's equations. Alternatively, one may obtaining it by starting from the definition of electric field $\mathbf{E} = -\nabla\phi - \partial_t\mathbf{A}$ due to the scalar ϕ and vector $\mathbf{A}$ potential gradients. The force density $\mathbf{f} = \rho\mathbf{E}$ is equivalent to the Newton's 2nd law of motion (Eq. 9). Thus when multiplying with $\mathbf{v}$ (cf. Eq. 8), the familiar form [8,27]

$$\mathbf{J}\cdot\mathbf{E} = -\rho\mathbf{v}\cdot\nabla\phi - \varepsilon\,\mathbf{v}\cdot(\nabla\mathbf{E})\partial_t\mathbf{A} = -\partial_t u - \mu^{-1}\nabla\cdot(\mathbf{E}\times\mathbf{B}) \quad (11)$$

is obtained where the definition of current $\mathbf{J} = \rho\mathbf{v}$ and the identity $\partial_t\mathbf{A} = \mathbf{v}\nabla\cdot\mathbf{A} = -\mathbf{v}\times\nabla\times\mathbf{A} = -\mathbf{v}\times\mathbf{B}$ have been used. In electrodynamics the 2nd law says that charge density ρ moves along the field lines of $\mathbf{E}$ with velocity $\mathbf{v}$ by consuming the scalar density $u = \rho\phi$ and dissipating light orthogonally (Fig. 4).

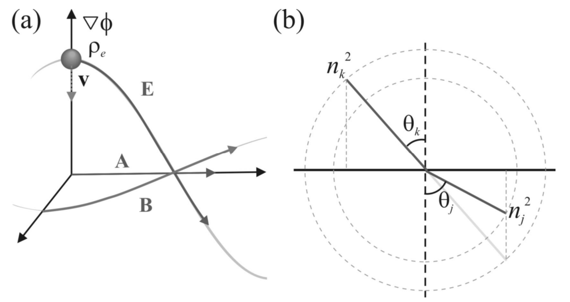

Fig. 4. (a) Kinetic energy flow as a charged density ρ_e moving at velocity $\mathbf{v}$ in an electric field $\mathbf{E}$ is balanced by the changing scalar potential $\rho_e\partial\phi/\partial t$ and the perpendicularly dissipated light $\varepsilon_o c^2\nabla\cdot(\mathbf{E}\times\mathbf{B})$. (b) When light propagates in a homogenous medium without sources and sinks $\nabla\cdot\mathbf{E} = 0$, the oscillating $\partial_t\phi$ balances the divergence of vector potential $c^2\nabla\cdot\mathbf{A}$ according to the Lorenz gauge. Energy is conserved in frequency shift when light refracts at a density boundary specified by indexes $n_k^2 > n_j^2$ relative to the universal reference $n_o^2 = c^2\varepsilon_o\mu_o = 1$.

In a medium without sources and sinks, e.g., in the vacuum density, Eq. 11 yields by the steady-state condition of constant $\partial_t 2K$ the Lorenz gauge $\varepsilon_o\mu_o\partial_t\phi + \nabla\cdot\mathbf{A} = 0$. Conversely, Eq. 8 delivers by the maximal-rate condition $\partial_t^2 2K = 0$ the familiar wave equation $\partial_t^2\psi = v^2\nabla^2\psi$. For example, when is crossing from a medium to a higher density of the refraction index $n^2 = c^2/v^2$, energy is conserved $\varepsilon\mu\partial_t\phi + n^2\nabla\cdot\mathbf{A} = 0$ along the path of least action, i.e., Fermat's principle of least time and light is refracting according to Snell's law.

III.5. Field Equations

The 2nd law of thermodynamics, when given as an equation of motion (Eq. 8), pictures by its directional spatial and temporal derivates the system as a curved energy landscape in evolution toward a stationary state evenness. The spatial densities are tied together to an affine manifold by flows of energy in mutual interactions. The evolving landscape is described in differential geometry by a non-vanishing Lie's derivative [28]. Evolution is non-commutative because the flows direct from heights to lows. The force is not collinear with the spatial (conserved) gradient $-\nabla U$ but departs by the temporal (non-conserved) part $i\partial_t Q$. In plain language a river, as it flows, is eroding its gorge.

The curved, non-Euclidean landscape can be approximated over a short spatial dx and over a short temporal dt coordinate by a short path $d\mathbf{s}$ on a slanted, Euclidean plane (Fig. 5). The L^2-norm $\|d\mathbf{s}\|^2 = d\mathbf{s}^* d\mathbf{s}$ is given by the familiar Lorentzian metric $dx^2 = ds^2 - c^2dt^2$ [29]. Thus the Lorentz transformation is recognized as an expression for the conservation of energy about a small space-time locus.

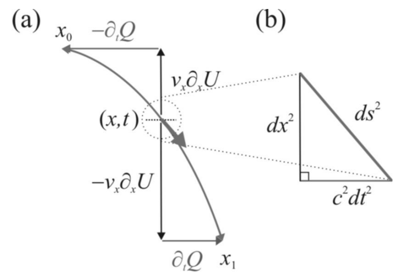

Fig. 5. (a) The free energy landscape is curved about a space-time locus (x, t) between x_0 and x_1 when the opposite spatial and temporal gradients are of unequal lengths. The difference forces evolution as a tangential flow of energy toward a steady, even landscape. (b) In a neighborhood (dx,dt) of the locus (x,t), the manifold can be regarded as Euclidean.

When the evolving, differentiable manifold (Eq. 8) is written in the form of a field equation, the spatial ∂_x and temporal ∂_t gradients are given as the 4-vector

$$\partial_\mu = (-\partial_t/c, \nabla) = (-\partial_t/c, \partial_x, \partial_y, \partial_z) \quad (12)$$

that acts on U and $\mathbf{Q}$, also given as the free energy 4-vector in the one-form space-time basis

$$A_\mu = (-U, \mathbf{Q}) = (-U, Q_x, Q_y, Q_z). \quad (13)$$

The curvature in the two-form $F = dA$ is represented by the covariant antisymmetric rank 2 tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & -F_x & -F_y & -F_z \\ F_x & 0 & R_z & -R_y \\ F_y & -R_z & 0 & R_x \\ F_z & R_y & -R_x & 0 \end{pmatrix} \quad (14)$$

where $d_t\mathbf{p} = \mathbf{F} = -\nabla U + \partial_t\mathbf{Q}/c$ and $\mathbf{R} = \nabla \times \mathbf{Q}$. The invariants $F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{F}^2 - \mathbf{R}^2)$ and $*F_{\mu\nu}F^{\mu\nu} = 4\mathbf{F}\cdot\mathbf{R}$ and $*F_{\mu\nu}*F^{\mu\nu} = 2(\mathbf{F}^2 + \mathbf{R}^2)$ contain the continuity in $\partial_t\mathbf{p}$, i.e., $\mathbf{F}$ and in the change in angular momentum $d_t\mathbf{L}$, i.e., torque $\boldsymbol{\tau}$. When the system is stationary without sources and sinks ($\nabla^2 U = 0$), there is no curvature. This particular condition $d\mathbf{F} = 0$ of the flat landscape yields the Maxwell's equations for light propagating in a homogenous medium, i.e., the Lorenz gauge, $\partial_\mu A^\mu = 0$. Likewise, when the system is closed ($dm = 0$), the law of motion $m\partial_t\mathbf{v} = -\nabla U$ governs the body with mass m on the least-action orbit $\mathbf{p} \times \boldsymbol{\omega} = m\mathbf{a}$.

The open, evolving system as the manifold in motion, is represented by $d_t 2K_{\mu\nu}$ equal to

$$F^{\mu\nu} v_{\mu\nu} = \begin{pmatrix} 0 & -v_x\partial_x U & -v_y\partial_y U & -v_z\partial_z U \\ \partial_t Q_x & 0 & v_y R_z & -v_z R_y \\ \partial_t Q_y & -v_x R_z & 0 & v_z R_x \\ \partial_t Q_z & v_x R_y & -v_y R_x & 0 \end{pmatrix} \quad (15)$$

where the 4-vector velocity $v_\mu = (-c, v_x, v_y, v_z)$. The tensor contracts to the 0-form $d_t 2K = \Sigma d_t 2K_{\mu\nu} = -\mathbf{v}\cdot\nabla U + \partial_t\mathbf{Q} + \mathbf{v} \times \mathbf{R}$. The overall change in $2K$ balances changes in U due to matter in motion and changes in $\mathbf{Q}$ due to radiation. When the system communicates with its surroundings exclusively via radiation, Eq. 15 is familiar from electrodynamics. Conversely, when the system is stationary over the period when to-and-fro flows vanish so that $\partial_t\mathbf{Q} + \mathbf{v} \times \mathbf{R} = 0$ and the stable orbits are governed by $\partial_t 2K + \mathbf{v}\cdot\nabla U = 0$ which is integrable to the virial theorem $2K + U = 0$.

IV. Conclusion

The 2[nd] law of thermodynamics, when given as the equation of motion for the flows of energy, is recognized in many familiar formulas of physics – as it should be for being the supreme law of nature. However, it has perhaps remained obscure that the holistic view of nature describes non-conserved systems in evolution along open and hence non-deterministic and even chaotic trajectories. This is in contrast to the reductionist account on conserved systems orbiting along closed, modular and hence deterministic tracks. Despite that the equation of evolution possesses no precise predictive power, the 2[nd] law delineates dispersal of energy along the steepest directional descents, equivalent to the paths of least time. This imperative imposes common characteristics and patterns of nature. It allows us to realize that numerous nested natural networks [30], such as organisms [31], ecosystems [32], economies [33] and communication systems [34] emerge and evolve to provide the paths of least action for the maximal dispersal of energy [13]. The 2[nd] law underlies also the ubiquitous standards [35], skewed distributions [36], such as populations of animals and plants, income partitions and gene lengths [37], and their sigmoid cumulative curves that are on log-log plots mostly power laws [38]. The characteristic intractability of natural processes is apparent, e.g., in protein folding [39] and ecological succession [40] just as it is in the three-body problem.

The 2[nd] law, since so central, is known besides physics in other disciplines but qualitatively and descriptively. The theory of evolution by natural selection has for long been regarded as an articulation of the 2[nd] law [41] but its firm connection has surfaced only recently from the statistical physics of open systems. The imperatives of evolution have also been recognized in socio-economic contexts [42,43] to alert us from what we are.

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I was inspired by the holistic and hierarchical view of nature as professor Stanley Salthe has described it.

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