

# Rotation of Galaxies within Gravity of the Universe

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**Abstract:** Rotation of galaxies is examined by the general principle of least action. This law of nature describes a system in its surroundings, here specifically a galaxy in the surrounding Universe. According to this holistic theory the gravitational potential due to all matter in the expanding Universe relates to the universal curvature which, in turn, manifests itself as the universal acceleration. Then the orbital velocities from the central bulge to distant perimeters are understood to balance both the galactic and universal acceleration. Since the galactic acceleration decreases with distance from the galaxy's center to its luminous edge, the orbital velocities of ever more distant stars and gas clouds tend toward a value that tallies the universal acceleration. This tiny term has been acknowledged earlier by including it as a parameter in the modified gravitational law, but here the tiny acceleration is understood to result from the gravitational potential that spans across the expanding Universe. This resolution of the galaxy rotation problem is compared with observations and contrasted with models of dark matter. Also, other astronomical observations that have been interpreted as evidence for dark matter are discussed in light of the least-action principle.

**Keywords:** cosmology; dark matter; free energy; gravitation; principle of least action; vacuum

## 1. Introduction

Today dark matter is mostly held responsible for the rotation of galaxies—yet no dark matter has been found. Therefore, we reason that there is motivation to analyze the rotation curve and other observations by other alternative approaches. Here we adopt the general principle of least action in its original form from Maupertuis [1–3] rather than attempting to match data with some specific model of galactic dynamics. We are particularly motivated to use this general law, because it supposedly accounts for any system in evolution toward energetic balance with surroundings. Moreover, the law has already proven its power in explaining other astronomical observations [4–7].

Observations have revealed that orbital velocities of stars and gas clouds increase from the galactic center and seem to attain a constant value as far away as can be measured [8–10]. This type of rotation curve differs drastically from that of planets, whose orbital velocities decrease with increasing distance from the Sun. Namely, the planets orbit according to Kepler's third law

$$\frac{r^3}{t^2} = \frac{GM_0}{(2\pi)^2} \Leftrightarrow mv^2 = \frac{GmM_0}{r}, \quad (1)$$

which says that an orbiter of mass  $m$  at a radius (semi-major axis)  $r = v\tau$  from a central mass  $M_0$  completes one cycle in a period  $t = 2\pi\tau$  with velocity  $v$ . The steady-state equation of motion for the planets (Equation (1)) is an example of the more general virial theorem  $2K + U = 0$  where kinetic energy  $2K = mv^2$  tallies potential energy  $U$ , whose source is customarily ascribed to the central mass (*i.e.*,  $U = -GM_0/r$ , where  $G$  denotes, as usual, the strength of gravity).

According to Equation (1), one would anticipate decreasing orbital velocities  $v$  with increasing distance  $r$ , because almost all of the galaxy's apparent mass is in stars, including a massive central

black hole, just as almost all of the solar system's mass is in the Sun. In contrast, the orbital velocities increase when moving away from the galaxy's central bulge, and far away from the luminous edge they tend toward an asymptotic value

$$v^4 = a_t GM_o, \quad (2)$$

which is proportional via a tiny constant of acceleration  $a_t$  to the galaxy's mass  $M_o$ . The asymptote follows from the empirical Tully–Fisher relation for a constant luminosity-to-mass ratio [11,12]. When trying to make sense of the flat rotation curve of Equation (2) by Equation (1), one is inclined to think that there has to be much more matter in the galaxies than has been detected. Hence this unknown form of matter has been coined dark.

This simple logic, as abridged above, seems impeccable, but no dark matter has been found. Since specialized models have not been ubiquitously successful we reason that also a general principle of physics is worth considering to make sense of the galaxy rotation. The value of a general law over specific models in providing explanations was acknowledged by Einstein (1946) so that “a law is more impressive the greater the simplicity of its premises, the more different are the kinds of things it relates, and the more extended its range of applicability” [13]. Likewise, the excerpt: “Rational Mechanics will be the science of motions resulting from any forces whatsoever and of the forces required to produce any motions” from *Principia* reveals that Newton also valued general concepts and profound principles over detailed data of specific systems as a source of understanding. Nevertheless, many a specialist today may find the old all-inclusive tenet odd, if not implausible, but to disregard the least-action principle without analysis would be imprudent.

The universal principle places the galaxy rotation in a general context by reminding us that not only galaxies in space but also rotational vortices in fluids present orbital velocities that increase from the eye of a whirlpool and eventually settle on a constant value that depends on the surrounding potential energy [14–16]. Furthermore, the galaxies may not be so special after all, since they display the same characteristics as numerous other systems in nature, namely power laws [17–19]. The distribution of luminous mass *vs.* distance is an example [20]. Moreover, spiral galaxies resemble other logarithmic spirals in nature. These universal characteristics of galaxies may at first appear all irrelevant to the rotation problem, however, the ubiquitous scale-free patterns [21] do not emerge from system-specific features but follow from least-time free energy consumption [22]. For these reasons we are not convinced that there is inevitably something so special about galaxies that their rotation would have to be accounted for by a substance as unknown as dark matter.

We are of course aware that evidence for dark matter does not only come from the rotation curves, but notably from measurements of how much a ray of light will bend when passing by a galaxy [23,24]. However, gravitational lensing has been calculated without dark matter in agreement with observations using the same least-time principle [4]. Moreover, we will demonstrate here that the general principle explains also the high velocity dispersion of galaxies in clusters. Furthermore, we recognize that the cosmic microwave background anisotropy power spectrum with acoustic peaks has been interpreted for portions of baryonic and dark matter [25], but note that this conclusion is a model-dependent interpretation of data. So, it is the rotation of galaxies where the dark matter conjecture is best and most directly examined from a large sample of well-observed galaxies in the nearby universe.

Lastly, we acknowledge that the rotation curves have already been modeled without dark matter by Milgrom, who modified the law of gravitation by including a tiny constant of acceleration  $a_t$  as a parameter [26–28]. However, we share the view of many critics that a good fit to data is not alone an explanation, but the physical origin ought to be understood [29]. Put differently, mere numbers mean nothing but the meaning emerges first from an interpretation. In Einstein's words, “Whether you can observe a thing or not depends on the theory which you use. It is the theory which decides what can be observed”.

## 2. Superior Surroundings

It is a trivial, hence a key observation that any system, irrespective of forces involved in the particular case, is at the mercy of its surroundings. This means, for instance, that a cyclone does not whirl without a temperature gradient, a nautilus does not develop its spiral shell without fueling food, and a sunflower does not grow its twirling inflorescence without energizing light. Admittedly these examples may appear to anyone, who is uninitiated to the universal principle, outwardly unrelated to the rotation problem. Yet, these scale-free characteristics follow from the least-time free energy consumption between the system and its surroundings [22]. Therefore, we will consider the possibility that also the rotation of a galaxy and motions of galaxies in clusters bears a relation to their surroundings.

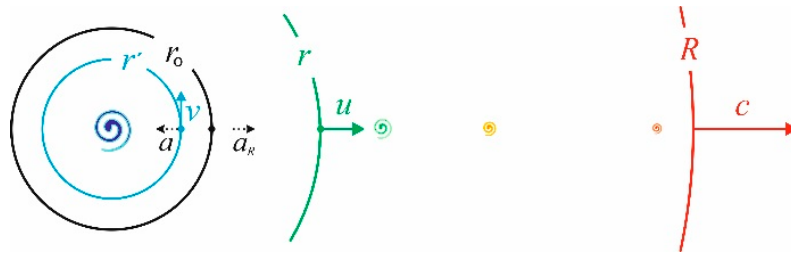
Indeed, the tiny constant of acceleration in the galaxy's velocity asymptote (Equation (2)) is indicative of the role of the universal surroundings. Namely,  $a_t$  is on the order of  $cH$  (*i.e.*, the speed of light  $c$  multiplied by Hubble's constant  $H$ ), which in turn relates to the inverse of the age  $T$  of the Universe [30]. We reason that this congruence is no quirk of the cosmos, but it reveals that the universal surroundings have a say on the rotation of galaxies as well as on their velocity dispersion in clusters of galaxies.

### 2.1. Gravity as an Energy Density Difference

We will proceed to show that the rotation of galaxies can be understood without dark matter when gravitation is considered as a force field as Feynman proposed [31]. When such a consideration is made, gravity can be understood as a force just like any other force, whose magnitude and direction is determined by the energy difference (*i.e.*, the free energy between the system and its surroundings) [4,6,7,32,33]. Namely, when the potential within the system exceeds the surrounding potential, the system will emit quanta of actions to its surroundings in a quest for leveling off the energy gradient. Conversely, when the potential within the system is below the surrounding potential, the system will absorb quanta from its surroundings to level off the energy gradient. Either way, the energy difference between the system and its superior surroundings causes changes in momenta. Therefore the system is driven toward steady-state trajectories (*i.e.*, toward the paths (e.g., orbits) on which the resultant of forces vanish).

According to this general definition of a force, gravity is an attractive force within a system of bodies when the surrounding potential is lower than that within the system. To attain balance the bodies will accelerate toward each other by releasing quanta (*i.e.*, carriers of the gravitational force, also known as gravitons) from the potential associated with the bodies to the sparser surroundings (*i.e.*, to the vacuum). So, an apple falls straight down toward the ground (*i.e.*, in least time), just like a nearby galaxy is moving toward the Milky Way because quanta escape from the energy-dense potential associated with the system of galaxies to the surrounding sparser free space (Figure 1). According to the same universal principle to consume free energy in least time, an exergonic chemical reaction will proceed forward from substrates toward products so that the system of reactants emits quanta of heat to its colder surroundings. The dissipative effect of gravity was recently demonstrated dramatically when propagating density perturbations, known as gravitational waves, were captured from a black hole binary merger [34].

Conversely, gravity is a repulsive force within the system of bodies when the surrounding potential is higher than that within the system. In a case such as this, to attain balance the bodies will move apart by acquiring quanta from the richer surrounding potential to the sparser potential within the system. So, an apple can be lifted up from the ground by consuming free energy (*i.e.*, fueling the potential associated with the two bodies with quanta that are, for example, captured from insolation). Likewise, a distant galaxy is moving away from us because the vast Universe fuels the space between the two galaxies with fluxes of quanta (Figure 1). By the same universal principle a chemical reaction will proceed backward from products to substrates when a system of reactants absorbs fluxes of quanta from its hot surroundings.



**Figure 1.** Schematic view of space that opens up from a galaxy (blue spiral) to the Universe of radius  $R = cT$  (*i.e.*, Hubble length at the age of  $T$ , expanding with the speed of light  $c$ ). At a radius  $r'$  from the galaxy's center, gravity is an attractive force because the energy density between the galaxy and a body (blue dot) exceeds that in the surrounding space. Hence, the body is subject to the acceleration,  $a$ , toward the center. When the body falls, gravitons are emitted from this system of two bodies to its sparser surroundings, and eventually, by gaining speed, the body may settle to an orbit (blue circle) with velocity  $v$  that balances the force by  $v^2/r$ . Far away from the galaxy's luminous edge within  $r' < r_0$ , the universal gravitational potential due to all matter dominates over the local potential of the galaxy, and hence the velocity profile is flat. Conversely, beyond  $r_0$  gravity turns to a repulsive force because out there the energy density of graviton influx from the surrounding sources, (*i.e.*, all other galaxies) in the Universe exceeds the efflux of quanta from the system of bodies. Hence, the distant body (green dot) at  $r$  will be subject to the universal acceleration  $a_R$  away from the center. So, it will recede with velocity  $u$  as the graviton influx from the vast space of surrounding sources produces the physical space (*i.e.*, the vacuum) between the two bodies. Accordingly, the total influx between all bodies from the combustion of all matter within  $R$  (red arc) to freely propagating quanta powers the universal expansion at the speed of light  $c$ .

In short, if the surroundings are neglected from the analysis, one cannot understand why the system is changing from one state to another, and one does not properly understand either what governs a dynamic or quasi-stationary state, such as a rotating galaxy.

The whole Universe is the surroundings of a galaxy. It must be taken into account. When there are energy gradients between the galaxy and its surroundings, these are understood by the least-time principle to decrease as soon as possible. This natural process leads to the observed characteristics. Namely, the large scale distribution of mass is uniform and the expansion of the Universe is symmetrical about any galaxy's center. From this perspective, it is no coincidence but a natural consequence that the vacuum's energy density  $\rho_E$ , on the order of  $10^{-9}$  J/m<sup>3</sup>, is in balance with the matter density  $\rho_m$  which is subject to the universal acceleration  $a_R$  within the radius of the Universe (*i.e.*,  $\rho_m a_R R = \rho_m (c/T) R = \rho_m c^2$ ).

According to the general definition of a force as an energy density difference, there is a certain distance about a galaxy where the efflux of quanta from the gravitational potential of falling bodies equals the influx of quanta from sources in its universal surroundings. When the net flow of energy from the system to its surroundings vanishes, the distance between the two bodies is steady. By the same token, concentrations of reactants do not change at a thermodynamic balance. In other words, at a stationary state, the resultant force is zero. According to astronomical observations, this zone of dynamic steady state for our Local Group of galaxies resides at a radius  $r_0$  of 1.0–1.5 Mpc away from the Group's center [7,35–37]. Obviously, only objects that are well within  $r_0$  of a given galaxy or a system of galaxies could be its orbiters. Naturally, the specific shape of a steady-state zone where inward and outward forces balance each other (e.g., for a group of galaxies) depends on the detailed distribution of mass, and hence the observed dynamics in clusters of galaxies is more intricate than that outlined simply by  $r_0$  for a single galaxy (Figure 1).

According to the least action principle, as well as according to modern physics, galaxies do not whirl in emptiness, but in the vacuum whose potential is embodied by gravitons. The vacuum energy density  $\rho_E = c^2/4\pi GT^2 \approx 10^{-9}$  J/m<sup>3</sup> is in balance with the gravitational potential  $U = GM^2/R$  due to all bodies, each of mass  $m_i$  in the Universe of total mass  $M = \sum m_i$ . The energy balance  $GM^2/R = Mc^2$  [31]

follows from the summation of the mass density  $\rho_m = 1/4\pi GT^2$  within  $R = cT$ , i.e.,  $M = \int \rho_m 4\pi R^2 dr = c^2 R/G$ . When this balance equation (i.e., the virial theorem  $2K + U = 0$  for the entire Universe) is rearranged to

$$\frac{R^3}{T^2} = GM \Leftrightarrow a_R = \frac{c^2}{R} = \frac{GM}{R^2}, \tag{3}$$

comparison of Equation (3) with Equation (1) relates the numerical value of the asymptotic acceleration per cycle  $a_t = a_R/2\pi = c/2\pi T = cH/2\pi \approx 10^{-10} \text{ ms}^{-2}$  to the age of the Universe  $T = 13.8$  billion years [38]. The value of  $a_t$  agrees with those values that have been obtained from fitting the asymptote velocity formula (Equation (2)) to the data [39]. This agreement means to us that the orbital motion of a body with velocity  $v$  at a radius  $r$  from the galaxy center balances the tiny acceleration by virtue of the curvature  $1/R = a_R/c^2$  of the huge, yet (here assumed) finite-size Universe. The length quantity  $R = cT = c/H$  can be also viewed as the horizon size, defining the largest volume with which can be causally connected to us and from which the gravitons now arriving can possibly originate.

Gravitation as a manifestation of the curvature is, of course, also at the heart of general relativity. Likewise, our reasoning about gravity applies equally to both a local and the universal curvature. Since the Universe is expanding, the asymptotic acceleration is time-dependent, and the proposed explanation of  $a_t$  could, at least in principle, be falsified by astronomical observations of the early Universe.

In the same way as the orbital velocity asymptote (Equation (2)) characterizes a galaxy with mass  $M_o$ , the recessional velocity asymptote of the expansion characterizes the Universe with total mass  $M$

$$c^4 = a_R GM. \tag{4}$$

This relation is obtained from Equation (3) by multiplying with  $a_R = c^2/R$ . The universal velocity asymptote (Equation (4)) can be rearranged to give the force of expansion  $F = Ma_R = Mc^2/R = GM^2/R^2 = c^4/G$  and the corresponding (negative) pressure  $p = F/4\pi R^2$  that powers the expansion. Likewise, the contribution of a single galaxy to the universal energy gradient (i.e., force) is obtained after rearranging Equation (2) to  $F_o = M_o a_t = v^4/G$ .

Gravitation when understood as the energy difference between the system of bodies and its surroundings, be it either way, displays itself also in Hubble’s law  $u = Hr$ , which serves to determine the distance  $r$  to a body that is receding with velocity  $u$ . The law can be rearranged by  $cH = c/T = a_R$  to a scaling relation  $u/r = c/R$ . According to the general principle, the scaling relation holds likewise for an approaching body, since the gravitational force is understood, like any other force, merely as the energy difference per distance. According to this holistic tenet, the space as the physical vacuum [7,32] between galaxies is emerging, not only when the distant galaxies are moving away from us, but also when the nearby galaxies and other close-by bodies are moving toward us. Thus, to account for the zone out there  $r' \approx r_o$ , where the body is neither receding nor approaching the scaling relations for velocity and acceleration can be rewritten as [7]

$$\frac{c}{R} = \frac{u}{r} = \frac{u' - u_o}{r' - r_o}; \frac{c^2}{R} = \frac{u^2}{r} = \frac{(u' - u_o)^2}{r' - r_o}. \tag{5}$$

Consequently, when the difference between the surrounding vacuum potential and the potential within the system is negative (i.e.,  $r' < r_o$ , in Equation (5)) the body will accelerate toward the galactic center because the sparser surroundings will accept the quanta that are released in the process. The magnitude of universal acceleration is the same for the approaching objects as it is for the receding ones, with only the sign of acceleration within  $r_o$  being opposite from that of beyond  $r_o$ .

The ratio of measured galactic to universal asymptotic velocities gives the ratio of a local mass  $M_o$  to the universal mass  $M$ , which in turn is available from the virial theorem for the Universe at the age of  $T$  (Equation (3)). By acknowledging  $a_R$  our estimates for the Milky Way  $M_o = 4 \times 10^{10}$  solar masses and for the Andromeda Galaxy  $M_o = 4 \times 10^{10}$  solar masses parallel those that are based on luminous matter

in the Milky Way [40] and the Andromeda Galaxy [41]. Thus, our analysis of the flat orbital velocities curve (Equation (2)) by the general action principle leaves no room for dark matter. Likewise, we understand that escape velocities of the Milky Way [42] build up to high values because the universal potential, not the putative potential due to dark matter, has to be also compensated. By the same token, high velocity dispersion of galaxies in clusters [43] can be obtained from the ratio of local to universal asymptotic velocities without more mass than has been deduced from the luminosities

However, if one applies the virial theorem to deduce masses in the clusters from velocities, but ignores from this equation of balance the universal gravitational potential due to the total mass of the Universe, erroneous estimates of the local masses will follow invariably [44]. Therefore, the universal gravitational potential due to all matter, communicated via the energy density of the vacuum, has to be included in the analysis of galactic rotation, just as it has to be acknowledged in all accurate accounts of gravity.

## 2.2. Velocity Asymptote

We understand that an orbiter at a distance  $r' < r_o$  from the galactic center is on a stable trajectory when its orbital velocity  $v(r)$  compensates both the galactic acceleration  $a_o = GM_o/r^2$  due to the central mass  $M_o$  within  $r$  (e.g., at the orbital radius of the Sun) and the universal acceleration  $a_R = 2\pi a_t = GM/R^2$  due to the centrally distributed total mass  $M = \Sigma m_i$  of the expanding Universe, *i.e.*,

$$\frac{v^2}{r} = a = a_o + a_t = a_o \left( 1 + \frac{a_t}{a_o} \right) = \frac{GM_o}{r^2} \left( 1 + \frac{1}{2\pi} \frac{M}{M_o} \frac{r^2}{R^2} \right). \quad (6)$$

Far away from the galaxy's luminous edge where  $a_t \gg a_o$  (Figure 1), the approximation  $v^2 a_o/r \approx a_t GM_o/r^2$  of Equation (6) is excellent. Therefore Equation (6) can be rearranged using  $v^2 = a_o r$  for the well-known asymptotic form (Equation (2)).

The flat tail of the orbital velocity curve indicates that the distant orbiter with velocity  $v$  at  $r' < r_o$  is on a least-time trajectory (*i.e.*, on a bound geodesic whose curvature  $1/r = a/v^2$  is dominated by the universal curvature  $1/R = a_R/c^2 = c^2/GM$  (Figure 1)). Conversely, when  $r' > r_o$ , the body is receding with velocity  $u$  along an open geodesic whose curvature is also  $1/R = a_R/c^2$ . So, any one body in the Universe is always subject to the tiny universal acceleration due to all other bodies, so that no body will move exactly along a straight line, which exists only in an ideal flatness without bodies.

At this point it is worth clarifying that Equation (6) is only a simple model without detailed mass distribution for the actual rotation curves. In other words, we acknowledge recent observations that reveal the flatness by Equation (2) as an oversimplification. A more matching phenomenology of rotation curves is available by including detailed mass distribution of luminous matter and halo [45].

Obviously the proposed insight to the rotation of galaxies prompts one to ask: Does the universal surroundings (*i.e.*, the gravitational potential due to all bodies in the Universe) display itself also in the orbits of planets? It does. Anomalously advancing perihelion precession, customarily attributed to the curved space-time of general relativity, has been found also by the least-action principle as a manifestation of the universal gravitational potential [4–6]. The planet's precession tallies the acceleration due to all matter in the Universe.

Yet, one may wonder how could the centrally distributed mass that resides outside of a galaxy possibly exert any net effect? It does because according to the virial theorem, the kinetic energy of a system is in a dynamic balance also with the universal gravitational potential due to the total mass of the Universe. At any moment on such a stable orbit this detailed balance of forces (*i.e.*, Newton's third Law) becomes apparent by differentiating the virial theorem

$$\int d_t (2K + U) dt = \int (\mathbf{v} \cdot d_t \mathbf{p} + \mathbf{v} \cdot \nabla U) dt = 0, \quad (7)$$

where it is implicit that momentum  $\mathbf{p}$  and acceleration  $\mathbf{a}$  are orthogonal (*i.e.*,  $\mathbf{p} \times \mathbf{a} = 0$ ). It is worth emphasizing that although the large distribution of mass about the galactic center is symmetric, the energy density of the Universe increases from the current position at  $r = 0$  toward the nascent Universe at  $R = cT$ , and hence there is indeed a gradient to be balanced by the orbital motion within  $r_0$ .

Similar to planets that are bound in the solar system, stars in globular clusters that are bound in a galaxy also do not display excessive velocities [46]. That is to say, the clusters of stars within a galaxy present no notable evidence of dark matter. We find this only natural because the surroundings of star clusters are dominated by the galactic potential, just like the planetary surroundings are dominated by the potential associated with the Sun. In contrast, dwarf galaxies, which have stellar contents comparable to the clusters of stars in galaxies, do display the galaxy-like rotational curves [47,48]. In fact, the dwarfs' velocity profiles, when interpreted by the contemporary consent, imply astonishingly high amounts of dark matter. This oddity also signals to us that dark matter is only a conjecture that follows from interpreting observations by an inaccurate tenet. Furthermore, there is no paralleling observation that a ray of light would bend astonishingly much when passing by a dwarf galaxy. Also mass distributions of early-types of galaxies are hard to model by lambda cold dark matter ( $\Lambda$ CDM) [49].

Consistently with conclusions derived from the least-action principle, clusters of galaxies do display high velocity dispersion [36,43,50] because these systems are exposed to the universal gravitational potential. Consequently, these systems are hard to model by localized dark matter [51] or by adding a tiny term to the law of gravitation [44]. Specifically,  $\Lambda$ CDM model does not account for the observations that dwarfs co-orbit the Milky Way in a plane as do those dwarfs about the Andromeda Galaxy. In contrast, the planar motion of dwarfs, as any other planar motion, appears to be a natural consequence of the central force, in this case  $F_0 = M_0 a_t$  due to the tiny universal acceleration. The force generates a torque  $\tau = \mathbf{r} \times \mathbf{F} = d_t L$  (*i.e.*, angular momentum  $L$ ) that is invariant over the orbital period. In other words, any action that displaces a body away from the center will be followed by a reaction taken by the rest of the Universe to restore the energetic balance. All in all, we conclude that the general virial theorem, also in the specific form of Kepler's third law, holds for the rotation of galaxies as well as for motions of galaxies in the clusters, but obviously only when all potentials, notably including that of the whole Universe, and associated energy differences are acknowledged in the balance with the kinetic energy.

Equation (6) is the renowned modification of the gravity law obtained when the acceleration  $a$  is multiplied with  $\mu = (1 + a_t/a_0)^{-1}$  [26,27]. Obviously, when the galactic acceleration  $a_0$  alone is used in Kepler's law, it is a very poor approximation for the galactic rotation. Likewise, velocities of bodies that are chiefly exposed to the universal energy density, such as velocities of galaxies in clusters, tally primarily the universal potential. Conversely, when the local acceleration is strong, it alone is a very good approximation (*e.g.*, for the planetary motion). When the universal acceleration is tiny relative to a local potential, it can, of course, be omitted from a practical calculation, but still *not from the explanation of how nature works*. By today, the universal radius  $R$  has grown so huge that the corresponding tiny curvature is easily masked by a local curvature.

It is worth emphasizing that the virial theorem  $2K + U = 0$  itself, even when including all potentials, is the special stationary-state case of the general principle of least action. It is easy to see that this special non-dissipative ( $d_t Q = 0$ ) equation of state follows from the general evolutionary equation [4,52]

$$d_t 2K = -\mathbf{v} \cdot \nabla U + d_t Q, \quad (8)$$

that equates changes in kinetic energy  $2K$  with changes in scalar  $U$  and vector  $Q$  potentials. Clearly, galaxies are not exactly stationary systems, but dissipative,  $d_t Q \neq 0$ . Stars are burning, and other celestial mechanisms, most notably black holes, are also devouring matter. It is this combustion of matter-bound quanta to freely propagating quanta that propels the expansion of the Universe. According to the least-time imperative, space is not an immaterial abstract geometry, but a substance that is embodied in quanta [3,32,53].

Moreover, according to the general principle, not only stationary motions but also dissipative processes pursue along geodesics (*i.e.*, least-time paths). For example, the orbital period of a binary pulsar decays with time along a parabola [54]. The quadratic relationship between the change in the period and the consumption of energy (*i.e.*, mass) follows from Equation (7). In other words, at any moment, the rate of evolution could not be any faster, and hence it is accounted for by a constant. Finally, at a free energy minimum state, the constant is zero.

### 2.3. Velocity Profile

A detailed account of the entire rotation curve of a galaxy requires detailed knowledge of the mass distribution. Earlier studies, where the mass distributions have been deduced from surface photometry and radio measurements, have proven that many velocity profiles follow Equation (6) [55]. The agreement is, in fact, impressive in comparison with dark matter halo models when considering that the only adjustable parameter is the stellar mass-to-luminosity ratio. Moreover, fine features in the observed profiles tend to get smeared out when curves are modeled by dark matter [56]. In some sense though one could say that the universal background potential due to all matter could be regarded as the omnipresent halo. Although space is dark, its substance, as we will shortly explain, is not mysterious; the vacuum is embodied with tangible quanta.

Thus, mathematically we have nothing to add to the functional form of Equation (6), but we are able to give physical meaning to this model using the least-time principle. In general, not only is the galactic rotation curve a sigmoid from the center to outskirts, but similar cumulative curves, also with damping oscillations, are found everywhere in nature [22]. These curves sum up from skewed nearly log-normal distributions [57] and appear on a log-log scale approximately as comprising pieces of straight lines. Also, the rotational curve, when modeled by the Sérsic profile [20]  $\ln I(r) \propto r^{1/n}$  for the surface brightness  $I$  vs. distance  $r$  from the galactic center, is a power law [58]. Sérsic index  $n = 4$  corresponds to de Vaucouleur's profile for elliptical galaxies [59]. For spiral disks and dwarf elliptical galaxies  $n = 1$  is a good model [60].

In any case, the slope

$$\frac{d \ln I(r)}{d \ln r} \propto -\frac{1}{n} r^{1/n}, \quad (9)$$

of brightness  $I$  vs. distance  $r$  is a straight line on a log-log plot. Eventually the whole profile compiles from a series of straight lines (*i.e.*, brightness follows a broken power law when the index  $n$  varies over a range starting from the central bulge to the luminous edge). Since brightness equals integrated luminosity, and luminosity, in turn, relates to mass, we conclude that the mass distribution also accumulates along a broken power law. Hence the orbital velocity  $v$  vs. radial distance  $r$  given by Equation (6) can be regarded as a profile comprising pieces of straight lines on the log-log plot.

In general, oscillatory behavior is common both in space and time when a system faces a sudden change in free energy (*i.e.*, a potential step). For example, laser light oscillates for a while when switched on. Likewise, chemical concentrations and animal populations tend to fluctuate when exposed to rich resources, before settling to a steady state. Moreover, the intensity of coherent and mono-chromatic light builds up in an oscillatory manner as a function of distance from an obstacle's edge. On astronomical scales, the change in potential from the dense active galactic nucleus to the sparse universal surroundings is a brisk change in energy density. Therefore, we expect the most massive and compact galaxies, as well as those that have been recently perturbed by mergers with other galaxies, to display velocity profiles with pronounced oscillations and asymmetry.

It is worth emphasizing that the power law is not merely a phenomenological model (e.g., for the velocity profile  $v(r)$  and mass distributions), but a consequence of the least-time free energy consumption. According to the principle in its original form by Maupertuis, the galaxies are regarded as powerful machinery for free energy consumption. These celestial engines (*i.e.*, stars, black holes, *etc.*) transform matter-bound quanta to free quanta (*i.e.*, photons). This characteristic action manifests



itself in the mass-to-light ratio that is constant over a broad range, at least over seven magnitudes in luminosity [61].

According to the least-time principle, galaxies evolve and merge to attain and maintain maximal free energy consumption in the changing and ageing universal surroundings. When a galaxy increases in mass by mergers, its realm  $r_o$  contained within the Universal curvature will extend even further out for it to devour even more matter to institute even more powerful machinery of free energy consumption, such as a gigantic black hole. Apparently by this powerful celestial mechanism baryonic matter is broken down into quanta that jet out in free propagation [62]. Star formation from gas clouds can also be regarded likewise (*i.e.*, as evolution in the quest of free energy consumption).

### 3. The Physical Substance of the Vacuum

This account for the rotation of galaxies and their velocities in clusters by virtue of the universal gravitational potential would be incomplete without an explanation of how the gravitational force is carried over from all those distant bodies. Their effect has long been argued for by pointing out that the amount of matter on ever more distant spherical shells is increasing as  $r^2$ , and hence is superseding the gravitational potential that is decreasing as  $r^{-1}$ . Thus the rotating galaxy, like an ice skater performing a pirouette, is an archetype of Mach's principle where the local motion is governed by the large-scale structure of the Universe. However, now we have to explain how does the mass out there influence the inertia here? So, what is the substance, if not dark matter or dark energy, that embodies and communicates both the local gravitational potential and the universal potential, known as the vacuum's energy density? In other words, we have to explain what the graviton is [33,53].

The free space characteristics, permeability and permittivity, which relate to the squared speed of light via  $c^2 = 1/\epsilon_o\mu_o$ , and their invariant ratio, the squared impedance  $Z^2 = \epsilon_o/\mu_o$ , suggest to us that the space is, after all, embodied by photons. At first the conjecture may seem absurd since space is not bright but dark. However, any two photons, when co-propagating with opposite phases, cancel each other's electromagnetic fields. This phenomenon is familiar from diffraction. The photons that are subject to complete destructive interference do not vanish but continue to propagate. By the same token, we reason, that free space is embodied by the photons on average in pairs of opposite polarization. These paired photons (*i.e.*, compound bosons) would be in this view the gravitons. Due to the opposing phases the paired photons do not display themselves as carriers of electromagnetic forces. Nonetheless the energy density in the "gas" of photon pairs will move to average out energy density differences. Thus, the paired photons act as carriers of gravitational force. The graviton, when understood as a compound boson comprising two photons with opposite phases, will readily move to attain and maintain the energy balance among all bodies in the Universe. Since both gravity and electromagnetism are carried by photons, their functional forms are similar, but their strengths differ greatly [3,32].

Perhaps it is worth stressing that by the photon-embodied vacuum we do not mean the old and abandoned luminous ether. The photon-embodied vacuum is not only a medium supporting photon propagation, but the paired photons themselves total the vacuum energy density which is in balance with the total mass of the Universe [31]. Likewise, the local energy density, known as the gravitational potential of a body, is embodied by paired photons whose density is in energetic balance with the body. Thus, gravity is the force (*i.e.*, the energy difference between the local density and the surrounding density). According to the least-time principle, any difference in energy will vanish as soon as possible, and hence objects will accelerate along geodesics by dissipating quanta from the rich local potential to the sparser superior surroundings. Conversely, objects would escape along geodesics when quanta of gravitation would flow toward a sparse local potential from the richer surroundings.

The photon-embodied vacuum is the omnipresent highly mobile substance that will adjust its density at the speed of light to any density perturbation. Thus, when a body moves relative to all other bodies, the photons embodying the vacuum will move to restore the energy balance. This reaction by

the vacuum to the action of a body manifests itself as inertia. By the same token, curvilinear motion is accompanied with inertial effects.

Furthermore, Hubble's law for the Universe,  $c = HR$ , when divided by the age  $T$  of the Universe, gives the expression  $cH = c^2/R = GM/R^2$ . This reveals that the expansion is powered by consuming the energy difference between the energy that is bound in the total mass of the Universe and the vacuum's energy embodied in the freely propagating quanta. The Universe is expanding because the quanta that are bound in the energy-dense matter are released by stars, black holes, *etc.*, to photons, obviously in the form of light, but mostly in the form of photon pairs without net polarization. These freely propagating quanta are diluting the density. Thus, energy in matter,  $E = Mc^2$ , fuels the expansion with power  $P = E/T = c^5/G$ . The least-time expansion along geodesics ensures uniformity at the largest scale, *i.e.*, solves the horizon problem. Since there is still free energy (*i.e.*, in the form of mass) to power the expansion, the present-day Universe is not exactly flat, but slightly curved due to its finite radius  $R = cT$ . Since  $R$  is huge, the Euclidean metric is an excellent approximation over many orders of magnitude.

Moreover, when the curvature of space is modeled, most notably, by the Riemann metric, the results are in excellent agreement with observations for many loci, but the constant-energy model does not account for the evolution of the energy density. This space-time notion of general relativity also remains abstract because space is not understood as a tangible substance embodied by the paired photons [33,53]. When the Universe is deemed to be infinite and flat by fitting data to the Friedmann–Lemaître–Robertson–Walker (FLRW) metric [63], the flatness in that model means that the average density equals the critical density of mass which is seen as necessary to eventually halt the expansion. However, here the geometry of the Universe is found to emerge from changes in energetics. It is worth clarifying that only when a system is in a free energy minimum state, such as a gas molecule in a stable orbit around a galaxy, can the equation of motion be transformed to a time-independent frame of reference, that is, solved exactly.

It is apparent from Equation (8) that the energy and momentum of the system of bodies are not conserved when the bodies are understood to accelerate toward each other so that paired quanta (gravitons) are emitted to the surrounding space. Likewise these quantities are not conserved when the bodies are understood to recede away from each other when the quanta are absorbed from the superior surroundings of the Universe to the local potentials. Presumably the MOND-model (Equation (6)) has been shunned in particular because in that model energy and momentum are not conserved. However, there is really no profound reason to insist on having conserved energy and momentum in a system that is open to its surroundings. One might maintain that the Universe as a whole would be a closed system by including everything, but such a thought is flawed because the photons themselves are open quanta of action. Namely, freely propagating photons are open paths that will adapt their energy to the surrounding energy density by shifting frequency, whereas quanta that are bound to closed orbits in matter cannot adapt without breaking their paths of symmetry [3,32].

When it comes to conservation laws, it would be the total number,  $n \approx 10^{121}$ , of quantized actions that is fixed in the Universe [3,32]. This elementary estimate for this invariant number of the basic building blocks follows from  $n = Mc^2T/h$ . This invariance is the essence of Noether's theorem (*i.e.*, that the total action  $\int 2Kdt = nh$  of the Universe is conserved). Planck's constant  $h = Et$  is the measure of a quantum of action that remains invariant under concomitant changes of energy and time. In other words, any change of state, for instance a displacement of a body relative to all other bodies, will break symmetry either by the emission or absorption of quanta. Yet, many familiar theories of physics are fixed in symmetry, and hence these models cannot account accurately for changes of state due to gravity or any other form of energy differences. Most notably, quantum electrodynamics that complies with Lorentz covariance yields a value of  $10^{113}$  J/m<sup>3</sup> for the vacuum energy density, which is in a flagrant contrast with observations.

#### 4. Discussion

The rotation of galaxies is difficult to understand when one attempts to match it with the orbital motion of planets. Mass would be missing when the focus is only on the galaxy because thereby its surroundings (*i.e.*, the whole Universe) are ignored. In this way one will erroneously conclude that the missing mass has to be in the galaxy, and since it is invisible, it has to be dark. Search for dark matter is further centered about the galaxy only because one thinks, by counting luminous matter, that a ray of light is bending more than it should. However, that gauge was miscalibrated because parallax was ignored when the degree of bending was deduced from the difference between a ray passing by the eclipsed Sun and a night-sky ray [4]. Therefore, the galaxy rotation problem cannot be solved satisfactorily by presenting an unknown substance or alternatively by introducing an impromptu modification to the law of gravitation. We believe that a proper comprehension entails correcting not one, but several misconceptions.

A brief account of history allows us to understand why physics turned away from the old general principle of least time to particular forms, such as that due to Lagrange. The general principle accurately describes systems in evolution toward energy balance with their surroundings, but it was shelved soon after appearing because the original equation did not meet the expectations of a computable law. At the time when physics emerged from natural philosophy, the non-dissipative form (*i.e.*, Lagrange's equation) became the standard, because physics, as the new powerful discipline, was expected, at least in principle, to be able to predict everything by calculation. Today we understand that the quest for a universal calculation method is futile. This is not because natural systems tend to be too complicated or too numerous in their details to be known exactly, but because intractability follows from the fact that everything depends on everything else. When a system changes from one state to another by dissipating quanta, its surroundings will also change by absorbing those very same quanta, and *vice versa*. Since the boundary conditions keep changing along with the motion, evolution is a path-dependent process. This is familiar from the three-body problem. As well, in galaxies we recognize signs of past processes, such as remnants of incorporated dwarf galaxies. Only at a stationary state, when there is no net flux of quanta, would a system orbit on a computable trajectory. Therefore, in the quest of calculating everything, physics curtailed its mathematical forms to models that conserve energy. Riemannian metric, for instance, complies with the conservation of energy. At energy balance the net force vanishes, so one tends to ignore the surroundings altogether and focus only on a system's constituents and mechanisms.

Customarily, when examining galaxy rotation, one takes Kepler's third law either as an accurate model that just needs more matter to account for the orbital velocity profile, or alternatively one takes Kepler's third law as an imprecise model that needs a modification to match the data. Even when one correctly recognizes the third law as a special case of the more general virial theorem, one will dismiss the surrounding potential when not realizing that the equation for the free-energy minimum state is itself a special case of the general least-time principle. So, when ignoring surroundings, one will ascribe the orbital motion as a balance between the centripetal and centrifugal forces, or more tacitly via a curved metric, but not as a thermodynamic balance between the system of bodies and its surroundings. The correct comprehension is that the outermost stars and gas clouds of a galaxy do not rip away by rotation because the sparse surrounding vacuum does not supply quanta with energy that would be needed for such a change in momentum. Conversely, one should explain that a distant galaxy is receding, because a huge flux of energy from the Universe enters between us and the distant galaxy. Eventually the recessional velocity will limit the speed of light when the distance between us the perimeter of the Universe is open to the flux from the whole Universe.

Naturally, one is inclined to omit the surrounding potential from the balance with kinetic energy when one cannot see how the distant bodies exert force here. The true trouble is that inertia appears to be instantaneous. Although the characteristics of the vacuum associate with light and although gravitation and electromagnetism have similar forms, one has not quite been able to grasp the idea of photons being the carriers of gravitational force [64–66]. Instead, modern physics imagines that

photons are virtual particles that will emerge from the vacuum and vanish into the vacuum [67]. However, when one does not see that the vacuum density is embodied by photons on average in pairs of opposite polarizations, one fails to understand inertia as the reaction taken by the Universe via the tangible photon-embodied vacuum to actions taken by a body, in order to regain an overarching energy balance. The inertial effects appear instantaneous because the vacuum embraces everything.

All in all, the prevailing but impaired comprehension of galactic rotation and the high velocity dispersion of galaxies in clusters follows from several deeply-rooted misconceptions. Most importantly, the failure to describe the omnipresent vacuum as a photon-embodied tangible substance that maintains energy balance with all matter in the Universe, has misled one to ignore the superior surroundings. Consequently, observations have become accounted for by overly complicated cosmological models tinkered with exceedingly abstract notions, most notably with dark matter. Today models that comply with data, at least partially, are mistaken as explanations, and hence alternative conclusions drawn from the general principle of physics tend to be contrasted against the prevailing specific models within a field rather than to be evaluated against observations.

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