



Quantum entanglement and classical correlation have the same form

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Abstract In agreement with the data, the quantum correlation between spins violates Bell's inequality by following a cosine curve when one analyzer is rotated relative to the other. In contrast, the linear correlation attributed to hidden variables has never been observed. Besides these well-established facts, we show here that classical covariance, Pearson correlation, between spins projected as up or down on the analyzer axes also follows the cosine form hitherto uniquely ascribed to the quantum mechanical expectation value. The common cause for the classical correlation is the conservation of intrinsic angular momentum that aligns the two spins antiparallel at the breakup. Thus, as long as the spins retain their orientations relative to each other, the measurement of one spin in a chosen frame of reference also discloses the opposite orientation of the other in that frame. Realizing that classical correlation has the same functional form as quantum entanglement sheds light on the foundations of modern physics and quantum computing.

1 Introduction

To date, measurements have proven beyond reasonable doubt that correlations between two particles, e.g., photons [1–3], neutrinos [4], and electrons [5], separated by a distance in space or time [6] outside causal influence violate Bell's inequality. When one analyzer is rotated relative to the other, the correlation between paired spins or paired photons follows a cosine curve ascribed to quantum entanglement, whereas the linear hidden-variable form has not been observed [7–11].

Regrettably, the hidden-variable model has been mistaken for the classical correlation, leading to the widely spread but incorrect impression that quantum correlations would be stronger than classical correlations [12]. However, we will show below that the conventional Pearson correlation between recordings of paired spins, or paired photons, also takes, by definition, the same cosine form as the quantum correlation of entangled particles. Furthermore, the ordinary correlation has been considered ruled out ever since the classical model of the EPR experiment [13] was mistaken to involve negative probabilities [14, 15]. However, we will demonstrate that the probabilities remain non-negative when the EPR experiment is correctly interpreted in terms of the conventional Pearson correlation.

The same cosine form is not surprising. Like quantum correlation, classical correlation invariably involves spins that are paired so that angular momentum is conserved or photons whose total polarization is conserved. Since the functional forms of quantum and classical correlation of paired particles are identical and thus indistinguishable, the difference lies in comprehending the physics behind the data.

Instead of entering into the age-old debate, Einstein, Podolsky, and Rosen (EPR) challenging Bohr and Heisenberg on whether particle property, such as spin, has or has not a definite value until measured [16–20], we will merely attest that classical, Pearson correlation shares with quantum entanglement the very same renowned non-linear form that violates Bell's inequality [13].

2 Correlation

Bell analyzed an experiment where spins, paired at the breakup antiparallel, σ and $-\sigma$, in random directions, are projected, $\sigma \cdot \mathbf{a}$ and $-\sigma \cdot \mathbf{b}$, either parallel or antiparallel to the unit vectors, \mathbf{a} and \mathbf{b} , that describe the two detectors, A and B [7]. The similarity measure between \mathbf{a} and \mathbf{b} is defined as the correlation, the inner product, $\mathbf{a} \cdot \mathbf{b}$ [21]. In agreement with the correlation estimated from a large number of coincident counts, the quantum mechanical expectation value, $\langle (\sigma \cdot \mathbf{a})(-\sigma \cdot \mathbf{b}) \rangle = -\mathbf{a} \cdot \mathbf{b} = -|a||b|\cos\theta$, discloses the angle, θ , between the detector directions [7].

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While Bell contrasted the result of quantum mechanics with a hypothetical outcome predetermined by hidden variables, we point out that the correlation between the a_i and b_i coincident counts, in total N , can be understood in terms of the ordinary correlation, the Pearson correlation coefficient,

$$r = \frac{\sum_{i=1}^N (a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum_{i=1}^N (a_i - \bar{a})^2 \sum_{i=1}^N (b_i - \bar{b})^2}}. \tag{1}$$

Since the spin directions vary randomly from one recording to another, the averages, $\bar{a} = \bar{b} = 0$ [22–24], and the correlation reduces to cosine similarity,

$$E = \frac{\langle a, b \rangle}{\|a\| \|b\|} = \cos \theta = S_c(\mathbf{a}, \mathbf{b}). \tag{2}$$

Thus, after rearranging $S_c(\mathbf{a}, \mathbf{b})$ for the inner product with the negative sign for antiparallel spins, $-\langle a, b \rangle = -\mathbf{a} \cdot \mathbf{b} = -\|a\| \|b\| \cos \theta$, classical correlation is found identical to the quantum mechanical expectation value.

2.1 Paired spins

As shown earlier, the ordinary correlation [13],

$$E = \frac{\langle (\sigma \cdot \mathbf{a})(-\sigma \cdot \mathbf{b}) \rangle}{\sqrt{\langle (\sigma \cdot \mathbf{a})^2 \rangle} \sqrt{\langle (-\sigma \cdot \mathbf{b})^2 \rangle}} = -\cos \theta, \tag{3a}$$

integrates the spin projections over the azimuth, φ , and polar, ϑ , angles,

$$\begin{aligned} \langle -(\sigma \cdot \mathbf{a})(\sigma \cdot \mathbf{b}) \rangle &= -\frac{\sigma^2 ab}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \sin \vartheta (\sin \vartheta \cos \varphi)(\sin \vartheta \cos(\varphi - \theta)) \\ &= -\frac{1}{3} \sigma^2 ab \cos \theta, \end{aligned} \tag{3b}$$

$$\langle (\sigma \cdot \mathbf{a})^2 \rangle = \frac{(\sigma a)^2}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \sin \vartheta (\sin \vartheta \cos \varphi)^2 = \frac{1}{3} (\sigma a)^2, \tag{3c}$$

$$\langle (\sigma \cdot \mathbf{b})^2 \rangle = \frac{(\sigma b)^2}{4\pi} \int_0^{2\pi} d\varphi \int_0^\pi d\vartheta \sin \vartheta (\sin \vartheta \cos(\varphi - \theta))^2 = \frac{1}{3} (\sigma b)^2. \tag{3d}$$

The fractions 1/3 in $\langle -(\sigma \cdot \mathbf{a})(\sigma \cdot \mathbf{b}) \rangle$ and $\sqrt{\langle (\sigma \cdot \mathbf{a})^2 \rangle} \sqrt{\langle (\sigma \cdot \mathbf{b})^2 \rangle}$, canceling each other out, derive from the projections, $\sigma \cdot \mathbf{a}$ and $-\sigma \cdot \mathbf{b}$, of spins in *three* dimensions (3d) into *one* dimension (1d) of the two detectors, \mathbf{a} and \mathbf{b} . In other words, while the unit vector norms are $\|a\| = \|b\| = 1$, the average of projections from spins, uniformly spread out in 3d, into 1d are $\sqrt{\langle (\sigma \cdot \mathbf{a})^2 \rangle} = \sigma a / \sqrt{3}$ and $\sqrt{\langle (\sigma \cdot \mathbf{b})^2 \rangle} = \sigma b / \sqrt{3}$.

However, since the spins are indivisible, they align *fully* along \mathbf{a} and \mathbf{b} according to their *projections*, $\sigma \cdot \mathbf{a}$ and $-\sigma \cdot \mathbf{b}$. In this way, the classical analysis yields two-valued, +1 or -1, instead of continuous [-1, 1] results [13, 14].

For random spin orientations, the up (+) and down (-) fractions, i.e., probabilities, covering the upper and lower hemispheres relative to \mathbf{a} , aligned along $\varphi = 0$, and relative to \mathbf{b} , aligned along $\varphi - \theta$, are equal and constant,

$$P_+(a) = \frac{N_+}{N} = +\frac{1}{4} \int_{-\pi/2}^{\pi/2} d\varphi \cos \varphi = \frac{1}{2}, \tag{4a}$$

$$P_-(a) = \frac{N_-}{N} = -\frac{1}{4} \int_{\pi/2}^{-\pi/2} d\varphi \cos \varphi = \frac{1}{2}, \tag{4b}$$

$$P_+(b) = \frac{N_+}{N} = +\frac{1}{4} \int_{-\pi/2+\theta}^{\pi/2+\theta} d\varphi \cos(\varphi - \theta) = \frac{1}{2}, \tag{4c}$$

$$P_-(b) = \frac{N_-}{N} = -\frac{1}{4} \int_{\pi/2+\theta}^{-\pi/2+\theta} d\varphi \cos(\varphi - \theta) = \frac{1}{2}. \tag{4d}$$

Obviously, the angle, θ , between the detector directions, a and b , does not show up in the sequence of counts recorded at a detector but first in the comparison of counts recorded at the two detectors. The joint probabilities

$$P_{+-}(a, b) = \frac{N_{+-}}{N} = +\frac{1}{4} \int_{-\pi/2+\theta}^{\pi/2} d\varphi \cos \varphi = \frac{1}{4} (1 + \cos \theta), \tag{5a}$$

$$P_{-+}(a, b) = \frac{N_{-+}}{N} = -\frac{1}{4} \int_{\pi/2+\theta}^{-\pi/2} d\varphi \cos \varphi = \frac{1}{4}(1 + \cos \theta), \tag{5b}$$

$$P_{++}(a, b) = \frac{N_{++}}{N} = +\frac{1}{4} \int_{-\pi/2+\theta}^{-\pi/2} d\varphi \cos \varphi = \frac{1}{4}(1 - \cos \theta), \tag{5c}$$

$$P_{--}(a, b) = \frac{N_{--}}{N} = -\frac{1}{4} \int_{\pi/2}^{\pi/2+\theta} d\varphi \cos \varphi = \frac{1}{4}(1 - \cos \theta), \tag{5d}$$

sum up the spins over the angular intervals where the product of cosines, $\cos \varphi \cos(\varphi - \theta)$, (Eq. 3b) does not change sign [13].

Accordingly, the coincident count combinations, ++, +-, -+, --, provide the estimate, $E = -(N_{+-} + N_{-+} - N_{++} - N_{--})/N = -\cos \theta$, for correlation as the difference between the antiparallel, $N_{+-} + N_{-+}$, and parallel, $N_{++} + N_{--}$, counts. In this way, it is understood that the coincident counts of numerous spins, paired oppositely at the breakup, examine the (cosine) similarity between the detector directions. For example, when the two analyzers are collinear, $\theta = b - a = 0, \pm\pi$, then $E = -1, +1$. Conversely, when the detectors are orthogonal, $\theta = \pm\pi/2$, then $E = 0$. And, when $\theta = \pm\pi/4, \pm3\pi/4$, then $E = -1/\sqrt{2}, +1/\sqrt{2}$.

It is worth emphasizing the joint probabilities (Eqs. 5a–5d) of the classical correlation, $E(a, b) = P_{+-}(a, b) + P_{-+}(a, b) - P_{++}(a, b) - P_{--}(a, b) = \cos \theta$, are not negative and add to unity $P_{+-} + P_{-+} + P_{++} + P_{--} = 1$. The non-physical negative probabilities derive from applying the hidden variable, λ , expectation value [25],

$$P_{\pm\pm}(\mathbf{a}, \mathbf{b}) = \int_{\Lambda} d\lambda \rho(\lambda) P_{\pm}(\mathbf{a}, \lambda) P_{\pm}(\mathbf{b}, \lambda), \tag{6}$$

erroneously for the classical correlation [13–15]. While the detector responses, $P_{\pm}(\mathbf{a}, \lambda) = \pm 1$ and $P_{\pm}(\mathbf{b}, \lambda) = \pm 1$, weighted by probability density, $\rho(\lambda)$, over the hidden variable space, $\lambda \in \Lambda$, do not depend on each other, as denoted by the multiplication, the classical analysis, as above, does not presume hidden variables, such as spin angles to the detector directions [13, 26], to account for the correlation. Instead, the total spin conservation is the common cause of the correlation. When the spins are paired antiparallel, irrespective of their angles to the detectors, the joint probabilities (Eqs. 5a–5d) are non-negative. This result is contrary to the incorrect inference that focuses on the angles between the spins and their detectors varying randomly from pair to pair while disregarding the invariant angle, π , between the antiparallel spins [14, 15]. Since we dissociate ourselves from the hidden-variable model (Eq. 6) altogether and find classical correlation to have the same form as quantum correlation, the Bell inequalities [7, 25], as functions of correlations, $E(a, b)$, obtained from the differences $\theta = b - a$ between various detector angles, a and b , are violated without reconsidering the test statistics [27].

2.2 Paired photons

Today, rather than paired particles, more often paired photons, \mathbf{p} and $-\mathbf{p}$, are projected, $\mathbf{p} \cdot \mathbf{a}$ and $-\mathbf{p} \cdot \mathbf{b}$, on the two-channel analyzers whose directions are defined by the Cartesian unit vectors, $\mathbf{a}_x, \mathbf{a}_y, \mathbf{b}_x, \mathbf{b}_y$ (Fig. 1). Then the photon phasor (cf. the Jones vector), $e^{i\phi}$, projections, $a_x = \cos \phi$ or $a_y = \sin \phi$, and the other photon phasor, $e^{i\phi+\pi}$, projections, $b_x = \cos(\phi + \pi - \theta)$ or $b_y = \sin(\phi + \pi - \theta)$, averaged over the random phase, ϕ , give the similarity measure,

$$E = \frac{\langle a_x b_x + a_y b_y \rangle}{\sqrt{\langle a_x^2 + a_y^2 \rangle} \sqrt{\langle b_x^2 + b_y^2 \rangle}} = -\cos \theta, \tag{7a}$$

where integrals over the polar angle, ϕ ,

$$\begin{aligned} \langle a_x b_x + a_y b_y \rangle &= \frac{1}{2\pi} \int_0^{2\pi} d\phi (\cos \phi \cos(\phi + \pi - \theta) + \sin \phi \sin(\phi + \pi - \theta)) \\ &= -\cos \theta, \end{aligned} \tag{7b}$$

$$\langle a_x^2 + a_y^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi (\cos^2 \phi + \sin^2 \phi) = 1, \tag{7c}$$

$$\langle b_x^2 + b_y^2 \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi (\cos^2(\phi + \pi - \theta) + \sin^2(\phi + \pi - \theta)) = 1. \tag{7d}$$

Thus, the result is identical to the quantum mechanics calculation $\langle (\mathbf{p} \cdot \mathbf{a})(-\mathbf{p} \cdot \mathbf{b}) \rangle = -\mathbf{a} \cdot \mathbf{b} = -\cos \theta$ [7, 13].

Similar to Eqs. 5a–5d, the probability terms in the experimental estimate of correlation $E = N_{x\bar{x}} + N_{\bar{x}x} + N_{y\bar{y}} + N_{\bar{y}y} - N_{x\bar{y}} - N_{\bar{y}\bar{x}} - N_{y\bar{x}} - N_{\bar{x}\bar{y}}/N = \cos \theta$, sum up the photons over the angular intervals where the products $\cos \phi \cos(\phi + \pi - \theta)$ and $\sin \phi \sin(\phi + \pi - \theta)$, (Eq. 7a) do not change sign [13].

Akin to convolution, correlation (Eqs. 7a–7d) shows how one output modulates the other upon pivoting one analyzer relative to the other. For example, when $\theta = 0, \pm\pi$, the two analyzers are collinear and $\langle a_x b_x + a_y b_y \rangle = -1, +1$, hence $E = -1, +1$. Conversely, when $\theta = \pm\pi/2$, then $\langle a_x b_x + a_y b_y \rangle = 0$, hence $E = 0$. And, when $\theta = \pm\pi/4, \pm3\pi/4$, then $\langle a_x b_x + a_y b_y \rangle = -1/\sqrt{2}$,

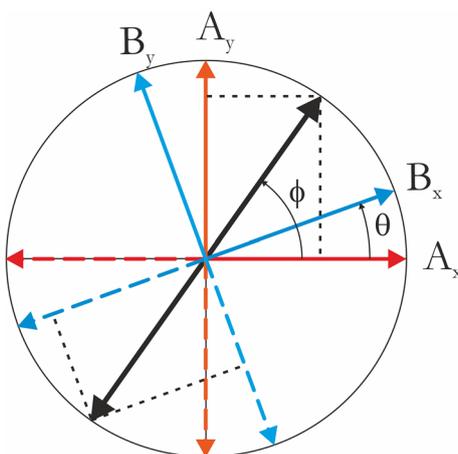


Fig. 1 Phases of two photons, propagating along the z-axis, point in opposite directions (black arrows) in the xy-plane. Each photon is counted after passing through an analyzer (A and B) whose phase-sensitive channels are perpendicular ($A_x \perp A_y$ and $B_x \perp B_y$). The corresponding axes (A_x & B_x and A_y & B_y) are at an angle θ relative to each other. The phase-sensitive detection distinguishes ϕ from $\phi + \pi$ by the signs of projections. Thus, one photon with phase, ϕ , projecting along A_x , is proportional to $\cos \phi$ and along A_y to $\sin \phi$, and the other photon with the opposite orientation, $\phi + \pi$, projecting along B_x , is proportional to $\cos(\phi + \pi - \theta)$ and along B_y to $\sin(\phi + \pi - \theta)$

$+1/\sqrt{2}$, hence $E = -0.71, +0.71$. Thus, one can show, as Bell did for the quantum mechanical expectation value [7], that neither classical correlation can be represented, accurately or arbitrarily closely, in the form of Eq. 6.

In practice, instead of the phase-sensitive detection described above, the photons, emerging pairwise from spontaneous parametric down-conversion (SPDC), are counted as having horizontal, H, or vertical, V, polarization after passing through a two-axis polarization analyzer [8, 13, 28, 29]. The $\pi/2$ shift between the orthogonal polarizations preserves total polarization, including the pump photon polarization [30]. In general, a polarization rotator placed on a photon path shifts the correlation by an angle of choice, δ .

Since the polarization analyzer cannot distinguish the photon with phase, ϕ , from the one with the opposite phase, $\phi \pm \pi$, in essence, the projection of a phasor with two heads is detected. Hence, the coincident counts vary quadratically $\cos^2 \theta = 1/2(\cos 2\theta + 1)$ at a double rate, 2θ , with half counts, $1/2$, compared to the phase-sensitive detection. Consequently, the averages are $1/2$ along the horizontal, $\bar{a}_x = \langle \cos^2 \phi \rangle$ and $\bar{b}_x = \langle \cos^2(\phi - \theta) \rangle$, and vertical, $\bar{a}_y = \langle \sin^2 \phi \rangle$ and $\bar{b}_y = \langle \sin^2(\phi - \theta) \rangle$, directions. Thus, correlation (Eqs. 1 and 6), by two-channel detection,

$$\begin{aligned}
 E &= \frac{1}{\pi} \int_0^\pi [(\cos^2 \phi - 1/2)(\cos^2(\phi - \theta) - 1/2) \\
 &\quad + (\sin^2 \phi - 1/2)(\sin^2(\phi - \theta) - 1/2)] d\phi / \\
 &\quad \left[\sqrt{\frac{1}{\pi} \int_0^\pi [(\cos^2 \phi - 1/2)^2 + (\sin^2 \phi - 1/2)^2] d\phi} \right. \\
 &\quad \left. \times \sqrt{\frac{1}{\pi} \int_0^\pi [(\cos^2(\phi - \theta) - 1/2)^2 + (\sin^2(\phi - \theta) - 1/2)^2] d\phi} \right] \\
 &= \cos 2\theta,
 \end{aligned}
 \tag{8}$$

has the same form as obtained by quantum mechanics [8, 9]. As above (Eqs. 5a–5d), the probabilities,

$$P_{HV} = +\frac{1}{4} \int_{3\pi/4}^{3\pi/4+\theta} d\phi \cos 2\phi + \frac{1}{4} \int_{7\pi/4}^{7\pi/4+\theta} d\phi \cos 2\phi = \frac{1}{4}(1 - \cos 2\theta),
 \tag{9a}$$

$$P_{VH} = -\frac{1}{4} \int_{\pi/4}^{\pi/4+\theta} d\phi \cos 2\phi - \frac{1}{4} \int_{5\pi/4}^{5\pi/4+\theta} d\phi \cos 2\phi = \frac{1}{4}(1 - \cos 2\theta),
 \tag{9b}$$

$$P_{HH} = +\frac{1}{4} \int_{-\pi/4+\theta}^{\pi/4} d\phi \cos 2\phi + \frac{1}{4} \int_{3\pi/4+\theta}^{5\pi/4} d\phi \cos 2\phi = \frac{1}{4}(1 + \cos 2\theta),
 \tag{9c}$$

$$P_{VV} = -\frac{1}{4} \int_{\pi/4+\theta}^{3\pi/4} d\phi \cos 2\phi - \frac{1}{4} \int_{5\pi/4+\theta}^{7\pi/4} d\phi \cos 2\phi = \frac{1}{4}(1 + \cos 2\theta),
 \tag{9d}$$

integrate the combinations of horizontal, H , and vertical, V , coincident counts [29] over the angular intervals without changes between H and V .

Since classical correlation and quantum entanglement share the same functional form, the Bell [7] or CHSH inequality [25] or other measures, e.g., uncertainty relations, entanglement entropy, and concurrence [31–34], cannot distinguish one from the other. Explicitly, the classical correlation, $E = \cos 2\theta$, inserted into the test statistic

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b') \quad (10)$$

with the A detector settings $a = 0$ and $a' = 45^\circ$, and the B detector settings $b = 22.5^\circ$ and $b' = 67.5^\circ$, violates the CHSH inequality. The four combinations correspond to the angle between the detectors, $\theta = b - a = b' - a' = 22.5^\circ$, $\theta = b - a' = -22.5^\circ$, and $\theta = b' - a = 67.5^\circ$. Thus, Eq. 10, yields

$$S = \cos(2 \times 22.5^\circ) - \cos(2 \times 67.5^\circ) + \cos(-2 \times 22.5^\circ) + \cos(2 \times 22.5^\circ) = 2\sqrt{2}, \quad (11)$$

ruling out local hidden-variable theories but not classical correlation.

3 Discussion

We have shown that classical correlation, as defined by the Pearson correlation, one of the most commonly used statistical measures of similarity, has the same form as quantum entanglement. Thus, in contrast to widely held belief within the physics community [15], but in agreement with the early work by Barut and Meystre [13], the classical correlation does not conform to the hidden variable linear form of Bell's theorem. Instead, it exhibits the inner product relationship, $E(\mathbf{a}, \mathbf{b}) = \mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}|| ||\mathbf{b}|| \cos \theta$, between two unit vectors, denoted as \mathbf{a} for one detector setting (Alice) and \mathbf{b} for the other (Bob), differing by the angle θ . This relationship could not but violate Bell's inequality.

The inner product correlation between vectors at two distant locations is not in conflict with local realism [22]. By classical physics, the spins paired due to a common cause of conserving momentum are like two clocks running at the same rate but set off by 12 h. Ever since the common cause that set the clocks with the 12-hour difference, the phasors are perfectly correlated without causal connection; checking one phasor does not determine but discloses the other in the chosen frame of reference. In other words, the clock faces are not without phasors but digits until the time zone, i.e., the frame of reference is defined. For example, let us consider Alice in Atlanta, noticing it is 8 am. Then, she knows immediately that for Bob in Beijing, it is 8 pm. Ever since and as long as the two chronometers are running in synchrony, the relationship between Alice and Bob is not arbitrary but inseparable.

Obviously, the measurement problem of how the superposition of states collapses to one outcome does not pertain to the classical account of the EPR experiment since the spin directions, despite being antiparallel, are undefined and indeterminate relative to an external reference until the measurement defines that frame of reference. This conclusion underscores that quantum mechanics is an effective theory, rendering interpretation issues pointless in the EPR context.

In retrospect, it is regrettable that classical correlation became erroneously associated with the hidden-variable expectation value [12], even though Bell did not use the word "classical" in his 1964 paper [7]. Furthermore, classical correlation became incorrectly identified with negative probabilities [14, 15], although Barut and Meystre only showed that when misapplied to classical correlation, the hidden-variable model leads to negative probabilities for certain detector settings [13]. This confusion is now cleared; classical probabilities are non-negative. Thus, the ordinary Pearson correlation due to paired spins or paired photons as indivisible entities can be recognized as an explanation of the EPR experiment.

It is worth emphasizing that our analysis does not discredit quantum mechanics as a way to calculate; it only discloses the two-particle wavefunction as an effective representation of oppositely paired spins rather than an interdependence between the quantum state of particles [35]. In other words, without interacting, one system cannot influence another, and contrary claims based on violations of Bell inequalities are incorrect [36].

When the correlation across space and time is not causal, i.e., not sustained with any physical substance, utmost care is taken to keep the paired spins or photons from decohering [37]. Accordingly, the behavior of macroscopic oscillators [38, 39] can be understood to display classical correlation since they were set in sync by a common cause. Thereafter, the oscillators are independent of each other as the measurement of the position of one and the momentum of the other more precisely than Heisenberg's uncertainty limit proved. Eventually, the correlation is lost because of noise, i.e., incoherent dissipation. After decohering, one oscillator is no longer in step with the other.

In conclusion, although Bell ruled out hidden variables to explain the observed correlation, his inequality did not exclude classical correlation due to a common cause, the conservation of intrinsic angular momentum aligning the spins antiparallel at the breakup. The same functional form of classical correlation and quantum correlation gives reasons to regard quantum mechanics, first and foremost, as an effective theory with well-known interpretational issues [40, 41] rather than a complete and comprehensive physical theory. Computing by correlations should be thought of accordingly.

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Declarations

Conflict of interest The author declares that he has no conflict of interest.

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