# Natural Classes and Natural Classification

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**Abstract** Categorization is a natural way for us humans to differentiate one object from another as well as to relate entities to each other. However, are there classes in nature independent of human categorization? And is there a fundamental way of classification free from human cataloging? We consider that all objects can be categorized based on their ultimate composition of elemental building blocks, quanta. Our conjecture parallels that of Noether's theorem but follows from statistical mechanics of open systems. We conclude that the natural categorization places objects to classes so that free energy is consumed in the least time. While the imperative is universal, any classification is subjective. We relate these resolutions to conventional methods of categorization.

**Keywords** Dissipation · Entropy · Free energy · Photon · The principle of least action · The second law of thermodynamics

## 1 Introduction

Are categories truly natural notions or only conceptual constructs? We, humans, tend to be so consumed in categorizing perceptions that we hardly attend to our category-making. What is the basis of our categorization, and why do we place objects in categories in the first place? In fact, often we presume that there would be distinct categories, for instance, as antonyms, when asking fundamental questions "What is life?" and "What is consciousness?" Perhaps resolutions to these profound

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questions (Sharma and Annila 2007; Annila 2016a) and others will first follow from thorough comprehension about classes and classification.

Aristotle's categorization of objects by successively narrowing questions such as "Is it animate or inanimate?" logically implies that one entity can be ultimately distinguished from another by an indivisible constituent. The ancient as well as modern atomism (Palmer 2012; Berryman 2008; Chalmers 2014) claims that everything comprises undividable basic building blocks. Indeed, humankind has progressed in making ever-finer distinctions manifesting today, e.g., as DNA-based taxonomy and lineages of elementary particles.

In terms of modern physics, the ultimate unit of increment is one quantum of action (Heisenberg 1927). The uncertainty principle excludes from categorizing any observation below the exactness of the quantum because the observation process itself will change its target at least by one quantum (Jordan 1934). In this sense, a natural class is defined by the basic entity whose properties are set (Dretske 1977).

This atomistic view was posited by Lewis in 1926 based on Planck's discovery of a constant and Einstein's interpretation of it as the quantum of light as well as by Noether's theorem that equates the number of quanta with system's energy and characteristic period of motion. We acknowledge that this old quantum theory was largely set aside when physics moved to modern quantum theory. Nonetheless, we are motivated to adopt the old concept and propose anew that the quantum of action is the basic constituent. This thesis means that a single quantum is the "natural" unit of classification. In the following, we will consider the consequences of this conjecture.

Although the single quantum can be regarded as the ultimate resolution of any object, many a categorization does not focus on the number of constituents but on functional differences among objects. The spectrum of functions in *scala naturae* is undoubtedly broad, but in terms of physics, all functions are alike. Namely, any process is some flow of energy (Sharma and Annila 2007; Du Châtelet 1759; De Maupertuis 1746; Mäkelä and Annila 2010). Thus, whether one entity can be distinguished from another by function depends on the subject's ability to discern differences in the flows of energy between one class of entities and another. In other words, there is no objective way of categorizing. Still, the flows of energy are not arbitrary either but comply with thermodynamics.

These preliminaries on the ultimate resolution and subjective character of classification imply on the one hand that there are natural categories distinct from each other by the number of quanta and on the other hand that objective and universal standards for categorization, albeit desired, are elusive. We motivate this insight by formulating a theory of classes and classification from the profound principles.

## 2 Hypotheses

The emergence of new classes and evolution to greater hierarchy are typical but not exclusive processes to biological systems (Salthe 1993; Ulanowicz and

Hannon 1987). Clearly, animate can distinguish, for instance, edible from harmful. Technological progress is characterized by ever-finer distinctions. The increasing competence in delineation is also reflected in increasing vocabulary. Specialized terminology meets specific needs. For instance, Sami languages in northern Europe have a wealth of snow- and ice-related words. Today English expands with words related to information technology. Applications of artificial intelligence, so-called expert systems (Jackson 1998), also depend on the ability to classify "correctly."

At this point, it is worth recalling that in philosophy, it has been debated whether natural classes exist or not (Quine 1970; Dennett 1991). When searching for the foundations of categorization, we assume no distinction between natural and artificial or other implicit categorizations (Du Châtelet 1759). Instead, we reason that everything can be categorized as we adopt the old atomistic tenet where everything is ultimately composed of basic building blocks, the building block we identify to be the quantum of actions (Pernu and Annila 2012; Annila 2010; Annila 2012; Annila 2016b). Our hypothesis is axiomatic and falsifiable. It can be proven wrong by showing that there is, in fact, an entity which cannot be broken down to the quanta of actions.

The quantum of light, i.e., the photon, is a familiar example of the quantum. Its attributes energy $E$ and period $t$ combine in an invariant measure known as Planck's constant:

$$h = Et. \tag{1}$$

The fixed quantity means that the photon is an indivisible entity. We only assume that everything that exists comprises the quanta. Thus, a compound entity, from now on referred to as a system, integrates its $n$ constituent quanta into an invariant known as the action (Noether 1918)

$$nh = \int 2K dt \tag{2}$$

over the paths of kinetic energy $2K$. This implies to us that conserved quantities in multiples of $h$ are natural categories. For example, the hydrogen atom in its ground state, defined by Eq. (2), is an action with a fixed number of quanta. When the atom absorbs one quantum of light, it will change from the ground state category to an excited state category. We acknowledge that in practice, it is not easy to keep track of all quanta in a given system in its universal surroundings (Annila 2016b; Lewis 1926; Annila 2015; Abbott et al. 2016; Grahn et al. 2018).

It follows from the atomistic axiom that we may formally describe any system in terms of quanta. We do this formally by placing the system's entities as compound quantized actions on levels of an energy diagram. In this scale-free manner, each entity can be assigned to a class by its energy attribute (Du Châtelet 1759). Energetically indistinguishable entities populate the same level in the diagram (Fig. 1). Accordingly, microstates, i.e., permutations of these identical entities, are energetically equivalent. We include dynamics of the system by flows of quanta from one level to another and from the surroundings to the system and *vice versa*.

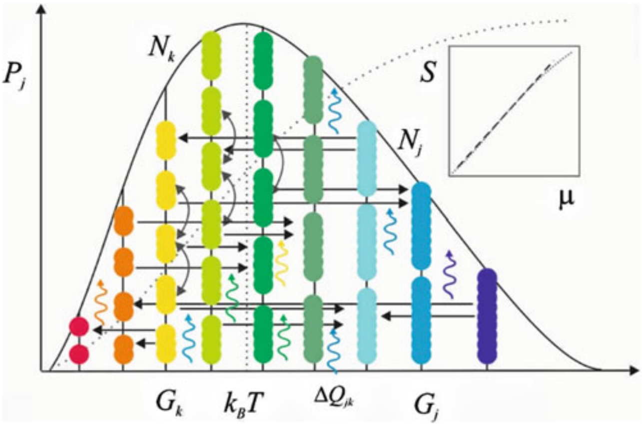


**Fig. 1** The system of classes is depicted in terms of an energy level diagram (Koivu-Jolma and Annila 2018). At each level, indexed by $k$, there is a population, i.e., a class of $N_k$ entities each with energy $G_k$. The size of $N_k$ is proportional to probability $P_k$. When an entity in the population $N_k$ transforms into an entity in the population $N_j$, horizontal arrows indicate paths of available transformations from one class to another. Vertical wavy arrows denote concurrent changes driven by energy in light. The vertical bow arrows mean the exchange of indistinguishable entities without changes in energy, and hence without a change in class. The system evolves, step by step, via absorptive or emissive $jk$-transformations that are mediated or catalyzed by entities themselves, toward a more probable partition of entities, i.e., a classification, eventually arriving at a stationary-state balance where the levels are populated so that the average energy $k_BT$ equals that in the system's surroundings. A sufficiently statistical system will evolve gradually because a single step of absorption or emission is a small perturbation of the average energy. Hence at each step of evolution, the outlined skewed quasi-stationary partition does not change much. This maximum-entropy distribution accumulates along a sigmoid curve (dotted) which is on a log-log scale (insert) a straight line of entropy $S$ vs. [chemical] potential energy μ

This allows us to describe the evolution of classification along with the system's evolution.

We exemplify our general nomenclature with a molecular system. The energy that is bound in a population [of molecules] is given by chemical potential $\mu_j = k_BT\ln N_j + G_j$ where $N_j$ is the number of entities [molecules], $G_j$ is the energy of one entity, and $k_BT$ is the average energy of the system at temperature $T$. The categorization of entities in terms of the energy diagram allows us to formulate the state of a system, i.e., to define categories. The change from one category to another involves a change in energy. The classification itself will also cause changes because at least one quantum must be obtained from the entity to quantify its class. It is insightful to speak in thermodynamic terms because then we are free from human categorization.

# 3 Theory

Classification entails that the classes are in some relation to each other. For example, individuals in a population can be categorized by body weight, in other words, put on the same scale relative to each other. Ultimately any relation can be given in terms of energy (Fig. 1), and hence, there is according to the second law of thermodynamics also an optimal occupancy of various classes. This free energy minimum manifests itself in scale-free patterns, i.e., nearly lognormal distributions, including logarithmic spirals, that accumulate along sigmoid curves and at times display oscillations and even chaotic trajectories (Du Châtelet 1759; Kapteyn 1903; Gaddum 1945; Limpert et al. 2001; Grönholm and Annila 2007). This conclusion about natural classification can be drawn from the probability theory of many-body systems (Sharma and Annila 2007; Du Châtelet 1759).

## *3.1 Definitions*

Let us consider the probability $P_j$ that a class, indexed with $j$, is populated by $N_j$ entities. For example, we may consider a species with individuals in an ecosystem or a molecular species in a chemical reaction mixture. First $P_j$ depends on energy $\mu_k = k_B T \ln N_k + G_k$ that is bound to the necessary substrates in numbers $N_k$ each with energy $G_k$. This means, for example, that for predators to exist, there must be preys. Obviously, the general formalism accounts for all species of a food web. This energy transduction network roots from the photon absorption of sunlight and it terminates at dissipation of photons to the cold space. Thus, $P_j$ depends also on the influx or efflux of energy, i.e., dissipation that couples to changes in the population from $N_k$ to $N_j$ and *vice versa*. Namely, the predators cannot consume the preys without some dissipation.

Likewise, a chemical reaction from substrates to products cannot proceed without either emission or absorption of photons. In fact, it follows from our axiom that no change of state can take place without either absorption or emission of at least quantum. The flux of quanta (photons) is denoted by the energy difference $i\Delta Q_{jk}$, which matches the energy difference $\Delta G_{jk} = G_j - G_k$ per entity between the $k$-substrates and $j$-products. The imaginary part merely indicates that the vector potential from the surroundings to the system or *vice versa* is orthogonal to the scalar [chemical] potential.

The probability $P_j$ for the population $N_j$

$$P_j = \left[ \prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T} \right]^{N_j} / N_j! \quad (3)$$

is obtained as the product of its $k$-substrates including an influx of photons that couple to the $jk$-transformations. For example, any chemical reaction is either endo- or exoenergetic. Thus, metabolism of a predator can be described accordingly. In Eq. (3) the division by factorial $N_j!$ enumerates the inconsequential exchange of identical entities that causes no changes in the classification scheme (Fig. 1). If any one vital $k$-ingredient is missing altogether from the product form $\Pi_k$, the $j$-population cannot exist, i.e., $P_j = 0$. Similarly, if no flux of energy couples from the surroundings to the system, the $jk$-transformation cannot take place. This means, for example, that when a vital nutrient is missing altogether, the species cannot proliferate even if everything else would be available. The index includes transformation stoichiometry by running from $k = 1$ to an unknown upper limit that is eventually reached when the system has attained thermodynamic balance with its surroundings. In a small chemical mixture, it might be possible to determine the stoichiometry of all conceivable reactions, but many a system is too big and diverse to imagine all possible evolutionary scenarios. For example, it is very difficult to predict how the metabolic system of a bacterium will respond to various agents, for example, intended to limit bacterial infection. Nonetheless Eq. (3) formally keeps track of all ingredients down to the precision of a single quantum.

The probability for the population of any other class can be expressed likewise. Thus, the total probability for the populations in all classes is the product of $P_j$:s:

$$P = \prod_{j=1} P_j = \prod_{j=1} \left[ \prod_{k=1} N_k \mathrm{e}^{-\Delta G_{jk}/k_B T} \mathrm{e}^{+i \Delta Q_{jk}/k_B T} \right]^{N_j} / N_j!. \tag{4}$$

In this manner, the total probability provides an energetic status, e.g., of a cellular system, ecosystem, or economic system. The status is high in energy-rich surroundings. Under such circumstances, the classification yields a high number of species or products distinguished from each other by numerous energy differences. Conversely, in surroundings that are low in energy, the most probable state of a system contains only relatively few entities. In other words, the probability is the system's energetic measure in relation to its surrounding systems, so that the highest value is attained at thermodynamic balance. Thus, natural categories are neither arbitrary nor algorithmic but relate to the surroundings in energetic terms.

The logarithm of $P$ (ln$P$), rather than $P$, is an additive measure to quantify the energetic optimality of a given categorization. Then, one classification can be compared with another by comparing the sums $\Sigma \ln P_j$. For example, a finer decimation of entities in distinct classes will yield a higher value than a coarse one. This means that the categorization is invariably a subjective process in accordance with observations. For historical reasons, entropy $S$ is defined as the logarithm of $P$

$$S = k_B \ln P = k_B \ln\left[\prod_{j=1}\left(\prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T}\right)^{N_j} / N_j!\right]$$
$$= \frac{1}{T}\left[\sum_{j=1} N_j k_B T + N_j\left(\sum_{k=1}\mu_k - \mu_j + i\Delta Q_{jk}\right)\right] \tag{5}$$

when multiplied by Boltzmann's constant $k_B$. It is the additive measure for natural classification. In Eq. (5) Stirling's approximation $\ln N_j! \approx N_j \ln N_j - N_j$ has been used. The approximation is consistent with the statistical character of a system. Specifically, if there were only a few objects, their categorization would be troublesome, to begin with.

It is worth emphasizing that entropy (Eq. 5), when multiplied with temperature $T$, identifies classes on the basis of two terms: first by energy $\Sigma_j N_j k_B T$ that is bound in the $j$-populations of the classes (Kondepudi and Prigogine 1998) indexed with $j$ and second by energy $\Sigma_j N_j(\Sigma_k \mu_k - \mu_j + i\Delta Q_{jk})$ that still is present between the system and its surroundings. The first term $\Sigma_j N_j k_B$ is the familiar entropy obtained from statistical mechanics for a closed or stationary system. Obviously, when all energy is bound in the various populations, the classes are steady and thus unambiguously countable. At this maximum entropy state, there is no net flow of energy carriers between the system and its surroundings, and hence neither a new class will appear, nor an old one will disappear.

Conversely, the second term $\Sigma_j N_j(\Sigma_k \mu_k - \mu_j + i\Delta Q_{jk})/T$ means that the classification system is open for evolution by consuming energy differences relative to its surroundings, i.e., forces that motivate classification. This flux of energy carriers from the system to its surroundings, or *vice versa*, leads to the increase in entropy until all energy differences have leveled off. The free energy term means, for instance, that there is a force that drives further or finer classification. Alternatively, there may not be enough free energy to maintain the current degree of classification, but the classes will be merged to regain balance with resources. This is, of course, common sense. A finer classification needs more resources than a course one.

### *3.2 Classification as a Process*

The natural evolution of the classification scheme will be obtained from the differential equation of motion for entropy (Eq. 5):

$$\frac{dS}{dt} = \sum\nolimits_{j=1} \frac{dS}{dN_j}\frac{dN_j}{dt} = \frac{1}{T}\sum\nolimits_{j=1} \frac{dN_j}{dt}\left(\sum\nolimits_{k=1}\mu_k - \mu_j + i\Delta Q_{jk}\right) \geq 0 \tag{6}$$

where the chain rule has been used. The two-term product reveals that the population $N_j$, of the class $j$, will change by $d_t N_j$ proportional to the driving force $A_j = \Sigma_k \mu_k -$

$\mu_j + i\Delta Q_{jk}$. Thus, the measure for classification can only increase, i.e., $dS \geq 0$, since $(A_j)^2 \geq 0$. In other words, the classification will evolve as long as there are motive forces for it and means to improve it. From this perspective, the predator is likely to evaluate its prey carefully when it must invest a considerable amount of resources in catching it. This is also common knowledge. The predator will not attack arbitrarily but consider which prey it will try to catch. Conversely, when surrounded by abundant resources, the degree of categorization is expected to be low. Then basically anything will do. For example, many whale species simply swim around with their mouths open and filter food through their baleen bristles, when they have found a rich school of fish or krill.

It is worth emphasizing that the classification will progress to define finer details only when such subtle differences contribute to the overall free energy consumption. Put differently, the finer classification must provide benefits that supersede its costs. Otherwise, it will not be adopted. Conversely, the classification scheme will evolve by abandoning classes when the distinction is energetically unfavorable or even immaterial. For example, many languages, when adapting to the modern way of life, are rapidly losing vocabulary related to the old rural lifestyle. At times the changes in surroundings are so big and rapid that the changes in categorization display oscillations and even chaotic characteristics. For example, words will acquire new distinct meanings among subpopulations, and hence due to decreased communication societal cohesion decreases overall.

Finally, when the classification has consumed all forms of free energy, the class structure has attained thermodynamic balance, i.e., $dS = 0$. The optimal classification has converged in a free energy minimum. It is Lyapunov-stable so that any perturbation $\delta N_j$ away from a steady-state population $N_j^{ss}$ will cause decrease in $S(\delta N_j) < 0$ and concurrently increase in $d_t S(\delta N_j) > 0$ (Strogatz 2000). In other words, the further away $N_j$ would be from $N_j^{ss}$, the larger will be the restoring force $A_j$. This balance manifests itself, for example, in maintaining consensus about meanings of words. The quest for the free energy minimum categorization is customarily understood so that a useful classification mechanism is such that knowledge accurately infers object properties and these properties accurately infer object classes (Corter and Gluck 1992). Likewise, many species in stable ecosystems have highly specialized diets.

It is worth pointing out that our approach for denoting a hierarchy in thermodynamics terms is not unique (Bar-Yam 2004a; England 2013; England 2015) and not the only option either (Allen et al. 2017). The emergence of new classes has been addressed in mathematical terms (Bar-Yam 2004b).

## *3.3 The Subjective Character of Classification*

The natural class structure that extends down to single quanta is obviously inaccessible in practice to a subject. Thus, one's categorization is invariably narrow and coarse grained. It limits to one's own observations and inferences as well as influences obtained from others. In other words, one's categorization is limited by resources and biased by past processes. This behavior is recognized as cognitive and confirmation biases as well as at the level of systems, as systemic or institutional bias (Nickerson 1998; Kahneman and Shane 2002; Anttila and Annila 2011). Nevertheless, the subjective classification is invariably governed by the second law of thermodynamics (Eq. 6). Put differently, the subjective classification, while narrow and coarse, is not arbitrary but energetically optimal for the subject. This revelation prompts us to analyze individual classification schemes for meanings as well as for inconsistencies.

One makes sense of perceptions by categorizing them. According to the second law of thermodynamics, making sense means ultimately consuming free energy (Annila 2016a; Anttila and Annila 2011; Annila and Salthe 2009). Conversely, from nonsense one cannot benefit [energetically]. Only some dissimilarity among observations will prompt categorization. Surely it makes a difference to distinguish an edible plant from a poisonous one. According to the thermodynamic tenet free energy motivates one to make distinctions of any kind. Conversely, when the reward for one in categorization is minimal, it will not take place. Approval for this stance is often sought by asking, "Who cares?"

It is intriguing that a subject may insist on making a difference among objects when there is no solid ground for it. For example, one tends to partition nature to animate and inanimate, although there is no single attribute that would warrant such a distinction. This is to say that many an illusory classification is motivated by quantitative rather than qualitative differences. The deceptive division is practical, but it leads to an inconsistent worldview. In terms of physics, inconsistency in classification is a tension, i.e., a force that finds no way to break out. Thus, the puzzle about "What is life?" prevails as long as one insists on having distinct classes for a living and nonliving against all evidence. Although science has abandoned vitalism eons ago (Wöhler 1828; Annila and Baverstock 2014; Annila and Kolehmainen 2015), this kind of fundamental questions are still deemed as philosophical. We wish to point out that they are, in fact, physical when everything is considered as being composed of quanta.

Also, the curious case when there is a difference, but the subject fails to make one, is also worth clarifying. For example, it is quite common that one fails to distinguish two somewhat similar sounds in a foreign language when the two are not distinct and present in one's native tongue. It takes extra effort to learn to hear the difference. Likewise, many other things are often placed in preexisting categories by presumptions and resemblances rather than putting effort into refining one's categorization. Thus, one easily loses opportunities to benefit from making the distinction between superficially similar perceptions. This also demonstrates that classification is a process governed by the energetic imperative of maximal efficiency, but efficiency is rated by the subject who performs the classification.

Actual disputes about definitions and meanings, i.e., differences in classification, are quite common among people. Although it may not be so obvious, the objective of a quarrel is to work out a common scheme of classification, i.e., an agreement on how to rationalize the state of affairs. In terms of physics, common categories allow a coherent and integrated consumption of free energy. First, when the optimal path along the resultant force has been agreed upon, it can be pursued. Of course, the agreement is motivated only when the gain in free energy consumption can be seen to exceed the energetic costs involved in the common category-making. In modern societies, these expenses are typically the costs of standardization (Annila and Salthe 2009; Annila and Salthe 2010). Therefore, those ones with least class structure are most apt to adopt a new classification whereas those with the already well-established classification scheme will find it unrewarding to invest in a new way of thinking or doing. By these examples, we wish to point out that physics has a say in social sciences as well when formulated for open, evolving systems (Koivu-Jolma and Annila 2018; Anttila and Annila 2011; Annila and Salthe 2009; Annila and Salthe 2010).

Finally, it is of interest to note that since Eq. (6) also describes oscillations and even chaotic trajectories, these characteristics are expected to manifest themselves also in categorization (Sornette 2006). The oscillations in categorization are, in fact, quite common. For example, many words in English will be categorized as either verbs or nouns, depending on the context. In general terms of physics, the context is the surroundings that ultimately dictate the meaningful classification, i.e., least-time free energy consumption.

Chaos in categorization is expected when the surroundings vary widely. When "rules" are repeatedly changing, it will be hard to root one scheme of categorization over and others. In other words, category-making fails. The chaotic behavior in categorization can be modeled by the logistic map (Eidenberger 2014). In turn, it has been shown to approximate the least-time free energy consumption. Chaos is typical when a whole class structure collapses. When relationships between classes are obscure, acts will be arbitrary.

We realize that our derivation of natural classes and classification from the principle of physics may, at first sight, appear somewhat remote to contemporary theories of categorization. Therefore, we will work out the correspondence with the most common tenets of classification.

# 4 Discussion

## *4.1 Correspondence with Conceptual Classification*

Aristotle's categorizing by narrowing questions successively can be put in an algorithmic form, known as conceptual clustering (Michalski and Stepp 1983; Carpineto and Romano 1993; Fisher and Pazzani 1991). The clustering algorithm predefines the path of categorization. In this sense, the algorithm mimics the evolutionary path toward the optimal categorization as given by Eq. (6). However, the conceptual clustering is a deterministic model, whereas the categorization process is non-determinate because the category-making itself affects the categories and *vice versa* (Biswas et al. 1998). Put differently, it is not possible to know in advance what will be encountered and how the encounters will, in turn, affect further encounters. Mathematically speaking variables cannot be separated in the evolutionary equation (Eq. 6), and hence, it cannot be solved (Du Châtelet 1759). We expect machine learning to benefit from this profound insight (Eidenberger 2014). In fact, machines are already thought by exposing them to large amounts of data, i.e., with experience, rather than by programming them to encounter conceivable situations.

Despite its disadvantage in complying with non-determinate reality, the algorithmic approach will suit many a purpose of categorization by being a handy model, i.e., computable in polynomial time. A perhaps more troublesome shortcoming of the algorithmic classification is the lack of energetically defined target function, i.e., the least-time free energy consumption. Then the class structure may evolve in a nonnatural way, for instance, by combining letters to words with no meaning. We expect the algorithmic classification to fail when meanings disperse widely. For example, when symptoms are diagnosed, it is not only subtle differences in observations and laboratory tests that matter but also the consequences of classification. Namely, when the categorization misses a fatal but rare disease, the difference in the data might well be insignificant, but the difference in the outcome is dramatic.

Surely, this problem of meaning in classification has been recognized. The quest for the free energy minimum has been modeled by assigning each class with utility whose maximization drives the clustering formation (Gennari et al. 1989; Lebowitz 1987). Thus, the utility maximization mimics entropy maximization, in fact also by its functional form when given by Kullback–Leibler divergence (Kullback and Leibler 1951). Nevertheless, the model's probability for two objects to be in the same or different category is not expressly given in energetic terms as in Eq. (3), but by phenomenological attributes. Also, it is worth stressing that the category utility sets in advance a deterministic layout. Thus, the method is biased, but its effectiveness is of great practical value.

The conceptual clustering as a classification method is closely related to data clustering. The probabilistic COBWEB algorithm (Fisher 1987; Fisher and Langley 1986) organizes observations into a classification tree. Each tree node represents a class and is summarized by a probabilistic attribute-value distribution under the node. This mode of organization corresponds qualitatively to the energy level diagram (Fig. 1), which can also be presented as trees and networks. On the one hand, the open structure allows one to describe any concept as well as to predict missing objects or to classify new objects (Iba and Langley 2011). On the other hand, there is no unambiguous principle to choose parameters of the algorithmic categorization that may even end up with classification produced by binary yes/no classification (Talavera and Béjar 2001). For example, an entity, say, a bacterial species, is recognized as a member of the class, i.e., a specific taxon, when the sum of predefined class attributes exceeds a given threshold. Obviously, it takes prior knowledge about the diversity of attributes to set a meaningful threshold. Moreover, meaningfulness itself will depend, for instance, on consequences of misclassification, for example, on a wrong diagnosis following from a mistyped bacterial strain. The problem of setting the thresholds is particularly pronounced in automated classification when the machine has no sense of meanings, i.e., consumption of free energy (Karnani et al. 2009). The poor sense of meanings does not limit to the machines.

### *4.2 Correspondence with Prototype Theory*

Prototype theory is a type of graded categorization, which groups identities based on prototypes (Osherson and Smith 1982; Lakoff 1989; Lakoff 1987). A prototype (Rosch 1983; Smith and Minda 2002) is defined as a stimulus that takes a salient position in a class, later redefined as the most central member of a class. Prototype theory is a step away from definition-based models. For example, prototype theory would consider a class like an atom consisting of different entities each with unequal status, e.g., a hydrogen atom is more *prototypical* of an atom than say a niobium atom. This approach is cognitive in the sense that it accepts that categories are graded and inconsistent, but as we argue, ultimately commensurable in energetic terms. The prototype theory can describe even abstract classes, but by our naturalistic tenet, everything is ultimately embodied by quantized actions. The inherent subjectivity of the approach can be exemplified by categories that are different for separate cultures (Smith et al. 1988).

Clearly, also the prototype theory parallels our thermodynamic theory of classes and classification. The most central member of a class is a natural notion for distribution whose central value is given by the average energy ($k_BT$). Moreover, the subjective character is also inherent in the natural classification.

The prototype theory can also be described in terms of dynamic systems theory where a given object is assigned with a weight determined by past conditions and depending on current conditions (Langacker 1987). Thus, a category reflects how it has been employed in the past. This way prototype systems allow for changes in meaning which are common to languages (Wittgenstein 1958). This path dependence parallels our natural classification.

The recursive nature of prototype systems resembles mathematical iteration. Consequently, outcomes inflate over time, and hence also category definitions keep changing. In other words, prototype systems are nonlinear due to feedback mechanisms. The nonlinearity, e.g., in Eq. (6), is also a characteristic of the natural classification. Expressively a small cause can produce a substantial, even a chaotic, effect.

## 5 Conclusion

Categorization is such an innate faculty of human beings that one hardly pays attention to it. In fact, the ability to distinguish one from another, as well as to group entities that are alike, appears to be vital for our survival. This evolutionary perspective implies that also other species behave similarly, and hence, we reason that the category-making is not distinctive to humans. Here we have extended this conclusion further by using the thermodynamic theory of evolution that there is an ultimate definition of a class by the quantum of action, which is the basic building block of nature. Moreover, we conclude that there is an optimal way to place objects and observations in classes. This imperative is known as the second law of thermodynamics. Thus, we understand categorization to equate ultimately with least-time free energy consumption, which is known in biological terms as survival.

Our comprehension of the ultimate classes and optimality of classification is convergent with observations that modern cultures aim for an even better understanding of the world by proceeding toward ever-finer decimation and by building ever-larger hierarchical systems and doing it ever faster. This holistic tenet provides an eye-opening viewpoint to human activities by revealing that they are after all not unique to humans and animates either.

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