

# Atomism revisited

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**Abstract** The ancient atomism inspires us to consider everything as being composed of indivisible entities, known today as quanta of actions. The quantum of light is the familiar single quantum in its open waveform. Likewise any other physical action is a geometric notion given in terms of energy and time. The quantized systems adopt geodesics, i.e., paths of least action in quest for energetic balance with surrounding quanta. This universal tenet can be related to quantum field theory so that the quantized entity, such an elementary particle, is the source for surrounding field quanta, expressly those that embody vacuum. The fine-structure constant as the ratio of two actions, corresponding to the electron and neutrino, allows us to deduce unambiguously characteristic symmetries of leptons, mesons and baryons. We exemplify the quantized structures of photon, neutrino, electron, proton and neutron as well as those of weak bosons and the Higgs boson. Moreover, we model some nuclei, among them chemically important <sup>12</sup>C, as high-symmetry complexes of nucleons. The elementary ingredients can be assembled to models of atoms to illustrate notions of quantum mechanics. Finally, we discuss the four fundamental forces and their relative strengths in the light of modern atomism.

**Keywords** elementary particles, free energy, fundamental forces, geodesic, quantum of action, the principle of least action

## 1 Introduction

It is well-known that the notion of atom, literally ‘uncuttable’, emerged in ancient Greek natural philosophy to account for everything as being composed ultimately of indivisible entities [1,2,3]. However, it is less well-known what follows from this universality [4] in terms of physics [5,6]. The early philosophers assigned the atom with only a few intrinsic properties, such as size and shape, as well as recognized only a few modes of interactions, such as atoms striking against one another, rebounding and interlocking in the universal void. Today physicists describe particles, i.e., constituents of elements, in terms of field quanta with energy attributes as well as recognize four fundamental interactions mediated by force carriers [7,8]. In contrast to the ancient atomism with its eternal elementary entities, modern physics portrays the force carriers as virtual particles that exist only for a short time dictated by the uncertainty principle [9]. Parallels and disparities between the old tenet and new theory inspire us to examine the elements of existence by regarding the quantum of action as the modern embodiment of the ancient notion of a-tomos.

At first sight it is perhaps not so obvious that there is need for revival of the ancient atomism, or any other perspective for that matter. Calculations of quantum chemistry comply with measurements, although in cases correspondence is limited by precision and computational requirements [10]. Further down toward the fundamental description of existence the Standard Model (SM) of particle physics classifies elegantly wealth of subatomic particles and accounts for electromagnetic, weak and strong nuclear interactions. Moreover, the theory appears self-consistent [11] and its calculations match well with measurements. Yet, SM falls short of being the complete theory of fundamental interactions since it does not include gravitation. By the same token the large

difference between the weak force and gravity, known as the hierarchy problem, remains a puzzle. Although the elementary particles are grouped to three generations of quarks and leptons as well as the force carriers are tabulated to gauge bosons and the Higgs boson, the origin of rule itself is somewhat of a mystery. In the same sense, the characteristic quark and antiquark composition of mesons and baryons as well as those of exotic hadrons is some kind of a riddle. Also neutrino oscillations and finite masses are at variance with SM [12].

The quantum field theory describes, in turn, interactions between particles in terms of interacting quantum fields. Feynman diagrams are familiar illustrations of various interaction processes mediated by virtual particles [13]. Despite its success the virtual particle model is worth reconsidering because it follows from perturbation theory, and hence interactions are assumed to be weak. This is typical of scattering processes, but the weak-limit approximation is not valid for bound states such as atoms [14]. Specifically the perturbation theory has not provided results compatible with experiments for the strong force that binds quarks into nucleons.

We are in no position to solve contemporary theoretical problems. Yet, we argue that it is inspiring and insightful to consider what can be deduced when considering the quantum of action as an indivisible basic building block of everything. This modern correspondence of the old atomistic can be elaborated with logic to describe elementary particles and their interactions as well as their complexes as atomic nuclei and finally the whole atom, all in agreement with observations and measurements. In this way our study completes earlier accounts on thermodynamics, evolution and emergence that follow from the same stance where the quantum of action is regarded as the modern embodiment of the ancient atom [15,16].

It turns out that our exercise will not expose anything fundamentally new, but it still provides a tangible viewpoint by rendering quarks and gluons as concrete constituents of elementary particles in the same manner as atoms are building blocks of molecules. Perhaps some specialists will find this portrayal somewhat astounding and as if falling short of mathematical rigor. However, we emphasize, it is the universal premise and ensuing logic as well as the geometric character of an action that distinguish from contemporary expectations, not the results themselves that comply with observations. Thus, we believe many will welcome this revelation of seemingly abstract and technical notions of elementary particle physics as inspirational in search for ways to comprehend constituents of existence. In a sense our paper also exemplifies the role of themata in science [17]. It is well understood but still often ignored that science is not solely guided by empirical information but in its effort to explain observations in a comprehensive manner general principles and universal tenets are invariably invoked [18].

## 2 The Quantum of Action

Much of physics entertains with the notion of energy. However, energy does not exist as such. It is the attribute of an action, e.g., that of the quantum of light [15,19,20]. The action identifies by its unit Js, or equivalently kgm/s-m, to a physical entity with geometric character having energy  $E$  on its period of time  $t$ , or equivalently momentum  $\mathbf{p}$  on its wavelength  $\mathbf{x}$ . In other words, the action is an integral over time, or equivalently over wavelength

$$A = \int E dt = \int \mathbf{p} \cdot d\mathbf{x} = nh, \quad (1)$$

where  $n$  denotes the number of quanta in terms of Planck's constant  $h$ . The equation says that everything is in motion either along open paths of evolution or along closed stationary trajectories [21]. The familiar invariance of steady-state motion is given by Noether's theorem [22]. In quantum theory the constancy of energy, i.e., invariance of observables under certain transformations, is expressed so that there is a unitary operator on the Hilbert space of states [7]. Conversely, symmetry will be broken when the system evolves from one state to another either by acquiring quanta of actions from its surroundings or losing them to its surroundings in quest of consuming free energy. The least-time free energy consumption, in turn, manifests itself as time asymmetry [21,23,24].

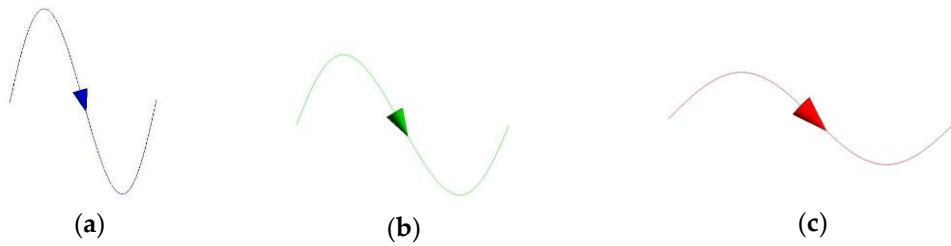
Hereafter we merely follow the old atomistic idea by considering everything in terms of actions and constructing everything from multiples of the elementary action ( $n = 1$ ). We present actions that comply with observations and measurements, but there could be also alternative actions that comply equally well with data. Then again uniqueness is no end itself either, e.g., resonances and oscillations are natural phenomena.

## 2.1 Photon

The photon [25] is the most familiar form of the quantum of action. We see light and we sense heat. The photon has energy  $E$  on its period of time  $t$  as well as momentum  $\mathbf{p}$  on its wavelength  $\mathbf{x}$ , so that the invariant product of these pair attributes is a geometric notion (Fig. 1). The elementary action integrates momentum over the action's path to Planck's constant

$$h = \int E dt = \int \mathbf{p} \cdot d\mathbf{x}. \quad (2)$$

The lowest ( $n = 1$ ) invariance implies that the photon is a manifestation of the basic building block of nature. The textbook form  $E = hf$ , where  $f = 1/t$  is the photon's frequency, is of course mathematically equivalent to Eq. 2, but it puts emphasis on the photon's energy attribute. This conventional view makes sense because it is the photon energy in interactions that determines what will happen, if anything. In contrast, our view by recognizing Planck's constant as the photon's measure, makes sense because it is the photon with its pair attributes that must either arrive to the system or depart from the system for something to happen (Figure 1).



**Figure 1.** The freely propagating photon is an open quantum of action whose pair-attribute product equals Planck's constant. Thus, the quantum of light when propagating from a higher (a) to lower surrounding energy densities (b,c), will lengthen its period of time, which appears as red shift (b,c), to maintain energetic balance with its surrounding quanta. (Illustration by Mathematica)

It is easy to understand that the freely propagating photon is massless when the mass is defined as Leonhard Euler did. The Euler characteristic, also familiar from the theorem of Gauss and Bonnet, is obtained by summing up geodesic curvature

$$k_g = \frac{\mathbf{n} \cdot \gamma'' \times \gamma'}{|\gamma'|^3} = \mathbf{n} \cdot \frac{\gamma''}{\gamma'^2} \times \frac{\gamma'}{|\gamma'|} \quad (3)$$

over the whole curve  $\gamma$ , i.e., along the quantized action at each point by calculating the cross product of acceleration  $\gamma''$  and velocity  $\gamma'$  and projecting it the surrounding curvature with normal  $\mathbf{n}$  [26,27]. The provided factorization helps to recognize the familiar curvature  $\mathbf{\kappa} = 1/r = \mathbf{a}/v^2$  as given in physical terms of acceleration  $\mathbf{a}$  and velocity  $\mathbf{v}$  multiplied with unit velocity vector  $\mathbf{v}/|\mathbf{v}|$ .

The universal surroundings is characterized by the tiny curvature of the Universe, i.e., by  $1/R$  where the huge radius  $R = cT$  at the current age of  $T = 13.8$  billion years. The geometric notion of mass  $m$  in terms of curvature when given in relation to the curvature of the whole Universe, complies with renowned  $E = mc^2$ . It allows us to recognize that the squared speed of light  $c^2$  is the (least)  $L^2$  norm of the vacuum [28]. For a symmetrical path the sum of  $k_g$  vanishes, and hence the photon is massless. Physically speaking  $k_g$  expresses how much surrounding quanta will depart from the energy density of the vacuum when at energetic balance in the vicinity of a particle [29,30].

The action characterized at any point by its three Cartesian components, (Eq. 3), implicitly identifies dimensionality of space as three. Time as the fourth dimension associates with changes in energy due to absorption or emission of quanta, when the stationary system opens up for evolution from one state to another.

The photon's open path entails that any system, ultimately the whole Universe, housing freely propagating photons is invariably an open system. In other words, one may imagine of enclosing a system accommodating also free photons, but such a thought is fictitious. Moreover, due to the photon propagation from the system to its surroundings or vice versa, energy of the system will change concomitantly with change in energy of its surroundings. Only at a dynamic stationary state the to-and-fro flows balance exactly. Therefore at the thermodynamic balance the photon exchange may well be modelled by virtual photons, because actually nothing happens. Otherwise the approximation, albeit convenient, does not hold.

## 2.2 Vacuum

We acknowledge insightful studies on ontological elusiveness of space [31], and at the same time admit our incompetence to operate with concepts other than concrete. Thus, we remark simply that when two photons of equal energy, and hence having equal periods of time, co-propagate but out-of-phase, their electromagnetic fields cancel each other out (Figure 2). At the complete destructive interference we see no light, but according to the atomistic tenet the photons do not vanish for nothing [15,25,32,33]. This conclusion is consistent with observations [34,35,36,37]. The sky is dark. The vacuum is black, but still full of paired photons giving rise to the energy density of space about  $n/m^3$ , which is, as known, approximately at balance with the energy density that is bound in the total amount of matter in the Universe [29,30,38,39]. Likewise, the gravitational field about a body comprises of photons that are out-of-phase in pairs and at energetic balance with the energy density bound in the body. Moreover, variations in the freely propagating paired-photon density above the universal background density are recognized as gravitational waves [40].



**Figure 2.** The co-propagating pair of photons is an open compound quantum of action. When the phase configuration is exactly out-of-phase, the net electromagnetic field vanishes (a). However, the photons themselves do not vanish for nothing but continue in propagation and carrying energy density. Conversely, when the phase configuration deviates from the complete destructive interference, electromagnetic fields manifest themselves (b). (Illustrations by Mathematica)

When the energy density of the vacuum and its local variations are understood to embody the paired photons, gravity can be comprehended as an energy density difference, i.e., a force just like any other. It is attractive for two bodies, when the surrounding energy density is lower than that bound in the system of the two bodies [29,30,33,41]. Then the surroundings will accept quanta from the system. The dissipative character of gravity is also obvious from the argument of reversibility. Namely, it takes work to restore a fallen object back on its initial height. Also the recently detected gravitational waves from the binary black hole spiraling to a merger, revealed an energy loss that corresponded to three solar masses [40].

Conversely, gravity manifests itself as a repulsive force, when the surrounding energy density is higher than that bound in the system of the two bodies. This too is obvious, since some work has to be done to pull two bodies apart. Here on Earth insolation is the common source of photons with

sufficient energy to do the work on the system of bodies. On the universal scale, a distant galaxy moves away from us, because the greater Universe supplies quanta, though mostly in pairs of the out-of-phase photons, between us and the distant galaxy from its numerous sources, most notably from stars and black holes. From this perspective space is understood to expand because the quanta bound in matter are converted to freely propagating photons constituting the vacuum [35].

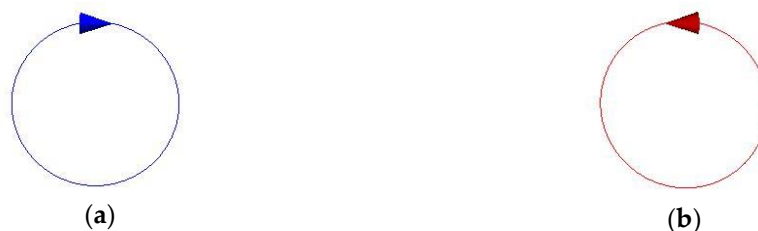
The out-of-phase configuration for the two co-propagating photons is the free-energy minimum state where their electromagnetic fields balance exactly, and hence it is the natural form of the background energy density. Conversely, it will require some force to move the two photons apart from their out-of-phase relation. This manifests itself as an electromagnetic field (Figure 2). When reasoning in this way, it is no mystery where the photons will emerge all of a sudden when an atom becomes ionized. They have been around all the time, but in the out-of-phase configuration, and hence manifesting only as surrounding energy density.

Our view of the vacuum as the paired-photon substance, makes it easy to understand that both gravity and electromagnetic forces follow the inverse square law, because the force carrier is the same for both forces, and hence also coupling effects are anticipated [42]. By the same token the norm of vacuum  $c^2 = 1/\epsilon_0\mu_0$  depends on the electromagnetic properties, denoted by permittivity  $\epsilon$  and permeability  $\mu$ . Also earlier it has been understood that  $c$ ,  $\epsilon$  and  $\mu_0$  are not fundamental constants but observable density-dependent parameters of the quantum vacuum, although the vacuum has been pictured to have a different embodiment than proposed here [43,44].

Of course, the vacuum's photon embodiment has been suspected for a long time, but one has not been quite able to put one's finger on it. To recognize the photon as an indivisible and indestructible quantum of action is a decisive conclusion [45,46,47]. Moreover, the tangible photon-embodied vacuum makes it easy to understand the two-slit experiment and its variants, where the interference involves also the photons embodying the vacuum [48,49]. The observed interference, even in the case when no more than a single particle is propagating at a time, is no different from the passage when a boat enters to a harbor through one opening of a breakwater while its backwash enters also through another opening, so that at the quay the boat rocks, i.e., interferes with waves that its own propagation generated. This reasoning is familiar from the old pilot wave theory [50,51].

### 2.3 Neutrino

Geometry of the photon is open, but it is easy to envisage that the quantum turns back to its beginning and closes to a loop. We associate the closed quantum ring to neutrino, to be precise, the perfectly planar geodesic is the electron neutrino. This conversion from the photon to neutrino exemplifies what Newton conceived about matter being ultimately made of photons [52]. The invariant measure of the quantum loop is  $h/2\pi = \hbar$  and the unitary group  $U(1)$  is neutrino's characteristic symmetry. The high-symmetry geometry reveals that the neutrino is its own antiparticle (Figure 3), just as the photon and antiphoton are the one and same particle.



**Figure 3.** Neutrino is the elementary action in its closed form (a). Specifically the electron neutrino is portrayed as a perfectly planar loop. The symmetric loop reveals that neutrino is its own antiparticle (b). (Illustrations by Mathematica)

The tiny mass of electron neutrino is understood via Euler's formula (Eq. 3) to arise from the minute difference between the perfectly planar ring and the almost flat Universe of curvature  $1/R$ . Conversely, muon neutrino is expected to be a bent ring to account for its higher mass. Likewise, tau

neutrino is expected to be still a more curved geodesic found in surroundings where energy density is much higher than in the vacuum. From this geometric perspective neutrino oscillations are means for the neutrinos to seek and maintain balance with surrounding energy density. Put differently, high flavor portions increase when energy density increases. This is indeed observed when comparing neutrinos that arrive directly to the detector from the Sun and those that pass through the Earth before the detection [53,54].

## 2.4 Electron and Positron

When resolving other particles as quantized actions, i.e., geometric entities, the fine structure constant  $\alpha$  is a revealing starting point. This number can be understood as a ratio of two actions [9,30]. When the charge of an electron  $e$  is given by Gauss law  $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137.036$  yields the action of an electron  $e^2/4\pi\epsilon_0\hbar c$  relative to the neutrino action  $\hbar$ . The numerical value of  $\alpha$  implies that the electron comprises of 138 quanta in a toroid form that already Andre-Marie Amperé proposed [55] (Figure 4). Due to the helical pitch, one quantum does not quite close one full loop, and hence a lag accrues along the torus and one extra quantum will be needed to close the curve of 137 loops.

This conclusion about the electron's quantized structure matches observations. The net number of loops amounts to the total charge. The magnetic moment amasses primarily from the large circle and its anomalous part  $\alpha/2\pi$  from the small loops, because due to the rising helix the small loops are not exactly perpendicular to the large circle. The ratio of the electron mass  $m_e$  and the mass  $M$  of the Universe can be computed from the toroid curve as the vector sum of signed curvature, just the way Euler did. The electron mass is minute, because the vector sum of signed curvature of any two quanta at the opposite faces of the torus is almost zero, departing from nil only due to the pitch.

The positron is just like the electron, but its charge is the opposite because its handedness is the opposite (Figure 4). This revelation of the elements of existence in terms of chiral quantized actions resolves the matter vs. antimatter asymmetry problem by regarding antimatter merely as the opposite standard of handedness [29,30,56].



**Figure 4.** Electron comprises of 138 quanta in a toroid ring of 137 loops (a). The positron is a toroid just as the electron but its handedness is the opposite (b). The net number of windings relate to the charge. The electron magnetic moment sums up to  $\mu_e$  along the curve from the differential magnetic moment  $\mu$ . The electron mass  $m_e$  is minute because the quanta at the opposite sides of the torus have opposite orientation apart from the toroid pitch, and hence the overall geodesic curvature is minute. When the electron  $e^-$  and positron  $e^+$  torus pack face-to-face, the two torus may open up and consume each other in annihilation for pairs of photons that co-propagate in the out-of-phase relation. In addition two easily detectable photons propagating in the opposite directions emerge from the annihilation to balance the opposite handedness. (Illustrations by Mathematica)

The Euler characteristic associates with the notion of mass so that the signed curvatures are summed along the action and projected onto the curvature of surroundings. The obtained quantity means physically speaking how much the photons in the surrounding vacuum have to curve, that is, to become denser, when near the particle. In the vicinity of the electron not that much, because the torus, apart from its pitch, is symmetrical. However, the toroidal winding forces the paired photons



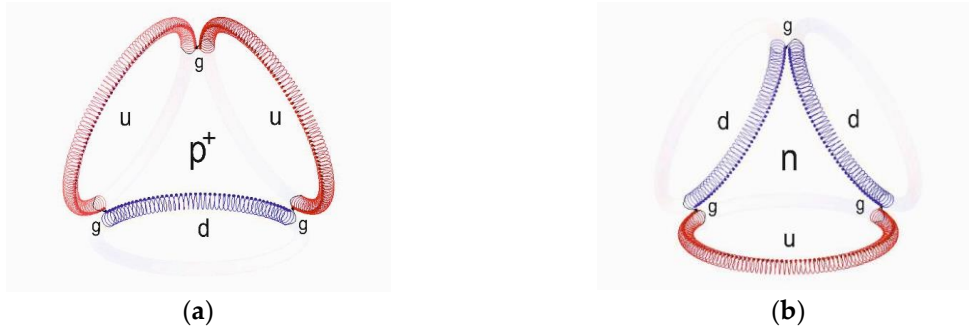
away from the minimum-energy out-of-phase relation, which manifests itself as an electromagnetic field about the electron (Figure 2b).

## 2.5 Nucleons

The electron torus structure suggests that the down-quark with charge  $1/3e^-$  comprises  $1/3$  of the electron torus, i.e., 46 quanta and accordingly that the up-quark with charge  $2/3e^+$  comprises  $2/3$  of the positron torus, i.e., 92 quanta [29,30]. When the quarks are connected to each other by gluons, i.e., short wavelength photons, structures of the proton and neutron are obtained unambiguously without alternatives (Figure 5). The closed directional loop of gluon-connected quarks in tetrahedral symmetry displays the familiar SU(3) characteristic of baryons in accordance with quantum chromodynamics [57]. The notion of color means, for instance, that the two up-quarks are distinct from each other in the signed geodesic of proton, because one of them precedes and the other succeeds the down quark. Moreover, the quark composition of baryons is easy to understand, because only a three-quark geodesic will close just as only three antiquarks will form a closed loop.

The models of proton and neutron comply with measurements. It is easy to calculate from the displayed structures their approximate magnetic moments  $\mu_{p^+} = 2.667\mu_N$  and  $\mu_n = -1.889\mu_N$  [30]. When angles between the quarks are slightly adjusted, e.g., due to Coulomb forces, we expect the calculated moments to converge toward the experimental values  $\mu_{p^+} = 2.793\mu_N$  and  $\mu_n = -1.913\mu_N$ .

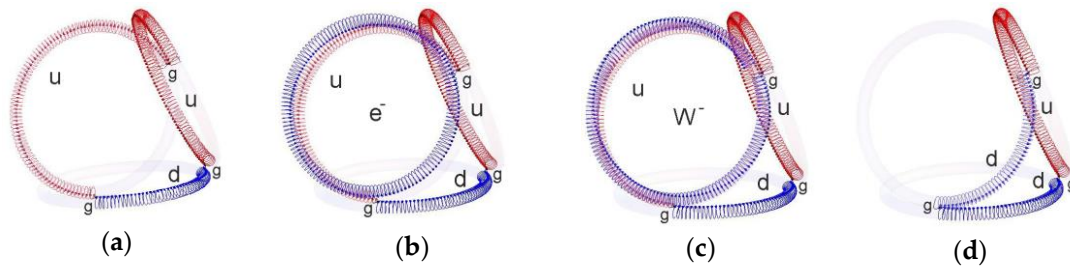
Also the plain mass of a nucleon  $937.54 \text{ MeV}/c^2$  can be calculated when knowing the electron mass, to yield elementary estimates  $938.82$  and  $938.22 \text{ MeV}/c^2$  [30] that agree well with measured values  $m_{p^+} = 938.27 \text{ MeV}/c^2$  and  $m_n = 939.57 \text{ MeV}/c^2$ . Again we expect these values to home in to the measured values when the structures are slightly adjusted to account for electrostatic effects between the quarks. The proton and the neutron are much heavier than the electron, because there is no curvature at the opposite side of the arcs of quarks to balance the vector sum.



**Figure 5.** Models of the proton  $p^+$  (uud) (a) and neutron  $n$  (udd) (b) having their up quarks (u) and down quarks (d) in tetrahedral symmetry follow unambiguously from the electron torus structure when the down quark is identified as the  $1/3$ -arc of the electron torus and the up quark is identified as the  $2/3$ -arc of the positron torus as well as gluons (g) are recognized as short wavelength photons. (Illustrations by Mathematica)

## 2.6 Electron Capture

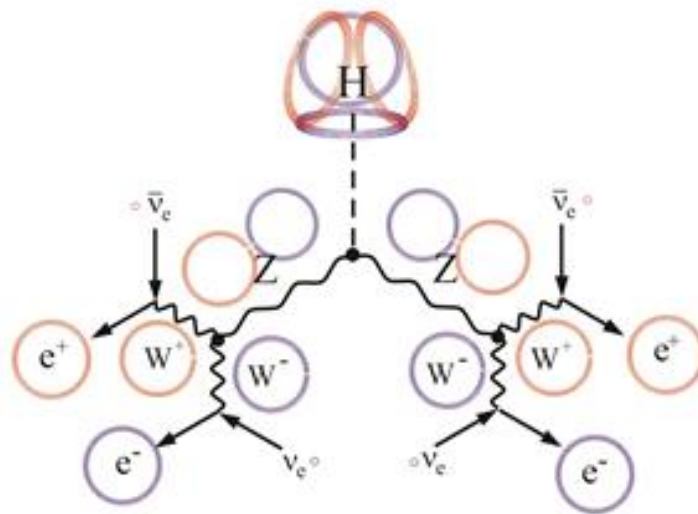
The quantized models of particles make it easy to illustrate how an atomic nucleus captures an electron so that a proton transmutes to a neutron [30,58]. When the electron comes close to the up-quark, it will open up to  $W^-$  boson by losing one of its loops with neutrino. The commencing annihilation consumes the up-quark altogether, so that  $1/3$  of an arc is left from  $W^-$  that subsequently closes to a down-quark, and thereby closing the action as the neutron (Figure 6).



**Figure 6.** The proton in a nucleus is the starting point of electron capture (a). Next an electron (blue torus) comes face-to-face to the up-quark (red  $\frac{2}{3}$  arc of torus) of a proton (b). When the electron opens up by losing one of its loops with neutrino, it will become a reactive  $W^-$  boson (blue open torus) (c). Subsequently  $\frac{2}{3}$  of  $W^-$  will annihilate with the up-quark (red  $\frac{2}{3}$  arc of torus). The remaining  $\frac{1}{3}$  of  $W^-$  (blue  $\frac{1}{3}$  arc of torus) will close to a down-quark that completes the transmutation from proton to neutron (d). (Illustrations by Mathematica)

## 2.7 The Higgs Boson

The quantized structure of many a particle can be inferred from its decay scheme. The decay of Higgs boson to a pair of Z-bosons [59] suggests to us that it is a perfect tetrahedron with four open torus (Figure 7), and hence the Higgs particle its own antiparticle. The mass of Higgs, as well as those of weak bosons,  $W^-$ ,  $W^+$  and Z is big because each torus ring is open by being short of one small neutrino loop. These tiny slots will accommodate only very high-frequency photons of the vacuum, i.e., high energy density, and hence the particle energy is high compared to the vacuum and it is also short-lived. According to the modern atomism the Higgs particle, apart from being a highly symmetric particle, is not special from other particles when considering the concept of mass as geodesic curvature (Eq. 3) [60].



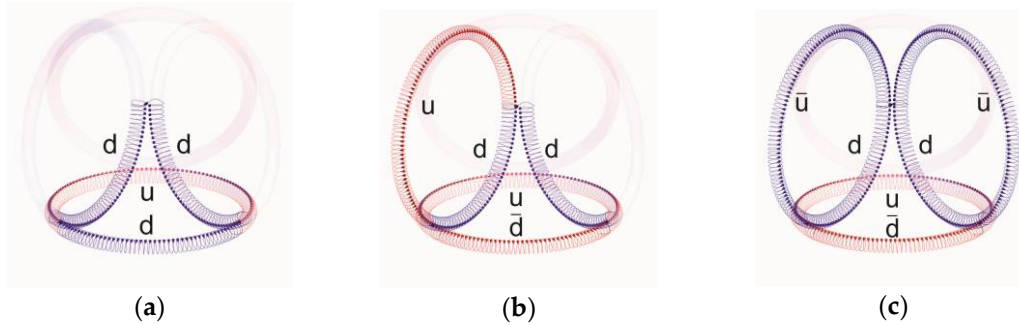
**Figure 7.** The decay scheme of Higgs boson (H) to two Z bosons, and further to W bosons and to electrons ( $e^-$ ) and positrons ( $e^+$ ) via neutrino ( $\nu_e$ ) absorption, implies that the Higgs boson is a highly symmetric tetrahedron of quanta (at top). Most of its mass, i.e., the particle's high energy attributes to tiny slots in the open rings that are matched by high-frequency photons of the vacuum. (Illustration by Mathematica)

## 2.8 Exotic Hadrons

A quark and antiquark combine to a meson. It is straight-forward to realize from meson masses that when the two-quarks are in a plane the configuration is known as the pseudoscalar meson and when they are coordinated along the tetrahedron faces the diquark is referred to as the vector meson. The triquarks are baryons. Four quarks, such as two quarks and two antiquarks, are easily envisioned to form the quantized path of a tetraquark on the tetrahedron's faces (Figure 8a), i.e., a dimeson with



compositions such as  $u\bar{d}-u\bar{d}$  or  $ud-dd$ . Likewise, one may imagine five quarks, such as four quarks and one antiquark, to form a pentaquark (Figure 8b), i.e., the molecule comprising of a baryon, e.g.,  $udd$  and a meson, e.g.,  $u\bar{u}$ . Finally it is easy to picture a hexaquark, i.e., a dibaryon, e.g., as a combination the neutron  $udd$  and antineutron  $\bar{u}\bar{d}\bar{d}$  (Figure 8c). Moreover, two neutrons could pack tightly on the four tetrahedron faces, for instance, in the core structure of a compact star. Thus the ‘exotic’ hadrons [61,62,63] when modeled as quantized actions, are not that exotic after all.



**Figure 8.** Examples of quantized models for a tetraquark comprising a dimeson, e.g.,  $ud-dd$  (a), a pentaquark encompassing a baryon and a meson, e.g.,  $udd-u\bar{d}$  (b) and a hexaquark containing a dibaryon, e.g.,  $udd-\bar{u}\bar{d}\bar{d}$  (c). (Illustrations by Mathematica)

The mass of an exotic hadron, similarly to the Higgs particle, we expect to stem primarily from geometric details to which the photons of the vacuum must adapt. Most notably any tiny slot relates to high energy. Conversely, the particle’s lifetime, despite its high energy, may be long enough to allow detection, when the reactive open end of a quark is not immediately accessible to a breakdown reaction.

## 2.9 Nuclei

The quantized structures of proton and neutron are the building blocks to assemble models of nuclei. The modeling is guided by measured quantities and observed nuclear processes as well as by symmetry arguments that typically relate to free energy minimum structures. However, the models that are presented here mainly serve to illustrate and exemplify the insight to nuclear structure provided by the modern atomism that considers everything to be composed of quanta.

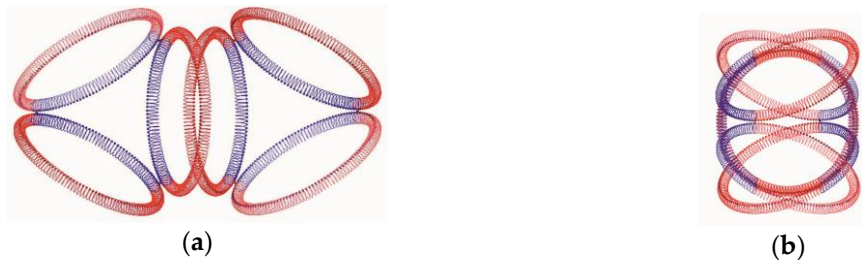
The models of proton and neutron can be assembled to an isospin doublet model of deuterium (Figure 9). In the compact high-symmetry configuration the proton and neutron are intertwined. In the model their magnetic moments, as calculated separately above, add up the total moment  $0.886 \mu_N$ . This value, by being rather close to the measured value  $0.857 \mu_N$ , implies to us that the model makes sense. Also it is apparent from the high-symmetry configuration that  ${}^2\text{H}$  has only a small electric quadrupole moment. Moreover, when the proton and neutron magnetic moments, as measured separately, are added together, the total moment is  $0.879 \mu_N$ . The deviation from the experimental value implies that deuterium is an admixture of the  ${}^2\text{H}$  compact low-energy state, indexed with spin  $s = 1$  and orbital angular momentum  $l = 0$ , as well as another extended high-symmetry state, indexed with  $s = 1$  and  $l = 2$ .

Considering the intertwined structure and elaborated model for electron capture (Fig. 6) it is of interest to note that the deuterium can be dissociated to proton and neutron by neutral current interactions with neutrinos. The cross section for this interaction is comparatively large, and hence  ${}^2\text{H}_2\text{O}$  is a very good neutrino target [64].



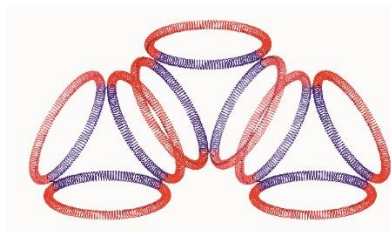
**Figure 9.** The measured electric quadrupole as well as the magnetic moment of  ${}^2\text{H}$  and those of proton ( $p^+$ ) comprising of quarks ( $uud$ ) linked by three gluons ( $g$ ) and neutron ( $n$ ) comprising of quarks ( $udd$ ) linked by three gluons ( $g$ ) imply a high-symmetry compact configuration where  $p^+$  and  $n$  are intertwined when viewed above (a) and from side (b). (Illustrations by Mathematica)

The model of  ${}^2\text{H}$ , when duplicated, assembles to a high-symmetry model of  ${}^4\text{He}$  (Figure 10) in agreement with measurements that reveal no magnetic and electric moments. Here we have made no effort to quantify the distance between two  ${}^2\text{H}$  units.



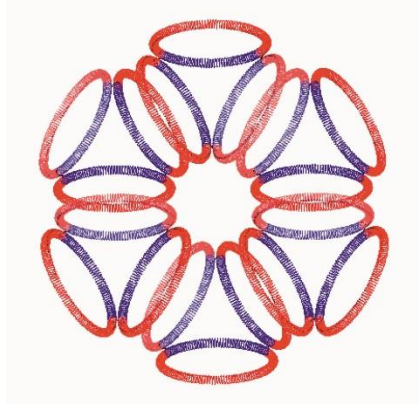
**Figure 10.** The model of  ${}^4\text{He}$  comprising two models of  $\text{H}^2$  in the high-symmetry compact configuration is viewed from side (a) and above (b). (Illustrations by Mathematica)

In a similar manner we propose a model for  ${}^6\text{Li}$  to account for its low electric quadrupole moment and nuclear magnetic moment. (Figure 11). Comparing the magnetic moment of  ${}^6\text{Li}$   $0.82 \mu_N$  with the value  $0.857 \mu_N$  of  ${}^2\text{H}$ , it seems to us there is a slight reorientation of the central unit away from the high-symmetry configuration. Such a spontaneous symmetry breaking to attain the free energy minimum state is the characteristic of many a system.



**Figure 11.** A model of  ${}^6\text{Li}$  comprises three models of  ${}^2\text{H}$  in a symmetry configuration. (Illustrations by Mathematica)

Analogously  ${}^{12}\text{C}$  can be constructed from two models of  ${}^6\text{Li}$  to comply with its electromagnetic characteristics (Figure 12).



**Figure 12.** The model of  $^{12}\text{C}$  comprising two models of  $^6\text{Li}$  in the high-symmetry hexagonal configuration. (Illustrations by Mathematica)

We expect studies of tetrahedron and truncated tetrahedron packing [65,66] to be useful when modeling isotopes with increasing number of nucleons. Among various arrangements the correct ones can be identified by comparing their properties with observables, most notably, magnetic moments, electric quadrupole moments, scattering cross sections, binding energy for the last neutron and excitation energy as well as stability. For example, it is well known that 20 tetrahedrons pack tightly to an icosahedron. The high-symmetry model complies with inert characteristics of  $^{20}\text{Ne}$ . This and other magic as well as double magic numbers are, of course, contained in the nuclear shell model [67]. Thus, the nucleons with quarks coordinated at tetrahedron faces illustrated here merely serve to give tangible insight to nuclear complexes in accordance with the established theory of nuclei.

### 3 The Model of an Atom

The scale-free formalism describes the atom likewise by its constituent actions of protons and neutrons forming the nucleus as well as by encircling electrons and quantized interactions (photons) that embody the enfolding space. However, to infer the exact number of quanta of an atom, as denoted in Eq. 1, is troubled by difficulties in detecting the paired photons without electromagnetic fields that constitute the vacuum and hence also embody the open imprecise realm referred to as the atom. In other words, since the constituents of an atom are interacting with the quanta of surrounding vacuum, there is no exact boundary for an atom. Instead the atom is an open system like another at thermodynamic balance with its surroundings actions. Accordingly the atom will respond to changes in surroundings, e.g., in electromagnetic fields, and changes in energy density on the whole.

In quantum mechanics [68] the notion of a wave function models this indeterminacy due to interactions with the surrounding actions. For example, the wave function  $\Psi(x,t)$  extends in space  $x$  and time  $t$  to account for the electron's influence on the vacuum. The elementary equation for a stationary motion is the renowned Schrödinger's equation

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \quad (4)$$

where energy in motion is at balance with potential energy denoted by Hamiltonian  $H$ . The formal solution of Eq. 4  $\Psi = \Psi_0 \exp(-iHt/\hbar)$  indeed complies with a closed orbit, i.e., a stationary action where the exponent's ratio of action  $Ht$  to the elementary action  $\hbar$  relates to the rate of precession. Conversely the complex conjugate  $\Psi^*$  denotes motion with the opposite sense of direction. Obviously, despite of the spatial and temporal spread of interactions, the electron of an atom will be found for sure, and hence the probability  $P = \int \Psi^* \Psi dx = 1$  sums up to unity.

The change from one state to another entails either influx or efflux of quanta, and hence concomitant change in energy. The open path from an initial state can be expressed by an evolutionary equation of motion for  $\Psi$  as well as by the concurrent change  $d_t P$  in the probability [21].

Since energy is not conserved along the open path, the evolving probability does not integrate to unity either. This means for instance, that when the atom absorbs a high-energy photon, there is no guarantee that the electron will remain bound. Eventually if the electron ends up on an excited orbital, the associated change in mass can be understood by Eq. 3 as a change in curvature, most notably in the paths of photons that embody the energy density within the atom.

When the wave function is understood to model the electron's influence on the photon embodied vacuum, it is easy to comprehend, for instance, that an s orbital extends to the nucleus even though the electron itself, as the quantized action, would never be there. In this way the modern atomism, when describing everything in terms of quanta, relates seemingly abstract theoretical notions to tangible constituents.

#### 4. Discussion

The models of elementary particles, given in terms of quantized actions, i.e., geometric entities, may at first sight seem astonishingly simple when compared with expectations, for instance, generated by quantum field theory. In particular if one is used to work with elementary particles only in terms of calculations and diagrams, the tangible representations may seem even naive. Then again the fundamental objects of string theory are open and closed strings that are superficially similar to the quantized actions. Some one hundred years ago also chemistry seemed opaque before the elements, each with its characteristic valences, materialized as models that today pupils use to build models of compounds. The concrete models of subatomic constituents serve the same purpose of comprehending by seeing, or better by having 3D models in one's hands.

The quantized actions corresponding to the particles comply with measurements, and hence the models serve to represent outcomes of the quantum field theory calculations in the same manner as the concrete atomic and molecular models assist to concretize quantum chemistry calculations. The quantized actions can be recognized as the sources of the quantum fields. The surrounding quantum field obtained from the quantum field calculation relates to the source's distribution of quanta in the same way as Gauss theorem relates the charge distribution of a source to the surrounding electric field. In other words, the distribution of field quanta is the response taken by the surrounding vacuum to the quantized source. Expressly the vacuum's paired-photon density distribution is at the energetic balance with the particle mass and the photon-phase distribution is at the energetic balance with the particle's electromagnetic characteristics. Interdependency of the photon density and phase distribution shows up, e.g., so that the proton charged radius is smaller when probed with muons than when with electrons [69]. In a sense, permittivity is higher around the heavy muon heavier than the light electron.

In the historical perspective the pilot wave of a particle, proposed by de Broglie [50], corresponds to the surrounding actions, i.e., paired photons that embody the vacuum. When considering calculations, it is worth emphasizing that Maupertuis' principle of least action is a non-determinate equation because driving forces and motions are interdependent [6,15,19]. This means in some cases that seemingly subtle changes in geometry may cause substantial changes in the particle's properties and vice versa. Therefore ensuing iterative calculations can at times be problematic without convergence, e.g., in cases of oscillations and resonant states [30].

The mass of a particle can be regarded according to Eq. 3 as a geometric response taken by the photon-embodied vacuum energy density. This notion parallels the meaning of Higgs field, but attributes no special meaning to the Higgs particle. The tangible models of particles make it easy to understand how the photons in the surrounding will adapt to the energy densities contained in various particles. In particular, the models illustrate how seemingly slight changes in the quantum structure might cause dramatic changes in the mass. For example, the transformation from electron to W-boson is accompanied with a huge change in mass. When the electron torus opens up by losing one loop for neutrino, a tiny slot will emerge. The vacuum will adapt to it by placing a short wavelength photon into the vacant slot, and hence the particle appears "heavy" in its surroundings. When such fine details matter, it is easy to guess that calculations of elementary particle masses, in

particular those of 2<sup>nd</sup> and 3<sup>rd</sup> generation, are tricky and the values are susceptible to subtle alterations in the particle's quantized geometry.

The quantized structures of elementary particles provide insight to the standard organization of elementary particles to three generations of matter as well as to the gauge bosons and the Higgs boson. Each generation shares the same geometry of its basic constituent. Expressly the first generation neutrino is the one-quantum planar loop. The 2<sup>nd</sup> generation neutrino is expected to be a more curved loop and the 3<sup>rd</sup> generation loop is still a more curved away from the plane. We suppose the curvatures of elementary bending modes of neutrino to match the ratio of muon and tau neutrino masses. We understand the 2<sup>nd</sup> generation particles to share the same basic constituent of the muon neutrino and the 3<sup>rd</sup> generation particles that of the tau neutrino. Therefore the corresponding masses will primarily reflect the curvature of the basic constituent that accrues along the geodesic. From this perspective there is no apparent reason why the particles should limit of three generations. However, higher curvatures correspond to higher energies, and hence to shorter lifetimes. It would be increasingly more difficulty to make unambiguous discoveries of the putative higher generation particles. From this perspective the recently found signal due to a pair of photons with energy 750 GeV, may well be similar to the Higgs particle (Figure 7) [70,71] but comprising elements of the 2<sup>nd</sup> generation whose higher curvature makes the tiny slots even smaller.

When everything is described in terms of the quantized actions, also all interactions can be described similarly, i.e., unified. Specifically, the strong interaction means the force, i.e., the energy difference that will be needed to break apart the quantized actions that are bound in nucleons and other hadrons. The weak interaction, in turn, concerns the force that is necessary to transmute one quantized action to another via weak bosons, for example, in beta-decay. The electromagnetic interaction means the force that is required to shift apart the phases of quanta that embody the surrounding energy density, most notably the vacuum. Finally gravitational interaction entails the difference between a local and the universal energy density, i.e., the free space embodied in the quanta. In this way gravity is quantized and compatible with other interactions. The modern atomism by its commensurable account on particles and interactions by actions relates them to each other via differences in energy densities. In particular, the huge ratio of electrostatic to gravitational coupling constants, i.e.,  $\alpha/\alpha_G = e^2/4\pi\epsilon_0 G m_e^2 = 4.17 \cdot 10^{42}$ , where  $G$  is the constant of gravity, relates to the ratio of the radius of the Universe and the radius of electron, and thereby provides insight to the hierarchy problem [30].

## 5. Conclusions

The ancient idea that everything is composed of indivisible elements can be expressed in modern terms of quantized actions. In this way elementary particles physics becomes as comprehensible as chemistry is today with its models of molecules comprising atoms. Expressly models of mesons, baryons and even exotic hadrons can be assembled from models of quarks and gluons as easily as models of molecules can be built from models of atoms with valences. When having the models of particles in hand one may demonstrate various nuclear reactions as simply as chemical reactions with models of substrates and products. Thus, the quantized models of elementary particles are not introduced to supersede calculations by quantum electro- and chromodynamics, just as the models of atoms were not manufactured to displace calculations and simulations of quantum chemistry.

The scale-free description in terms of quantized actions benefits from the principle of least action which maintains that the quanta will adopt the paths of least time in the prevailing surrounding energy density. While this is true, it is not so obvious in practice how one would calculate a specific geodesic. However, often the particle's properties and decay schemes suggest at least one structure, often only few alternatives to be considered as a model. Eventual ambiguity among the alternative models is, of course, characteristic of any inverse problem, but the models themselves may suggest experiments to remove the ambiguity.

In the end it may not even be the most relevant task to construct detailed models of increasing complexity. More and more versatile systems tend to have more and more alternative paths open for

evolution in quest of attaining balance with the surrounding energy density [5,6]. The quest for thermodynamic balance, i.e., evolution to diversity is observed already at the level of elementary particles, for instance, as admixtures of states, resonances and oscillations as well as emergence of novel particles in extraordinary conditions with increasing energy [30,71,72]. Thus for many a substance, i.e., a system it would be only an elusive and futile aim to nail down anyone of its specific stationary states, i.e., certain symmetry. After all there is nothing permanent except change.

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