Threads of time

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Abstract The concept of time's arrow is examined using the principle of least action as given in its original non-Abelian form. When every entity of nature is considered to be composed of quantized actions, such an entity will change, either by absorbing quanta from surrounding actions or by emitting quanta to the surrounding actions. In natural processes, quanta disperse from high energy density actions to low energy density actions in quest of consuming free energy in least time. We propose that the flux of quanta embodies the flow of time, and therefore the irreversible consumption of free energy creates time's arrow in a fundamental physical sense. The cosmological arrow of time results from universal processes that take place, most notably in stars and other celestial systems, where matter, i.e., bound actions, combust to photons, i.e., freely propagating actions. The biological arrow of time manifests itself in maturation processes where quanta absorb to emerging functional structures, leading eventually to aging processes where quanta, on balance, emit from disintegrating organs. Mathematical analysis of an evolutionary equation of motion, given in general terms of a spontaneous symmetry breaking process of actions, reveals the reason why future paths – and the future itself – remain inherently intractable.

Keywords action, free energy, irreversibility, space, the principle of least action, time.

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1. Introduction

The origin of the time's arrow is considered as one of biggest puzzles of science. The past distinguishes from future as the arrow of time, the essence of the Second Law of thermodynamics, associates with a change from one state to another. Thus the notion of time presupposes the concept of state [1]. The state, in turn, is defined by Noether's theorem with the action of a system [2], and a change of state to another is governed by the principle of least action [3]. From these foundations of physics we will find that the arrow of time as a physically embodied process involving the basic building blocks of nature, i.e., the quanta of action. We hope that this minimal materialism may sharpen the seemingly abstract ideas and convoluted conversation about the notion of time on-going in various disciplines [4,5,6,7].

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2. Irreducibility

Noether’s theorem relates a stationary state of a system to the characteristic symmetry of a conserved action [2]. It defines the stationary system in terms of the action as:

$$\mathsf{A} = \int_{t_1}^{t_2} 2K\, dt = \int_{x_1}^{x_2} \mathbf{p} \cdot d\mathbf{x} = h \int_{x_1}^{x_2} \nabla \varphi \cdot d\mathbf{x} = nh, \;\; n \geq 1 \tag{1}$$

that totals n multiples of quanta, h, from the integration of energy in motion, $2K$ [8], over a defined interval $t_2 - t_1$ that spans all periods of the system’s characteristic motions. Equivalently, the number quanta sums up to nh from the integration of momentum, $\mathbf{p} = m\mathbf{v}$, along its bound and oriented path, $\mathbf{x}$, e.g., along a circle of radius $r = x/2\pi$. For example, a photon that integrates momentum along its wavelength can be regarded as an action. In fact, the photon embodies the absolutely least action, comprising only a single quantum. Also other carriers of force as well as particles with mass are defined by Eq. 1 as actions composed of quanta in some integer numbers [9]. The concept of action is a powerful way to define the stationary system since the symmetry of action represents all motional modes. Conversely, when the system changes from one state to another either by absorbing or emitting quanta, the symmetry will break. Concurrently with the change in energy also eigenmodes and eigenvalues will change.

The nested natural hierarchy of entities in its holistic entirety [10,11] can be represented as systems within systems where levels of densely interacting actions are immersed in a surrounding system of sparser actions. For example, a star as a rich source of energy evolves from one state to another by emitting photons as quantized carriers of energy to its energy sparse surroundings. Another example of a compositional hierarchy of actions as sources, carriers and sinks is Earth in its surroundings. High energy quanta from the hot Sun are absorbed by biota as well as by various inanimate mechanisms, and subsequently processed in a series of transformation steps from one state to another in food webs and other energy transduction networks, and eventually quanta of thermal waste are dissipated into the low energy density of cold space. The conservation of quanta (First Law of thermodynamics) implies that the Universe is a system that is closed to some large but definite number of quanta. The Universe’s average mass density, when integrated over the time of expansion T = 13.7 billion years to the total mass M, provides an estimate for the number of quanta $n = Mc^2T/h$ to be on the order of 10^{121} quanta [12,13].

The definition of a stationary action by Eq. 1 will yield the familiar form of angular momentum, $L = mr^2\omega$, when the precession of energy along the least-action path, $d\mathbf{x} = d(\mathbf{r}\varphi)$, is given by the angular velocity, $\omega = \partial_t\varphi = \mathbf{v}\cdot\nabla\varphi$, where φ is the phase relative to a reference frame, e.g., that of an observer [14]. The characterization of a state in terms of a stationary action with distinct symmetry is irreducible because at least one quantum of action is required to distinguish one state from another. The irreducibility of action can be recognized also in the identity of indiscernibles [15]. The quantization of the action is familiar from Bohr’s atomic model, but also, in general, trajectories are modular [2,16]. However, the discrete character of nature is not obvious from a stationary system whose phase of dynamics relative to an observer is arbitrary. Therefore a steady-state trajectory appears to be continuously differentiable. Accordingly, in the limit of vanishing Planck constant the continuum model of classical physics fails to discern one state from another. In other words, a photon in the limit of zero energy cannot drive a change of state.

According to Noether's theorem stationary-state dynamics, irrespective of motional complexity, are on closed and modular trajectories where total angular momentum is conserved. This conservation of quanta means that there exists a norm. If the norm of the physical state is fixed, the time evolution operator of quantum mechanics would be unitary. In other words, quantum mechanics describes by the time evolution operator precession of phase relative to the frame of an observer as the isoenergic system moves from one configuration to another. Since the various configurations have the same energy, they cannot be distinguished from each other by any means and thus belong to the same state. It follows that when energy of the system is constant a unitary transformation can be found which will place the system's equation of motion in the principle frame where $\omega = 0$, i.e., time is not a parameter. This frame transformation merely demonstrates Noether's theorem which says that the isometry of time represents the conservation of energy. Time-independence is, of course, the definition of a stationary state. As well, the coordinate transformation from the frame of an observer to the principle frame of the system reveals that the flow of time related to the steady-state dynamics is embodied in the stationary flow of energy on a bound path.

The repository of a steady stream of quanta embodies a coordinate of space [9,17]. For example, a hydrogen atom embodies a locus of space where an electron orbits a proton. Likewise planets orbiting the Sun embody the solar system. The steady circulation of energy, $2K = \mathbf{v}\cdot\mathbf{p} = mv^2$, on the bound path has a sense of rotation relative to the observer, but since there is no net flux of energy from the system to its surroundings within the orbital period, t, motion will return invariably to the position of an initial phase. The recurrence of motion means that the steady-state dynamics is reversible [18]. So we conclude that, while the thermodynamic stationary flow of energy embodies the flow of time, that steady flow by itself does not have the character of time's arrow, i.e. irreversibility.

3. Irreversibility

At the change of state, the action will break its symmetry, so that the modular path of motion will open up either to acquire or expel at least one quantum, i.e. a photon [9,19]. The acquired or discarded quanta constitute a net flux of energy between the system and its surroundings. According to the Second Law of thermodynamics the quantized flux directs spontaneously from high energy density actions to low energy density actions. Hence, we reason that the direction of net flux from one repository of quanta to another embodies time's arrow in any particular spontaneous process. If a step of time can be identified with a change by one quantum, it follows that time flows physically in steps. However, the height of a step depends on the frequency of the absorbed (or emitted) quanta.

The net flux of energy from the surroundings to a system or from the system to the surroundings embodies causality, whereas reversible flows at a thermodynamic stationary state will cause nothing. When the energy of the surroundings is higher than that of the system, the system is forced to absorb quanta. Conversely when the energy of the surroundings is lower than that of the system, the system will emit quanta. It follows from the conservation of quanta that the change in momentum due to absorption or emission leads to a change in the coordination of action relative to other actions that embody other coordinates of a common space. In the context of causation, it is by no means a new notion to embody irreversibility in a net flow of energy [20,21,22,23,24].

Evolution from one state to another is a step in an irreversible series of absorption or emission of quanta [25] that directs the system toward equilibrium with its surroundings. In accordance with the Second Law of thermodynamics, when a system is higher in energy density than it surroundings, then it will evolve by discarding quanta to the sparser surroundings. Conversely, when the system is sparser in energy density than its surroundings, it will evolve by acquiring quanta from the denser surroundings. Eventually energy differences of any kind, i.e., free energy, must level off if a system were to attain thermodynamic equilibrium in its surroundings. For example, a chemical reaction will progress either by emitting or absorbing heat toward the stationary state where dissipation ceases. Also in general any other change of state is elicited by the surrounding energy density in quest of the thermodynamic stationary state.

Evolution is physically not a random (stochastic) process, but would vary its course so that free energy will be consumed in the least time. According to the principle of least action in its original form [3,17] the change in the integrand

$$d_t 2K = \mathbf{v} \cdot d_t\mathbf{p} = \mathbf{v} \cdot m\mathbf{a} + v^2 d_t m = -\mathbf{v} \cdot \nabla U + i d_t Q \tag{2}$$

is minimum. The conservation entails that changes in kinetic energy, $2K$, balance changes in scalar, U, and vector, Q, potentials. It is worth emphasizing that the commonly used Lagrangian form of the action principle without net dissipation can describe only stationary-state dynamics, i.e., motion from one configuration to another energetically indistinguishable one. Hence the Lagrangian dynamics, or any other formalism where Hamiltonian is unitary, displays no irreversibility.

The differential equation of evolution (Eq. 2) is convenient for mathematical manipulation, but the gradients should be accurately denoted as quantized differences. In other words an evolving system experiences a net force, $\mathbf{F} = d_t\mathbf{p}$, due to energy differences relative to its surroundings. The change in momentum, $d_t\mathbf{p} = m\mathbf{a} + \mathbf{v}d_t m$, results from the scalar potential gradient, $-\nabla U = m\mathbf{a}$, bound in matter (e.g., gravitational, electric or chemical potential, $\mu_j = dU/dN_j$, of a substance, indexed with j) and from the energy flux contained in propagating light, i.e. the vector potential gradient. The photons stem from changes in mass, $dm = dQ/v^2$, given in relation to a medium with index of refraction, $n^2 = c^2/v^2$. Ultimately all fluxes originating from $dm = dE/c^2$ will terminate in the universal surroundings. The lowest energy density is characterized by the permittivity, ε_o, and permeability, μ_o, of free space, which define the norm by the squared speed of light, $c^2 = 1/\varepsilon_o\mu_o$. Noether's theorem implies that when the energy of the system is not conserved, the isometry of time does not hold either. Thus, we conclude that the arrow of time is embodied in a net physical flow of quanta from the system to its surroundings or vice versa. The flow of quanta directs irreversibly to diminish the energy density imbalance between a system and its surroundings.

At a stationary state the forces, i.e., gradients of energy are on the average independent of time whereas during evolution forces are time-dependent because then the gradients of energy will be being consumed by the flows of energy that bring the system from one state to another. The net force will herd the system from one state to another along the least-time path in accordance with Newton's 2nd law [26,27]. Moreover, contrary to the common conception, Newton did not advocate for the notion of absolute time, instead only noted that the absolute stance is an ideal but convenient frame to do mathematical calculations [28]. In this context it is also worth recalling that Newton did not employ differential equations but geometry that allowed him to speak for the a-tomistic, i.e., quantized worldview.

When recalling the work-energy theorem, $\mathbf{F}\cdot d\mathbf{x} = d(2K)$, it is apparent that the work done on a system by its surroundings will cause a change in entropy, i.e., $d(2K) = TdS$. Since the system will change its momentum while moving along the resultant of time-dependent forces, $\mathbf{F} = d_t\mathbf{p} = T\nabla S$, its evolution will direct as well along a gradient of produced entropy. Thus, the least-time consumption of free energy will in general have the same trajectory as the least-time increase in entropy [26,27] in accordance with the basic maxim of thermodynamics [30,31]. Accordingly at the end point of evolution, the state of free energy minimum must be the state of physical entropy maximum. Temperature, T, is a meaningful statistical parameter to associate with a system [32] when the system is big enough to absorb or emit energy, $đQ$, without a marked change in the average energy, k_BT. The inexactness of the differential, $đQ/T = dS$, representing the 2nd law of thermodynamics, suggests that evolution from one state to another is quantized and will be path-dependent. Accordingly, the value of the antiderivative, i.e., the integral, cannot be computed from the initial and final states of a given system. Thus, the least-time imperative in its original form [3] can in fact be subsumed in the Second Law of thermodynamics, which in this context is commonly referred to as the maximum entropy production principle [cf. e.g., 33,34].

When recalling that entropy, $S = k_B\ln P$, is the system's logarithmic probability measure of bound and free energy [17,35], an irreversible natural process, albeit non-holonomic, is according to the Second Law of thermodynamics a probable process ($d_tP \geq 0$). The probability as a physical measure of the bound and free energy is changing due to the irreversible consumption of free energy when the system is changing from one state to another. Since the energy of the system is changing, there is no norm to normalize P to unity [36]. Accordingly, for the evolving system, there is no unitary transformation that could possibly remove time-dependence from the equation of evolution (Eq. 2). Thus, we again conclude that the irrevocable flow of quanta from bound to freely propagating photons physically embodies the irreversible arrow of time. When the system has attained a thermodynamic stationary state in its surroundings, the probability is stationary and then it could be normalized. At the stationary state the particles merely exchange momenta in interactions.

Quantum mechanics describes the time evolution operator by the change from one configuration to another, but the gauge invariant theory has no means to account for changes in energy of the system. This is the reason why quantum mechanics produces conceptual problems when speaking about detection where at least one quantum has to be emitted from the system to the detector or vice versa. In other words, the mere observation will produce a change of state both in the system and its surroundings. Hence no state can be defined more precisely than by one quantum h. In the context of quantum field theory transformations are described in terms of operators that are ordered with respect to time. Nonetheless, a mere transformation from a state at $t = -\infty$ to another at $t = +\infty$ is a holonomy of the gauge connection. The future will differ from the past when the probability of a particular history in the exhaustive set of histories is computed as a trace from the density matrix of the initial system subject to the time-ordered projection operators [37], but only when the initial and final states do not commute [38]. In other words the quantum field theory, even with time-ordered operators but without net dissipation of quanta from the system to its surroundings, cannot account for irreversibility, i.e., time's arrow. Moreover any other theory that complies with invariant gauge is limited to description of a state, and so cannot account for evolution by spontaneous symmetry breaking [9].

4. Intractability

The familiar relation $E = hf$ holds when a photon influx and efflux of energy, E, per quantum, h, drives repetitive changes of state with frequency, f. Likewise, a corpuscular flux of kinetic $2K = mv^2 = n\hbar\omega$ will produce changes of state in quantized steps of $n\omega$ where $\hbar = h/2\pi$ and ω is the angular frequency. The quantized flux that is conveniently but erroneously denoted by the continuous differential, d_t, will break the time-independent symmetry of mv^2 to evolution, $d_t(mv^2) = \mathbf{v}\cdot d_t\mathbf{p}$. If a quantum was indefinitely dividable, the symmetry of a system could stay the same irrespective of energy influx or efflux. Although a change in energy is invariably associated with a change in state, the L^2 (Euclidean) norm of a stationary state, contained in mv^2, remains appealing to many working in a discipline that aims primarily at making predictions because the norm complies with a modular, hence computable, trajectory.

Because the universal least-time imperative describes path-dependent processes, the Second Law of thermodynamics cannot be a locally deterministic principle. In general there is no way to a solution, e.g., by the way of integrating the equation of evolution (Eq. 2) to a closed and definite form, because the driving forces of motion cannot be conceptually separated from the motions while they are occurring [36]. For example, when a reservoir drains via two outlets, the flow through one will affect the other by lowering the common level of water that drives the flows. The differential form implies that the differentials at the branching points of any path are inexact. Quantization relates well to this intractable path-dependence because the quantum cannot be divided indefinitely among alternative paths of free energy consumption.

The non-deterministic and holistic character of natural processes due to the common universal surroundings manifests itself as well in the double-slit experiment. When a photon or an electron, or any other projectile, propagates in a non-zero energy density at least as large as that of the vacuum, this projectile will perturb surrounding energy densities prior to its traversal through to a plate pierced by two slits. Then the induced perturbations will propagate and traverse through the slits and subsequently re-interact with the projectile to yield an interference pattern [40]. In the sense of Mach, when everything depends on everything else, causal relations are not one to one [17]. Thus, when a quantum is absorbed from a common reservoir of potential energy by one process, the same quantum cannot also be taken up by another.

Despite the non-deterministic character of natural processes [41,42,43,44], the least-time quest to consume free energy will have been directing the courses of energy flows so as to result in the rules and regularities that we see in nature [45,46,47,48,49]. The path-dependence of statistical processes can also be formulated as the statement that among all conceivable worlds the most probable must be the actual one [50], in the sense that Leibniz found the actual world to be the best of all possible ones.

Many mathematical models of evolution are forced toward solution by imposing boundary conditions [51,52]. However, when fixed conditions are imposed, a mathematical model of the state change will violate the conservation of quanta because the net flux of energy from the system to its surroundings will change the surroundings that define the boundary conditions. Consequently, it is not only we who do not know where nature is tending locally, but nature in its entirety, and any subsystem of it, cannot ‘know’ these details, but varies its paths among alternatives at all levels of the natural hierarchy as it discovers the least-time paths. The flows of energy will naturally select those evolving paths that will decrease free energy in the least time [53]. Other paths will receive less flow of energy and will eventually run dry when draining the common reservoir of energy. Thus, we conclude that the quantized flows of energy from the system to its surroundings along least-time, intractable, paths literally embody threads of time.

5. Interrogations

The notion of time’s arrow relates to many studies of the evolving nature. Therefore various queries, depending on one’s education and profession, may come up when inspecting the above physical portrayal of the time’s arrow.

A physicist may question, how the universal arrow of time relates to the expanding Universe? According to the principle of least action various natural processes must be breaking down actions of high energy density in the form of matter constituting the present Universe to actions of low energy density, ultimately to photons, in the quest to attain universal equilibrium in the “zero-density” surroundings. The lowest possible symmetry of unitary group number one U(1) would characterize the ultimate equilibrium – the Universe comprised of only extremely cold photons. It will be fully symmetric when comprised only of photons propagating evenly in all directions. In other words, time’s arrow ceases at the heat death. The imperative of least-time consumption of free energy would manifest itself also in homogeneity at the largest scale because any local excess, just as any deficit of energy density, will generate motions that will consume the forces [54,55]. Thus the universal arrow of time is embodied in the universal flux of quanta from bound repositories of matter to freely propagating photons.

Moreover, what renders time relative? Each system is subject to the influx or efflux of quanta when interacting with its surroundings. A quantum received by one subject cannot be received by another. Therefore the flow of time as the flux of quanta is sensed by each subject – for example a detector or an observer. This flux will change the coordination of a subject relative to other subjects as well. In this perspective there is no absolute space and no absolute time. Indeed Einstein noted that time is relative, but, of course, he did not say that time is physically embodied in the flows of quanta.

Finally, why, then, does a clock run faster in a weaker gravitational potential, such as in an orbiting satellite compared to the surface of the earth? A weaker gravitational potential is a sparser surroundings and therefore it will accept more readily the quanta that the clock will emit as it runs. In this sense the clock is like any other heat engine whose rate of revolution depends on the energy difference between the fuel and exhaust. For example, the gravitational blueshift of light that approaches a celestial body indicates an increasing energy density due to the local gravitational potential. The thermodynamic tenet makes no distinction between one surroundings and another, but acknowledges that an engine of any kind will revolve with a rate that is proportional to the energy difference between the potential of its fueling source and the potential of the sink for its thermal waste.

A biologist, in turn may wonder, why order and organizations can emerge from natural processes where entropy is increasing rather than decreasing? According to the naturalistic tenet adopted here, order and organization are viewed as the means by which free energy is consumed, not ends in themselves [35,53]. Therefore there is no direct correlation between entropy and disorder. This common misconception stems from Boltzmann's formulation of statistical mechanics, which applies only overall, globally to isolated systems [56]. An isolated system at equilibrium is invariably conserved, hence it is incompetent to evolve further, and is able only to change temporarily its isoenergic internal configurations, referred to as microstates. In a stationary state without net influx or efflux of energy no new species will emerge. Conversely a net flux of quanta from surroundings to the system is necessity for emergence within that system [57]. An open system will become disordered via exchange of quanta with incoherent surroundings, while such a system can become ordered via exchange of quanta with coherent surroundings [58,59]. However, if there is no net flux, neither will the free energy decrease nor will entropy increase. It would violate the conservation of quanta for a system to spontaneously decrease in entropy beyond local fluctuations while its surroundings increase in entropy, because the quantized flux traverses through the common interface between a system and its surroundings.

How did natural selection guide the primordial evolution of dissipative structures without genes? All actions are subject to the irrevocable consumption of free energy, which leads inevitably to the irreversible increase of entropy. Evolution will take its irrevocable direction of energy dispersion irrespective of the energy transduction mechanisms involved [25,60,61,62,63], and regardless of how complicated these mechanisms might be, or to what degree its complete dissipation might be delayed, e.g., when passing through a food chain. Entropy not only increases by necessity, but it will increase in the least time, consistent with local constraints. Living structures exist in order to dissipate energy gradients not susceptible to simple conduction. They dissipate energy gradients as fast as they can consistent with maintaining their operational forms to consume free energy also in the future. Consequently when there is variation in mechanisms, irrespective of whether prebiotic or genetic, the flows of energy will naturally select to channel through those mechanisms that will consume free energy in the least time. Life emerged and flourished as functional structures [11], therefore, without demarcation from its surroundings [64]. Selection naturally operates in like manner, guided by the Second Law, on economic and cultural evolutions [65,66].

Finally, how can we justify the thermodynamic tenet, making no distinction between animate and inanimate entities? Animate and inanimate processes both display the same scale-free characteristics, namely growth and decline along non-deterministic, sigmoidal curves. Moreover, distributions of both kinds of population sizes and event occupancies are skewed, and their cumulative curves follow power laws [45,46,47,48,49]. Thus, evolutionary unfolding involves not only living species but all kinds of entities. The structures of storms themselves will evolve. Irrespective of how small the energetic cost might be to represent a concept, physical embodiment makes the concept subject to the laws of nature [67]. Hence distributions of information, such as words and letters, are skewed with long tails as are other distributions of nature [49]. Accordingly, evolution should not be conceived only in specific genetic terms but also in more encompassing general terms, of which the Maupertuis action is the most comprehensive, applicable to all events of all kinds.

Since a natural process is a path-dependent series of state changes, decisions about how to consume free energy will also affect the future set of choices [68]. When a subject has consumed all sources of free energy, it will be left with no alternative paths, and hence has no freedom to choose. Conversely, the presence of resources requires the exercise of responsibilities. To live is to process, and thinking itself involves temporal flows of energy. Indeed, many puzzles and peculiarities can be reasoned and rationalized using the principle of least time, representing the supreme law of nature seems indeterminate as to how the Universe came to its existence. Perhaps this question, also involving chronological expectations, will be found to have been ill-posed as the mystery breaks.

6. Conclusion

In this paper we argue that time, including its directionality, is not an abstract notion but is embodied in net quanta that an evolving system absorbs from its high energy density surroundings or emits to its low energy density surroundings. The universal arrow of time results from diverse natural processes where quanta that are bound in the high energy densities of matter break free as photons that embody the universal vacuum. The natural bias for the irrevocable consumption of free energy is expressed by the Second Law of thermodynamics, as mediated by the principle of least action.

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