



Natural thermodynamics



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HIGHLIGHTS

- Thermodynamics is founded on the notion of quantum.
- Entropy is derived from statistical mechanics of open systems.
- The principle of increasing entropy equals the imperative of decreasing free energy.

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ABSTRACT

The principle of increasing entropy is derived from statistical physics of open systems assuming that quanta of actions, as undividable basic build blocks, embody everything. According to this tenet, all systems evolve from one state to another either by acquiring quanta from their surroundings or by discarding quanta to the surroundings in order to attain energetic balance in least time. These natural processes result in ubiquitous scale-free patterns: skewed distributions that accumulate in a sigmoid manner and hence span log–log scales mostly as straight lines. Moreover, the equation for least-time motions reveals that evolution is by nature a non-deterministic process. Although the obtained insight in thermodynamics from the notion of quanta in motion yields nothing new, it accentuates that contemporary comprehension is impaired when modeling evolution as a computable process by imposing conservation of energy and thereby ignoring that quantum of actions are the carriers of energy from the system to its surroundings.

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1. Introduction

Thermodynamics speaks about nature in energetic terms. Since energy can be assigned to anything, thermodynamics addresses everything. However, energy as such does not exist. It is an attribute of its carrier. Energy is carried, for instance, by a photon. Thus, when quantifying the state of a system, its carrier composition – not its energy – ought to be examined in the first place. Likewise, when describing the system in transformations from one state to another, motions of carriers – not changes in energy – ought to be explained in the first place. This assertion that the carrier is a more fundamental notion than energy may though seem superfluous, because the equation for the carriers in motion, as will be shown below, turns out to be equivalent to the equation for energy in motion [1–6]. Yet, we will argue that thermodynamics, as a concise and consistent theory, follows naturally from the concept of the quantum.

Fermat's principle says the photon will propagate along the path of least time. When moving along that path, the photon will, by carrying energy, diminish an energy difference, known as free energy, between the state of departure and the state of arrival. In other words the photon's least-time trajectory trails along the energy gradient where the change in momentum

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points along the force, i.e., $\mathbf{F} = d_t \mathbf{p}$. This means that Fermat's and Newton's accounts on motions are equivalent, as they should be.

Moreover, it is not only the photon but any other embodiment of energy that moves alike along a geodesic, i.e., the trajectory where free energy will be consumed in least time. For instance, water runs down along the steepest descents on a hill slope thereby consuming free energy in the form of a gravitational energy gradient in the least time. If this were not the case, then there would be a change in momentum, i.e., an effect without a force, i.e., water would linger without a reason. Likewise, a stock of animals grows by consuming surrounding chemical potentials in the form of food in the least time. Of course, it may though appear to an observer as if this were not the case, when a force, i.e., a reason goes unrecognized. For instance, browsers' fear for predators may redirect them on a detour. When that force is taken into account, the observed behavior is indeed found motivated, i.e., to obey the least-time principle.

The universal least-time imperative, known as the principle of least action, is the generalization of Fermat's principle. So it turns out that transformations from one state to another of any kind whether given in terms of action or in terms of energy are equivalent, as they should be. Nevertheless it is worthwhile to ask, what exactly is the action that carries energy?

The quantum of action, specifically the quantum of light, is the most elementary action. Its unit, joule seconds (Js), informs us that energy E and a period of time t are attributes of the quantum of action so that their product is an invariant measure known as Planck's constant

$$h = Et. \quad (1)$$

This equation is, of course, mathematically equivalent to the familiar textbook form

$$E = hf \quad (2)$$

where $f = 1/t$ is the photon's frequency. Yet the former (Eq. (1)) stresses that the photon is the basic building block of nature with energy and time as its compound attributes whereas the latter (Eq. (2)) places emphasis on the photon's energy attribute. The units imply that Planck's constant is not only an invariant number but it measures a physical entity, namely the quantum of action.

Here we adopt the old atomistic tenet by regarding the quantum of action as the invariant and elementary unit of nature [7]. This tenet differs from that in modern physics where the quantum of light, i.e., the photon, as a gauge boson, is deemed also as a non-conserved virtual particle [8]. Put differently, according to the atomistic idea the photon is the indivisible quantum of action whereas the modern doctrine declares that energy is quantized. This tension between the old and new ways of thinking about the physical basis requires for a resolution.

2. The notion of quantum

Let us begin by postulating that indeed everything is ultimately composed of the quanta of actions, and explore its consequences. However, first we had better make sure that the postulate is not in an obvious conflict with observations. Since even one exception to the rule would jeopardize the tenet's consistency and logic, we must analyze also phenomena and substances that are traditionally remote from thermodynamics.

2.1. Correspondence with observations

Every chemical reaction will either emit or absorb at least one photon, as light or heat depending on energy of dissipation (Eq. (2)). Nuclear reactions dissipate likewise, albeit then the quanta carry higher energy. Also annihilation of a particle with its antiparticle yields photons. Conversely, pair production proceeds from photons to particles. All this implies to us, just as Newton conceived, that matter is ultimately made of photons [9]. This tenet can be falsified, for instance, by presenting a material entity whose annihilation would yield something that, in turn, would not annihilate to mere photons.

This resolution of everything being composed of the same basic building blocks is motivated further by models that present elementary particles in terms quantized actions [5,10]. These models reproduce particles' measured properties and comply with their oscillations and decay schemes. Still some changes of state, such as a body falling from a height on the ground, are not outwardly dissipative. Yet, we continue arguing by considering the reverse reaction that also the transformation from one place to another entails either absorption or emission of quanta of actions. When the body is lifted up from the ground back up on the height, some form of free energy will be consumed. Here on Earth the required fuel for the reverse transformation is ultimately produced by absorbing photons from insolation. Thus, the reversibility in transformation points to the irrefutable conclusion: photons are emitted when the body is falling down because photons are absorbed when the body is lifted up. This resolution raises another question. Why are the emitted photons not detected?

To answer, let us examine a well-known phenomenon involving photons where no light is observed. Namely, the double slit experiment yields bright and dark bands for constructive and destructive interference. At a dark band where two photons have combined with opposite polarizations, we see nothing. The cancellation of electromagnetic fields, i.e., forces, however, does not mean that the photons themselves would have vanished into nothing. The net force vanishes when all forces balance each other. When there is no net electromagnetic force, nothing will drive detection, i.e., a charge-coupled transformation in the detector. Nevertheless, the lack of electromagnetic energy gradient does not imply a nil potential.

Namely, Aharonov–Bohm variant of the double slit experiment demonstrates that an added vector potential alters the diffraction pattern. Since the diffraction pattern exists already without the additional potential, the coherent conclusion is that the space, known as the vacuum houses already some energy density. So, the obvious thought is that the photons in pairs of opposite polarization, for example emitted when the body fell down, embody the vacuum [11]. Is this inference about the character of vacuum correct?

The question about the substance of vacuum is of particular importance for thermodynamics because dissipation of any kind ultimately ends up to the all-embracing vacuum. The vacuum energy density ρ_E is on the order of nJ/m^3 [12]. Curiously, the value matches the average energy density of matter in the Universe. In other words the gravitational potential of the total mass M within the radius R of the Universe equals, via $GM^2/R = Mc^2$, energy in mass, where G is the gravitational constant and c is the speed of light. We reason that this universal balance is by principle no different from thermodynamic balance in a reaction chamber, where reactant concentrations and surrounding conditions, most notably temperature, depend on each other. The Universe is apparently not quite in a perfect balance, because it is changing, i.e., evolving from one state to another. The energy density is diluting by expansion as matter is being combusted by stars, black holes and other mechanisms to the freely propagating photons. Thus the space is emerging from these transformations that emit photons, surely as light, but apparently mostly as the photon pairs without net polarization, thus imperceptible yet carrying energy. Therefore the space is as dark as a destructive interference band. This resolution about the vacuum is important for thermodynamics, because the vacuum is the ultimate sink for transformations of any energy source. Moreover, it allows us to describe gravity in thermodynamic terms.

When a local gravitational potential is understood, alike the universal potential, embodying the photon pairs, gravity can be comprehended as a force, just as any other force [5,11]. Namely, free energy, as a general capacity to do work, is a force of any kind, i.e., an energy difference per distance. Bodies will by moving from one state to another consume energy density gradients that exist between the system of bodies and its surroundings. Although it is not so easy to detect the escaping energy embodied in the oppositely paired photons, the notion of photon-embodied space does explain observations [13–15]. Moreover, the photon-embodied vacuum appears as a natural way to explain why the electromagnetic characteristic of the free space, namely permittivity and permeability relate to the speed of light via $c^2 = 1/\epsilon_0\mu_0$, as well as why Coulomb and gravitational potential have a similar function form.

Thus we conclude that the postulated unity of everything given in terms of the quantized actions complies with observations that range from microscopic interference of quanta to cosmic expansion. Next we will show how thermodynamics follows naturally from the universal notion of the quantum action.

2.2. Entropy as a measure

Statistical mechanics pictures thermodynamics of macroscopic systems to result from behavior of microscopic constituents. We employ this renowned probabilistic theory of many-body systems to describe evolution from one state to another. It depicts the state of a system in terms of an energy level diagram. Even when without of exact knowledge of how a given system comprises of its elementary entities, i.e. from the quanta of actions, the level diagram is exact by including everything. Therefore it allows us to deduce unambiguously the probability for each entity to exist in its contemporary surroundings. Subsequently, as advised by statistical mechanics, we will take the logarithm of the probability P to obtain entropy $S = k_B \ln P$, where k_B is Boltzmann's constant. Entropy obtained in this manner is the measure of a state [4,6]. When equipped with the entropy formula, we are in position to derive its changes, i.e., to derive the second law of thermodynamics, i.e., the principle of increasing entropy.

In the diagram (Fig. 1) each entity, with a distinct energy attribute, populates a distinct level. Conversely, energetically identical, in other words, indistinguishable entities occupy the same level. Synthesis and degradation of entities via dissipative transformations of any kind, such as chemical reactions, are depicted by horizontal arrows that draw from one scalar potential level to another and by vertical wavy arrows that draw flows of energy from the surrounding vector potential to the system or vice versa. Conversely, energetically inconsequential non-dissipative exchanges of identical entities, i.e., permutations, are indicated by vertical bow arrows.

The level diagram can be exemplified for a chemical reaction as follows. Each population of identical molecules occupies one particular level, whose chemical potential $\mu_k = k_B T \ln N_k + G_k$ where N_k is the number of molecules, G_k is energy of one molecule and $k_B T$ is the average energy of the system at temperature T . The synthesis of a population comprising N_j molecules from its substrates in numbers N_k couples either with emission or absorption of photons whose energy matches the energy difference ΔG_{jk} per molecule between the substrates and products. This influx or efflux of energy, i.e., dissipation is denoted by $i\Delta Q_{jk}$. The imaginary part merely indicates that the vector potential from the surroundings to the system or vice versa is orthogonal to the scalar [chemical] potential. This convention about the orthogonal potentials manifests itself, for instance, so that when an electron falls down along the electric potential gradient, light will emit in an orthogonal direction. This explicit distinction between the scalar and vector potentials is also contained in Poynting's theorem.

The probability P_j^1 for one entity to exist at the level indexed with j depends on its ingredients, each of which is available on a level indexed with k , as well as on the flux of energy from the surroundings to the system that couples to the transformation from k to j

$$P_j^1 = \prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T} \quad (3)$$

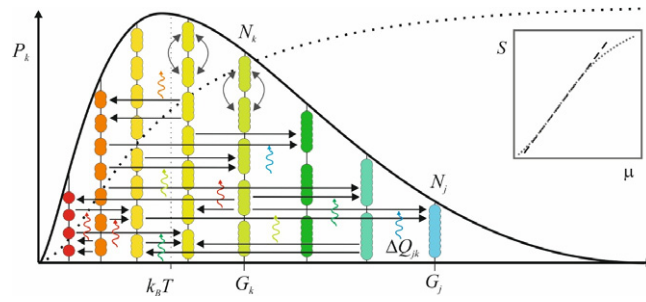


Fig. 1. The system is depicted in terms of an energy level diagram. At each level, indexed by k , there is a population of N_k individuals each with energy G_k . The size of N_k is proportional to probability P_k . When an entity in the population N_k transforms to an entity in the population N_j , horizontal arrows indicate paths of transformations which are available for changes in the potential energy bound in matter and vertical wavy arrows denote concurrent changes driven by energy in light. The vertical bow arrows mean exchange of indistinguishable entities without changes in energy. The system evolves, step-by-step, via absorptive or emissive jk -transformations that are mediated or catalyzed by entities themselves, toward a more probably partition of entities eventually arriving at a stationary-state balance where the levels are populated so that the average energy $k_B T$ equals that in the system's surroundings. A sufficiently statistical system will evolve gradually because a single step of absorption or emission is a small perturbation of the average energy. Hence at each step of evolution the outlined skewed quasi-stationary partition does not change much. This maximum-entropy distribution accumulates along a sigmoid curve (dotted) which is on a log-log scale (insert) a straight line of entropy S vs. (chemical) potential energy μ .

where the product form \prod_k ensures that if any one vital k -ingredient is missing altogether, the j -entity cannot exist, i.e., $P_j^1 = 0$, as well as, that if no flux of energy can couple from the surroundings to the system, the transformation from the k -entities to the j -entity cannot take place. Stoichiometry of transformations from k to j is included in the degenerate indexing that runs from $k = 1$ to an unknown upper limit at the energetic balance.

The probability P_j for the population N_j of j -entities is obtained simply from the product of P_j^1 's, i.e.,

$$P_j = \left[\prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T} \right]^{N_j} / N_j! \quad (4)$$

where the division by factorial $N_j!$ enumerates the inconsequential exchange of identical entities (Fig. 1). Then the total probability for the system of populations is obtained from the product of P_j 's, i.e.,

$$P = \prod_{j=1} P_j = \prod_{j=1} \left[\prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T} \right]^{N_j} / N_j! \quad (5)$$

where the product form \prod_j guarantees that if any one population j is missing altogether, then that particular kind of a system cannot exist.

Finally, it remains to take the logarithm of P to obtain the formula for entropy

$$\begin{aligned} S &= k_B \ln P = k_B \ln \left\{ \prod_{j=1} \left[\prod_{k=1} N_k e^{-\Delta G_{jk}/k_B T} e^{+i\Delta Q_{jk}/k_B T} \right]^{N_j} / N_j! \right\} \\ &= \frac{1}{T} \sum_{j=1} N_j k_B T + N_j \left(\sum_{k=1} \mu_k - \mu_j + \Delta Q_{jk} \right) \end{aligned} \quad (6)$$

where Stirling's approximation $\ln N_j! \approx N_j \ln N_j - N_j$ has been used. The result (Eq. (6)) defines entropy S when multiplied with temperature T , as a measure both of energy $\sum_j N_j k_B T$ that is bound in all populations each having entities N_j and of free energy $A = \sum_j \sum_k \mu_k - \mu_j + i\Delta Q_{jk}$ that resides between the system and its surroundings to be consumed by available mechanisms of transformations between the j - and k -entities. This is to say that the term $S_b = \sum_j N_j k_B$ in Eq. (6) sums the fraction of entropy that is bound to the populations. It is, of course, the familiar entropy obtained from statistical mechanics for a closed system. Namely, the stationary system cannot gain or lose energy by acquiring or dispatching quanta to its surroundings, and hence its energy is altogether bound in the populations. Conversely, when the carriers of energy move from the open system to its surroundings to consume the energy difference, i.e., free energy A , also known as affinity, both the system and its surroundings will undergo transformations from one state to another. This flux of energy carriers from the system to its surroundings or *vice versa* leads to the irreversible increase in entropy. The term $S_f = (\sum_j \sum_k \mu_k - \mu_j + i\Delta Q_{jk})/T$ in Eq. (6) denotes the fraction of entropy that is still free for transforms to become bound.

Considering importance of definitions in communication, it is worth relating common terminology with the terms used here. The textbook thermodynamics defines a system as isolated when it cannot acquire or lose energy in any form. Here, isolation means that the system cannot acquire or lose any quantum as the carrier of energy. However according to the holistic tenet, the isolated system is an illusion because all systems in the Universe are coupled to all other systems at

least via gravity. In other words, all systems are embedded in interaction with the common vacuum. The textbook defines a system as closed when it cannot acquire or lose matter in any form but is only open to radiation. Here, no difference is made whether the system acquires or loses quanta in a form of matter or radiation because both comprise ultimately of photons. This claim is falsifiable by showing that annihilation of matter and antimatter would yield something else besides the photons, or conversely demonstrating that pair production of matter and antimatter would require something else besides the photons. Moreover, the common distinction between matter and radiation used in the definition of a closed system is reasoned to be inaccurate because in any reaction, such as in a chemical reaction, mass as an attribute of matter does change. So, the change in mass implies that matter is in fact not an invariant which is inconsistent with the definition of the closed system. Finally the textbook defines a system as open when it can acquire or lose energy in any form. Here, the meaning is the same, when understanding the quanta are the carriers of energy.

2.3. The least-time evolution

The entropy formula (Eq. (6)), as the equation of a state, can be differentiated to obtain the equation of motion for transformations from one state to another, i.e.,

$$\frac{dS}{dt} = \sum_{j=1} \frac{dS}{dN_j} \frac{dN_j}{dt} = \frac{1}{T} \sum_{j=1} \frac{dN_j}{dt} \left(\sum_{k=1} \mu_k - \mu_j + \Delta Q_{jk} \right) \geq 0 \quad (7)$$

where the chain rule has been employed. The result about the irreversible entropy increase is common sense. Explicitly, a population N_j will increase, i.e., $d_t N_j > 0$ when there are resources, i.e., free energy $A_j = \sum_k \mu_k - \mu_j + i\Delta Q_{jk} > 0$ to be consumed for the growth. Conversely, the population will decrease, i.e., $d_t N_j < 0$ when the resources are over-depleted, i.e., free energy $A_j = \sum_k \mu_k - \mu_j + i\Delta Q_{jk} < 0$ whose consumption drives downsizing. Be it either way, when the two terms of the product have the same sign, then the product is positive, i.e., $dS > 0$. When evolution has attained thermodynamic balance, then $dS = 0$. The state of free energy minimum is stable. Any perturbation δN_j away from a steady-state population N_j^{ss} would lead to decrease in $S(\delta N_j) < 0$ and concurrent increase in $d_t S(\delta N_j) > 0$. This Lyapunov stability criterion for homeostasis says that the further away N_j would be from N_j^{ss} the larger will be the restoring force A_j [16]. In this manner the evolutionary equation (Eq. (7)) states the second law of thermodynamics.

In short, entropy will increase until the maximum has been attained. Specifically, it would be unnatural to claim that entropy of a system could possibly decrease at the expense of an entropy increase elsewhere, because it would entail that quanta of actions would emerge from nothing or that they would vanish to nothing. We emphasize that the entropy increase follows from the consumption of free energy. Therefore, the equation of evolution (Eq. (7)) reveals that the notions of entropy and its increase do not communicate anything more than what can be expressed in energetic terms. Specifically, entropy (Eq. (6)) does not relate to disorder. This unnatural and ill-founded connotation follows from Boltzmann's derivation of entropy for a system of constant energy and invariant particle number. This definition means that the system cannot change its state, i.e., to evolve by acquiring or losing quanta. Moreover, coherence or incoherence is not an end itself but merely a consequence of evolution toward the free energy minimum state. Put differently, a system, such as an organism, will organize itself to consume free energy, i.e., the energy difference between it and its surroundings. Likewise, another system, such as a grain salt, will disorder itself by dissolving in water to consume free energy, i.e., the energy difference that resides between the crystal structure and the surrounding water.

The evolutionary equation (Eq. (7)) says that entropy will increase. Yet, it is only implicitly that entropy will increase in least time. We make this least-time entropy increase explicit by taking a convenient, but approximate, continuum limit of the evolutionary equation. Using the continuum definition of chemical potential $\mu_j = (\partial U / \partial N_j)$ in terms of the scalar potential U , and likewise for dissipation $Q_j = (\partial Q / \partial N_j)$ in terms of the vector potential Q , we obtain

$$\begin{aligned} T \frac{dS}{dt} &= -\frac{dU}{dt} + i \frac{dQ}{dt} = -\frac{dV}{dt} \\ \Leftrightarrow \frac{d^2 K}{dt^2} &= -\mathbf{v} \cdot \nabla U + i \frac{dQ}{dt} \end{aligned} \quad (8)$$

where we have used the definitions of velocity $\mathbf{v} = d_t \mathbf{x}$, and spatial gradient $\nabla = d/d\mathbf{x}$ as well as a shorthand notation V for the total potential that combines both the scalar U and vector Q potentials. Moreover, to equate the change in entropy with changes in kinetic energy, i.e., $TdS = d^2K$, can be motivated, for instance, so that the force \mathbf{F} will cause a change in momentum \mathbf{p} , i.e., a change of state quantified by entropy according to $d_t(2K) = d_t(mv^2) = \mathbf{v} \cdot d_t \mathbf{p} = \mathbf{v} \cdot \mathbf{F} = Td_t S$.

When dividing Eq. (8) with velocity, the resulting equation reveals that the force that directs down along the potential energy gradient $\mathbf{F} = -\nabla V$ is equivalent to the path's direction up along the entropy gradient, $\mathbf{F} = d_t \mathbf{p} = T\nabla S$. So, we conclude that the second law of thermodynamics (Eqs. (7) and (8)) and Newton's second law of motion are equivalent expressions for transformations from one state to another in the continuum limit, as they should be. It is worth emphasizing via

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m\mathbf{a} + \mathbf{v} \frac{dm}{dt} = -\nabla U + \frac{\mathbf{v}}{c^2} \frac{dE}{dt} = -\nabla U + i\nabla Q = -\nabla V \quad (9)$$

that dissipation $dQ = (v^2/c^2)dE$, eventually to a medium that differs from the vacuum's energy density by index $n = c/v$, stems from the change in mass d_m according to the mass–energy equivalence $dm = dE/c^2$. If this dissipative term of Newton's second law of motion, present in its original complete form [17], is omitted from the description of an evolving system, as it often is when writing $\mathbf{F} = m\mathbf{a}$ in textbooks, conceptual conundrums will follow. Then the bookkeeping of quanta is imperfect. Of course, the change in mass is minute in many transformations, for instance, formation of a covalent bond amounts up to about one per mill of the electron mass m_e . Yet, the concurrent dissipation is readily sensed.

This tenet that any change of state is invariably coupled with emission of at least one quantum to the surroundings or absorption of at least one quantum from the surroundings, can be falsified by presenting a counterexample, namely a change of state without any dissipation. Of course, a single quantum emitted from the change of state may easily escape detection and hence compromise the falsification. On the other hand the emission of quantum can be inferred by running the reverse reaction where the absorption of quanta is necessary to restore the initial state. In other words, the proposed tenet can be falsified by running a reaction without any input or output of energy.

It is also worth clarifying that here the mass–energy equivalence $E = mc^2$ is not obtained from special relativity, but it is the special case for vacuum of the general *vis viva* $2K = mv^2$ applicable to any surrounding medium [18]. Today's definition $K = 1/2mv^2$ limits the description to the motion of a body and thereby neglecting concomitant changes in surroundings. It is obvious that a herd of cows will by grazing consume chemical potential of a pasture. Likewise, a rock rolling from a hill top to a valley will diminish gravitational potential of a height.

Finally, the differential equation of evolution (Eq. (8)) has an integral counterpart, i.e., the action

$$L = \int 2K dt = \int (-\mathbf{x} \cdot \nabla U + iQ) dt = nh \quad (10)$$

which enumerates by n the quanta of actions, each of measure h . The action (Eq. (10)) re-expresses Noether's theorem: Every differentiable symmetry of the action of a physical system has a corresponding conservation law [19]. Conversely, symmetry will be broken when the system evolves from one state to another by acquiring or discarding quanta, i.e., $d_t n \neq 0$. Evolution proceeds along the least-time path where the change in integrand, $d(2K) > 0$, takes place as soon as possible. In other words, there is no mechanism available for the transformations to happen any faster. Finally, when the system has arrived at the stationary state, $d(2K) = 0$, where all transformations have completed to a thermodynamic balance. At the balance net dissipation vanishes and the virial theorem $2K + U = 0$ holds. So, we conclude that the principle of least action in its original form á la Maupertuis (Eq. (10)) is equivalent to the principle of increasing entropy, i.e., the second law of thermodynamics, and the Newton's second law of motion. There is one law of nature.

3. Consequences

The equation of evolution (Eqs. (7)–(9)), when derived from the postulate that everything is composed of the quanta of actions, is nothing new. During the course history the basic law of nature appeared in three equivalent forms, namely as the old principle of least action á la Maupertuis, the 2nd law of thermodynamics due to Carnot and Newton's 2nd law of motion in its original form. Yet, it seems to us that today the basic law of nature goes by somewhat unappreciated, and hence, for instance, the notion of entropy production is obscure. It seems to us that the quest of determinism has led to imprecise theoretical description of nature.

The contemporary form of least action devised by Lagrange as well as Newton's 2nd law, when reduced to $\mathbf{F} = m\mathbf{a}$, are incomplete descriptions of transformations by neglecting dissipation. Of course, this shortcoming has been somehow sensed but not rationalized since *ad hoc* terms have been added to the steady-state formula to model dissipation but not to explain dissipation [20]. These tinkered models do not comply with conservation of quanta, and hence they serve only as approximations of natural processes. In contrast when the correct equation of evolution (Eqs. (7) and (8)) is used, it is clear that entropy increases hand-in-hand with decreasing free energy. From this perspective the notion of entropy generation, both maximal and minimal, is out of the picture.

Above all the natural law in the form of evolutionary equation (Eqs. (7) and (8)) is motivated by observations. It yields the ubiquitous characteristics of nature concisely and consistently. Conversely the proposed tenet can be falsified by presenting data that does not display the universal patterns.

3.1. Scale-free patterns

Data from numerous phenomena, when plotted on a log–log scale, follow approximately straight lines, i.e., power laws [21,22]. Moreover, distributions of animate or inanimate populations, when plotted on a semi-log scale, are nearly normal [23–26]. These log-normal distributions accumulate along sigmoid curves, which in turn when plotted on log–log scales, are mostly straight lines (Fig. 1). Even a sharp phase transition when zoomed in is a sigmoid curve. So the scale-free patterns of power laws, skewed distributions and sigmoid curves are not only ubiquitous but actually they all present only one and the same pattern with varying values of parameters. Here the objective is not to model this universal pattern by various mathematical functions, but to clarify the cause of universality.

The origin of ubiquitous patterns can be identified with the least-time free energy consumption by analyzing the equation of evolution (Eq. (7)) [6]. In practice, it is not changes in entropy S but changes in populations N_j that are monitored. The

rate $d_t N_j$ of a population change is consistent with the conservation of quanta, when proportional to the free energy terms A_{jk} , i.e.,

$$\frac{dN_j}{dt} = \frac{1}{k_B T} \sum_{k=1} \sigma_{jk} (\mu_k - \mu_j + i\Delta Q_{jk}) = \frac{1}{k_B T} \sum_{k=1} \sigma_{jk} A_{jk} \tag{11}$$

where the parameter $\sigma_{jk} > 0$ for the conductance of energy carriers summarizes characteristics of a particular mechanism that transforms the k -entities to j -entities or vice versa. Substitution of Eq. (11) to Eq. (7) reproduces $dS \geq 0$, i.e., the 2nd law. So kinetics given by Eq. (11) is consistent with thermodynamics. It means that all transformations run downhill, i.e., by consuming free energy. In other words, we refute the common description of motion over an energy barrier, whereby kinetics appears as independent from thermodynamics. Instead we claim that all rates of changes are proportional to free energy A_{jk} according to Eq. (11) and that all reaction mechanisms, including catalytic mechanisms, are characterized by conductance σ_{jk} . This stance can be proven wrong, for example, by showing that kinetics is not proportional to free energy. The conductance for flows of energy in the catalyzed reaction is simply larger than the conductance via those mechanisms that facilitate a non-catalyzed reaction. The catalyst affects only the rate (Eq. (11)), not the energy landscape. It is the flows of energy that cause changes in the energy landscape. In other words, when the reaction takes place nothing moves on the energy landscape but the landscape changes as substrates transform to products. Narrow passages for flows of energy embodied in reactant are bottlenecks, not barriers, since energy flows always downhill.

The ubiquitous sigmoid course of a transformation can be understood from Eq. (11) as follows. Initially a nascent population will grow nearly exponentially when the free energy resources seem unlimited. Likewise, a mature population will decline nearly exponentially when the surroundings potential is almost zero, and hence free energy is large but negative. In other words, when the change in the free energy

$$\frac{1}{k_B T} \sum_{k=1} \frac{d}{dt} (\mu_k - \mu_j + i\Delta Q_{jk}) = \frac{1}{k_B T} \frac{dN_j}{dt} \sum_{k=1} \frac{d}{dN_j} (\mu_k - \mu_j + i\Delta Q_{jk}) = \frac{d}{dt} \ln N_j \approx \sum_{k=1} \sigma_{jk} = \sigma_j \tag{12}$$

can be taken almost as a constant σ_j , the population growth or decline, depending on the sign of free energy, will be nearly exponential. Likewise, when the population has almost attained thermodynamic balance with its surroundings, all free energy is nearly consumed and hence its change can be taken approximately as a constant σ_j . Thus, the remaining residual growth or decline, depending on the sign of free energy, will be a decaying exponential function.

At any time between the exponential initial increase and final decay the population change consumes free energy as soon as it becomes available, and hence the product and substrate potentials can be taken almost as equal, i.e.,

$$\mu_j \approx \mu_k + i\Delta Q_{jk} \Leftrightarrow N_j \approx \prod_{k=1} N_k e^{-(\Delta G_{jk} - i\Delta Q_{jk})/k_B T} = N_1^j \prod_{1 \leq m, n \leq j} e^{-(\Delta G_{mn} - i\Delta Q_{mn})/k_B T} = \alpha_j N_1^j \tag{13}$$

where the j -entity of the quasi-stationary system has been denoted in a recursive manner because it results from multiplicative operations, indexed by m and n , from the basic building blocks, i.e., from the quanta in numbers N_1 . In other words, the change

$$\frac{dN_j}{dt} = \frac{dN_j}{dN_1} \frac{dN_1}{dt} = j\alpha_j N_1^{j-1} \frac{dN_1}{dt} = \frac{jN_j}{N_1} \frac{dN_1}{dt} \Rightarrow d \ln N_j = j d \ln N_1 \tag{14}$$

in the central region, where $-\Delta G_{nm} + i\Delta Q_{nm} \ll k_B T$, follows a power law.

The above analysis discloses that the evolutionary equation of motion accounts for the ubiquitous patterns without postulating any boundary conditions to model data. In fact, no boundary conditions ought to be imposed because the carriers are moving from the system to its surroundings or vice versa, and thereby the surrounding boundaries co-evolve with the system. Admittedly, for many systems the changes in the surroundings, most notably in the vacuum, are in practice negligible but if neglected from the equation of motion, conceptual conundrums will rise.

Many systems evolve along a series of sigmoid curves, each punctuating [27] when a new mechanism appears in the system. Yet, the overall course follows to a good approximation a sigmoid curve too. This recurrent feature emphasizes further the scale-free character of natural processes. A new mechanistic species of transformation will advent and propagate when it will yield faster increase in entropy than could be attained by increasing the populations of existing species. In the context of speciation, known also as bifurcation, it is worth mentioning that the appearance of a new species will change the conditions of all other species in the system. Therefore the stem species, in particular, will face substantial evolutionary forces too when the new species branches out. Often the stem species is operating in the same environment and consuming largely the same resources as its new rival. Therefore, to escape extinction, the stem species is forced to specialize further in energy transduction to distinguish it from its branching rival. Fossil records reveal that the struggle for existence ended in extinction. By the same token, the system with any two species with similar phenotypes is unstable. As a result of mutual competition one of them will eventually be excluded from the ecosystem [28,29].

The interdependence between the change and evolutionary force, i.e., free energy can be modeled to the first-order by the approximation that gives a recurrence relation [30]

$$\frac{1}{k_B T} \frac{dN_j}{dt} \sum_{k=1} \frac{d}{dN_j} (\mu_k - \mu_j + i\Delta Q_{jk}) = \frac{1}{N_j} \frac{dN_j}{dt} \approx \sigma_j - \beta_j N_j \Rightarrow N_j (n + 1) = N_j (n) [\sigma_j - \beta_j N_j (n)] \tag{15}$$

for the population N_j at the consequent steps, indexed by n and $n+1$. When the parameters σ_j and β_j of the quadratic relation (Eq. (15)) are assigned with values that relate to a sufficiently statistical system, evolution will converge to a stationary point where the j -entities have either survived and settled to stasis or all gone extinct. However, when the free energy consumption via a mechanism with σ_{jk} exceeds the average energy so that $(\Delta G_{jk} + i\Delta Q_{jk})/k_B T > 1$, oscillatory or even chaotic motions will follow [6]. It is a common observation both in ecosystems and economic systems that when free energy will be consumed by a mechanism, say by a species or by a stock broker, at a rate that exceeds the rate at which its resources, i.e., free energy is replenished, the populations will invariably fluctuate.

When the slope of a sigmoid curve for transformation from the initial state to the final state is plotted, a skewed distribution curve will be obtained. Many natural distributions have this nearly log-normal shape. It can be understood as follows. The distribution at thermodynamic stationary-state condition $d \ln P = 0$ of Eq. (13) is found linear on a semi-log scale

$$\ln N_j = \ln \prod_k N_k e^{-(\Delta G_{jk} - i\Delta Q_{jk})/k_B T} = j \ln N_1 \sum_{1 \leq m, n \leq j} -A_{mn}/k_B T \propto j \ln N_1, \quad \text{when } A_{mn} \ll k_B T \quad (16)$$

where each j -entity in the hierarchy (Fig. 1) emerges via evolutionary processes from the basic constituents N_1 . The condition $A_{mn} \ll k_B T$, as above, excludes oscillatory and chaotic motions. Each population N_j in a distribution embodies an energy density

$$\phi_j = N_j e^{G_j/k_B T} = N_1^j e^{j(G_1 + i\Delta Q_1)/k_B T} = e^{j(\ln \phi_1 + i\Delta Q_1/k_B T)} \Leftrightarrow \ln \phi_j = j \ln \phi_1', \quad (17)$$

where the number of quanta $j\Delta Q_1$ that have been incorporated in the assembly of ϕ_j are denoted by ϕ_1' . Likewise, the population of an adjacent $(j+n)$ -class houses the density

$$\phi_{j+n} = \exp[(j+n)\phi_1'] = \phi_j \exp(n \ln \phi_1') \Leftrightarrow \ln \phi_{j+n} = (j+n) \ln \phi_1'. \quad (18)$$

This form yields a distribution of energy densities $\phi_{j-n \dots j+n}$ over adjacent classes $j-n \dots j+n$ about ϕ_j according to

$$\phi_{j+n} = \exp[(j+n)\phi_1'] = \phi_j \exp(n \ln \phi_1') \Leftrightarrow \ln \phi_{j+n} = (j+n) \ln \phi_1'. \quad (19)$$

The functional form is normal according to the central limit theorem when $n \ll j$. This condition of small variation is effectively the criterion of classification. For example, related species represent relatively small variation in n compared with j , and hence the distribution of species within a genus is almost lognormal. Conversely, any two species will differ considerably from each other when the number of multiplicative steps differs considerably, and hence the two species are not classified in the same distribution. The natural distribution departs from the log-normality both at low and high ends where the characteristic mechanism of energy transduction becomes increasingly ineffective (Fig. 1). Therefore the sigmoid cumulative curve departs from the power law at both the low and high ends.

When the stationary-state condition holds approximately, i.e., $A_{jk} \ll k_B T$, the system evolves gradually, and hence the natural distribution retains its nearly log-normal characteristic. When the system grows by absorbing quanta with energy from its surroundings, the skewed distribution will shift toward higher classes. Conversely when the system declines, its distribution will shift to lower classes. This characteristic dependence of a distribution on its average energy $k_B T$ is familiar, for instance, from Maxwell–Boltzmann and Planck's distributions but equally well it accounts for distributions of ecosystems and economic systems.

The presented connection between the ubiquitous patterns and the least-time consumption of free energy can be proven false by presenting a system whose evolution from one state to another does not yield the common characteristics. Such a system would be regarded here by the adopted holistic tenet as unnatural.

3.2. Non-determinism

The equation of evolution (Eq. (7)) clarifies why natural processes cannot be predicted precisely. The non-determinism follows because the flows of energy consume energy differences that in turn affect the flows [31,32]. Mathematically speaking interdependency means that when the variables cannot be separated, the differential equation cannot be solved. Conversely, the equation of evolution cannot be integrated to a closed form because the integration limit is changing during integration, i.e., during evolution. The boundary condition is not invariant because the surroundings is changing as the evolving system acquires quanta from its surroundings.

We emphasize that the non-deterministic character of natural processes does not ultimately stem from complexity of the system. Simple systems are non-deterministic too. This is apparent from the three-body problem. Moreover, our inability to make precise prediction does not ultimately follow from our limited knowledge of the system, but from the interdependency. In fact we cannot measure the state of any system more precisely than by one quantum of action, i.e., h , simply because the detector has to receive at least one quantum from the system to sense it, or conversely the detector has to grant at least one quantum to the system to infer it. In other words, the system and the observer are coupled to each other so that one cannot change without a corresponding change in the other.

Mathematical models of natural processes, such as Markov's chain implemented with Monte Carlo sampling from a probability distribution mimic non-determinism by indeterminism but do not reproduce it accurately [33]. Surely many

forces are small and tend to fluctuate, to give an impression of randomness, but in nature, according to Eq. (7), there is nothing truly random; all changes of state consume free energy. In other words indeterminism, that is a change in momentum without force, is ruled out. So, there is this sense of teleology in the least-time principle, but trajectories cannot be determined because everything depends on everything else. In other words, the final state does depend on the taken path, not only on the initial state.

Previously non-deterministic factors in biological evolution have been ascribed to lack of knowledge about initial conditions in models of chaos and to emergent factors in models of complexity. Here non-determinism is found to follow from holism, i.e., everything depends on everything else. When everything is accounted for with the ultimate precision of the basic building block, there is no room for non-determinism to rise from lack of knowledge or from unaccounted emergent factors. The least-time imperative for the flows quanta between the system and its surroundings, in turn, ensures that no quantum is lost for nothing or created from nothing during evolutionary courses.

3.3. Emergence

Natural processes produce from small and simple ingredients complex products whose properties cannot be reduced to mere combinations of the ingredients' properties. This irreducibility, namely that the whole is greater than the sum of its parts, has been regarded somewhat mysterious. However, when everything is composed of actions, then also interactions that combine the ingredients will change at the dissipative changes of state toward the products. In other words, also fluxes of quanta from the surroundings to the system, or vice versa, are ingredients of a reaction. Even a single photon can make a great difference in dictating properties of ingredients and products. For example, at a chemical reaction electron orbitals of an atomic substrate will open up either to acquire quanta from the surroundings or to expel quanta, and close anew as molecular orbitals of a product. Thus, the system of atoms will invariably emerge with new characteristics of the molecule, when quanta from the surroundings integrate into the system's existing ingredients [34]. Due to the energy input the new qualities associated with new eigenvalues and eigenmodes cannot be reduced to the former free constituents. Conversely, no new property can materialize from constant-energy permutations of pieces. Moreover, division into strong and weak emergence is unnatural.

Often animate are associated with absorptive processes which are described as self-organization whereby novel properties are gained. Conversely inanimate are often associated with emissive processes which are seen as disintegration whereby old qualities are lost. However, all processes either way obey the same principle that directs any system along least-time trajectory of free energy consumption toward balance in the respective surroundings. In other words, thermodynamics makes no distinction between animate and inanimate, thereby dissolving the question about life and its origin [35–37] as misapprehension.

4. Conclusions

Thermodynamics speaks in energetic terms about a system and its surroundings. General enough, yet the theory, as we know it today, does not state that energy is only an attribute, not an embodiment of its own. It is the quanta of actions that embody the system and its surroundings as well as inter-actions. From this standpoint nature is in evolution, when the quanta move from the system to its surroundings or vice versa to diminish energy differences in least time. When this natural law is formulated mathematically it turns out to be the 2nd law of thermodynamics which in turn is found equivalent to Newton's second law of motion and to the principle of least action in their original complete forms. So, actually nothing new was gained by emphasizing the notion of quantum, as the basic building block of everything. However, if the conservation of quanta is not appreciated, the photons are perceived to appear from nothing or disappear for nothing, and hence the old general law of nature will be violated. This is the case with *computable* models that aim at mimicking specific evolutionary data because the computability, i.e., the ability to solve an equation of evolution from one state to another, means that energy of the system is invariant [31]. This stationary condition, however, is in contradiction with the objective to describe an evolving system.

It is perhaps worth emphasizing that imprecise predictions of evolutionary courses do neither ultimately stem from our incomplete knowledge nor from complexity of a system. Non-determinism is a character of nature. It follows from the fact that everything depends on everything else, and hence a change of the system imposed by its surrounding will, in turn, change the surrounding affecting the system and so on. For example, chaotic motions can be in many cases modeled quite well by deterministic equations of motion that are sensitive to minute differences in initial conditions. Yet, in reality it is not variation in initial conditions that yield the dispersion of outcomes, but every step along the paths will affect the outcomes. In other words, diversity in nature does not originate from subtle differences in an initial state but follows from the path-dependent natural processes.

Certainly shortcomings both in mathematical models as well as in biological narratives of evolution have been recognized early on and analyzed [38–41]. It has been understood that evolution entails changes both in energy and time, but since much of modern physics is formulated to conserve energy, and hence to retain also time as invariant (Eq. (1)), it calls for revision in the fundamentals. We acknowledge that this need for a change in paradigm has been spoken earlier by many. Yet, it appears to us that the change from one state to another as the elementary step of evolution described by Newton's

2nd law of motion in its original complete form (Eq. (9)) and equivalently by Maupertuis' principle of least action (Eq. (8)), has not been reconsidered. When without the firm physical foundation of evolution embodied in mathematical exactness, abstract and erroneous notions about entropy flourish as well as unfounded ideas about animates differing from inanimate in some fundamental way thrive [41,42]. Perhaps by now many deem the holistic account as an illusory objective, instead substitute understanding with modeling. In fact, when accustomed with the diversity of mathematical models, it may seem today unbelievable that only one law could possibly account for all phenomena. True enough doubts will prevail because it is inherent in the universal law that its universality is impossible to test everywhere. On the other hand one exception is enough to falsify the tenet. We continue to search for it.

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