Asymptotic safety versus triviality on the lattice

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arXiv:1908.04605

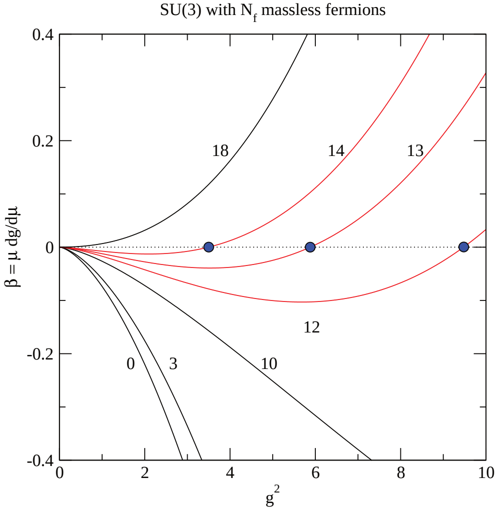
Bridging perturbative and non-perturbative physics

The standard picture:

Consider *2-loop perturbative* β-function of SU(N) + N_f fermions:

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

- Small N_f: $\beta_0 > 0$, $\beta_1 > 0$
  running coupling, confinement and χSB (QCD-like)
- Medium N_f: $\beta_0 > 0$, $\beta_1 < 0$
  IR fixed point, no χSB [Banks,Zaks]: **conformal window**
- Large N_f: $\beta_0 < 0$
  Asymptotic freedom lost
  - $\rightarrow$ Landau pole
  - $\rightarrow$ Theory is **trivial**



Does this really happen at large N_f?

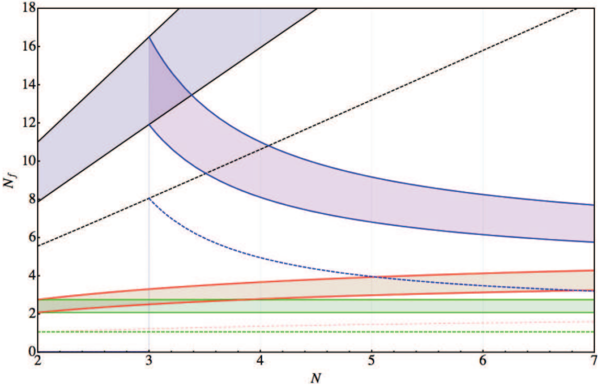
Consider SU(N) gauge with N_f (fundamental) fermions:

- Standard lore: as the asymptotic freedom is lost, theory has a Landau pole.
- However: $N_f \to \infty$ calculations suggest that there may be an UVFP at strong enough coupling (Asymptotic safety) [Antipin,Sannino 17], see also [Gracey 96]

In this talk: first attempts to study the behaviour on the lattice

- SU(2) gauge with $N_f = 24$ and 48 at $m_{\mathrm{fermion}} = 0$
- Measure the evolution of the coupling constant
- Use similar methods as used earlier within the conformal window

Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

2-index symmetric

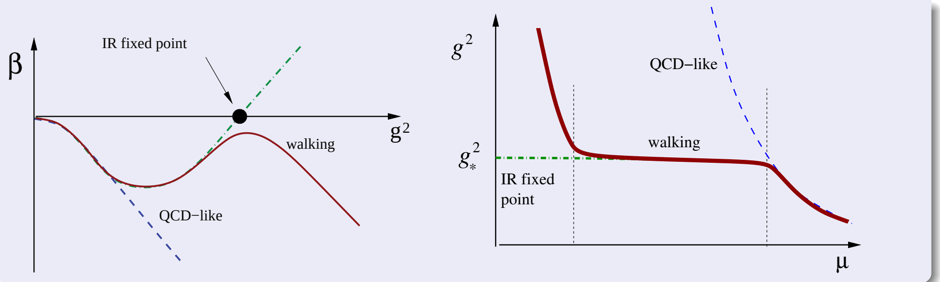
adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- *Building BSM models using higher reps: easier to satisfy EW constraints* *[Sannino,Tuominen,Dietrich]* $\rightarrow$ recent interest

Walking coupling

- Just below the conformal window β-function *may* get close to zero at finite coupling

$\Rightarrow$ The coupling evolves slowly, *walks*.



- Building blocks for strongly coupled BSM scenarios
- CP3-Origins very active!

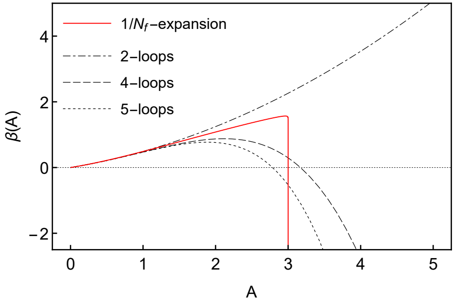
Large N_f:

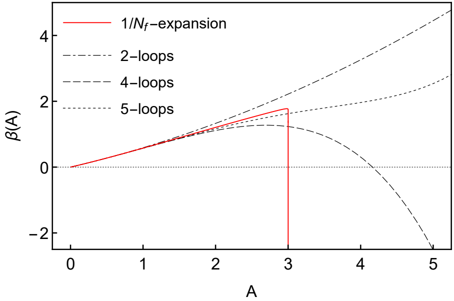
Let $A = N_f \frac{\alpha}{2\pi}$ (for fundamental fermions).
At large N_f:

$$\frac{3}{2}\frac{\beta(A)}{A} = 1 + \frac{H_1(A)}{N_f} + \frac{H_2(A)}{N_f^2} + \ldots$$

$H_1(A)$ has a logarithmic singularity at $A = 3$
$\Rightarrow$ β-function vanishes, UVFP.
[Antipin, Sannino 17; Gracey 95; Litim, Sannino 14]

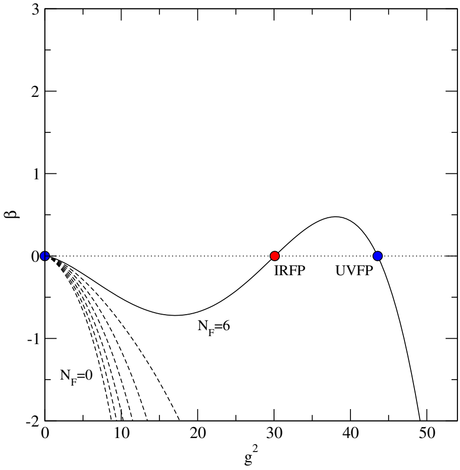
SU(2) with $N_f = 24$ (top) and 48 (bottom):
Large-N_f result compared with 2-loop and 5-loop $\overline{MS}$.





Cartoon of β-function

- We illustrate possible UVFP behaviour using perturbative 4-loop β-function for SU(2)+N_F fermions:
- This is just a cartoon!



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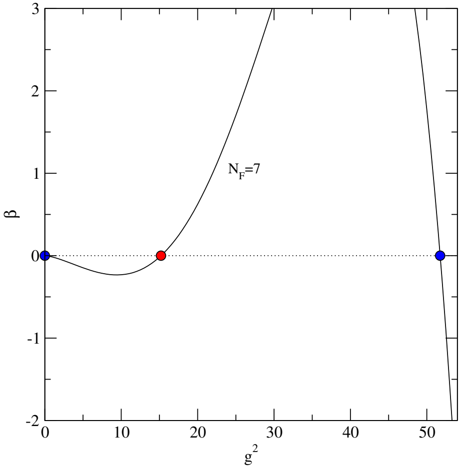

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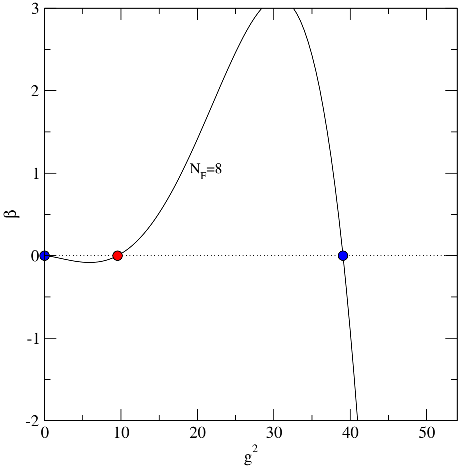

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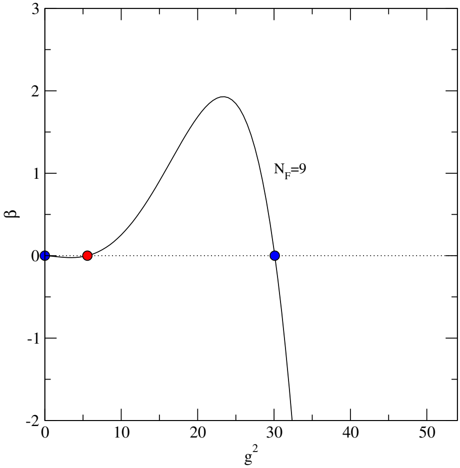

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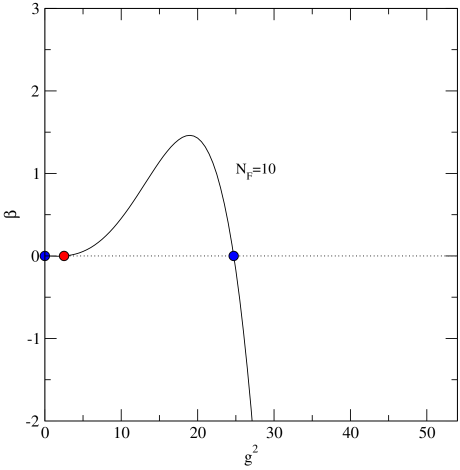

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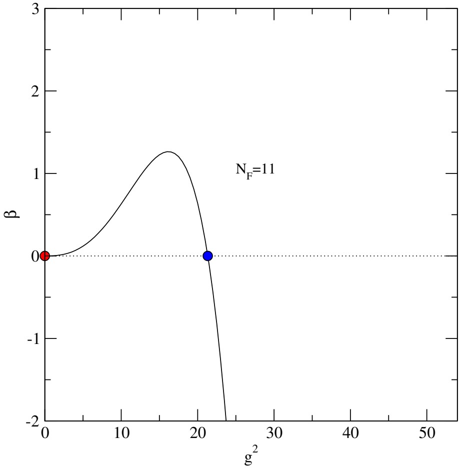

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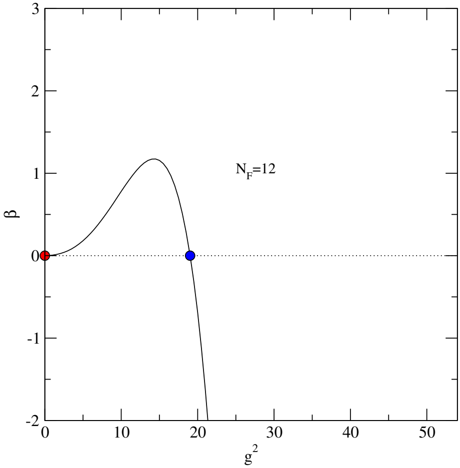

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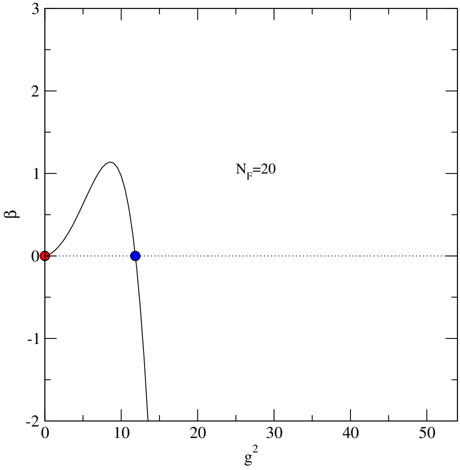

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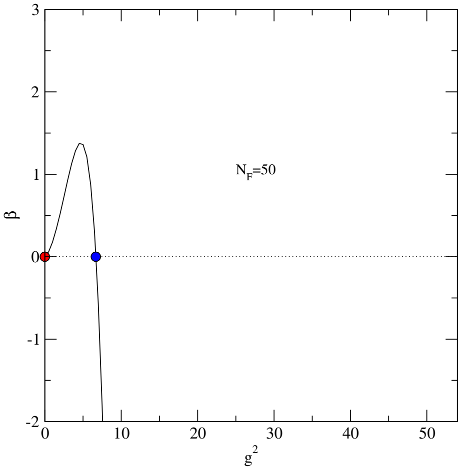
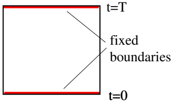


- Note: this does not work at 3-loop or 5-loop level $\rightarrow$ perturbation theory cannot be trusted
- However: if there is a fixed point, we can expect it to move to smaller coupling as N_f grows

How to study the coupling on the lattice?

We use methods successfully used to study conformal window in SU(2) + $N_f = 6$ and 8:

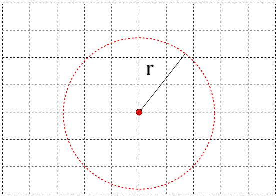
1. Wilson-clover action with HEX smearing
2. Dirichlet boundary conditions in time:
   - Allows $m_{\text{fermion}} = 0$
   - Tuned using axial Ward identity



3. Measure coupling through **gradient flow** [Fritzsch,Ramos]:
   - Cool

$$\partial_t A_\mu = D_\nu F_{\nu\mu}$$

smooths gauge over radius $r \approx \sqrt{8t}$.
We use $\sqrt{8t} = cL$, with $c = 0.3$ (+ other values).



   - Define the gradient flow coupling as

$$g^2_{\mathrm{GF}} = \frac{t^2}{\mathcal{N}} \langle E(t) \rangle$$

where $E = -\frac{1}{4}\langle F_{\mu\nu} F_{\mu\nu} \rangle$.

How to study the system on the lattice?

4 Step scaling function:

$$\Sigma(u, s, L/a) = g^2_{\rm GF}(g_0^2, sL/a)\Big|_{g^2_{\rm GF}(g_0^2, L/a)=u} \tag{1}$$

$$\sigma(u, s) = \lim_{a/L\to 0} \Sigma(u, s, L/a), \tag{2}$$

Step scaling tells us how much the coupling evolves as length scale is increased by a constant factor s.

System size is increased by the same factor: finite volume effects cancel

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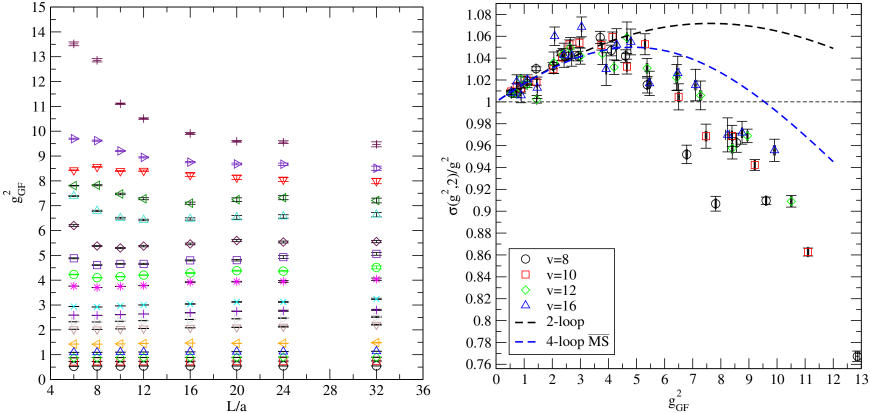
Note: coupling constant definition is not unique on the lattice! (Except near $g = 0$, universality).

The existence of a FP is universal.

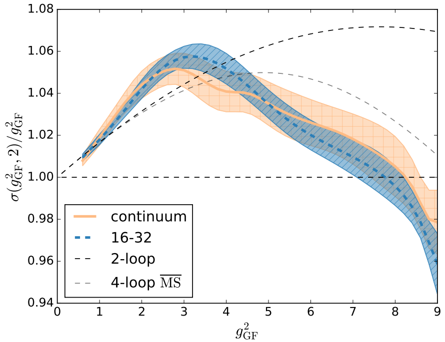
Example: what happens at $N_f = 8$?

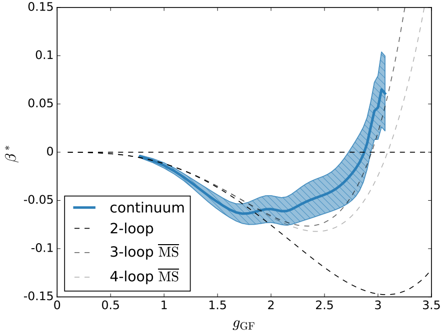
[Leino et al. 17]

Measured gradient flow coupling and step scaling:



$N_f = 8$ Interpolation to continuum



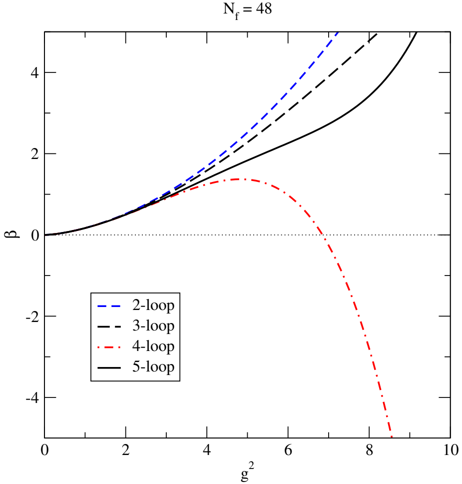


IRFP at $g^2_{\rm GF} \approx 8$

Works well! Let us now try the same method for large N_f

What to expect at large N_f?

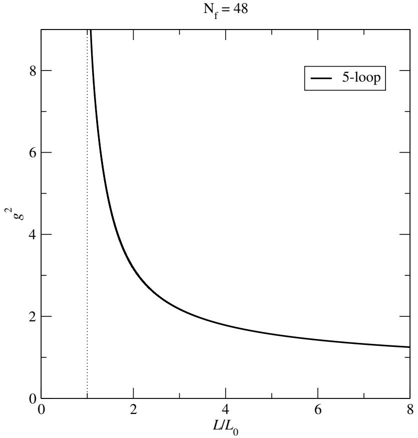
Perturbative $\overline{MS}$ β-function for $N_f = 48$:

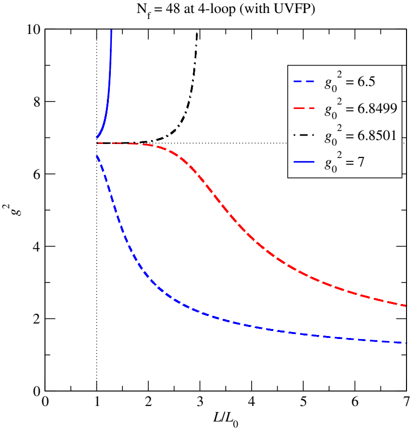


At 4 loops, there appears an UVFP, at 5 loops Landau pole.
We can take these as "toy models" for UVFP and Landau pole

What to expect at large N_f?

Integrate the 5- and 4-loop β-functions to obtain evolution as a function of length scale L:



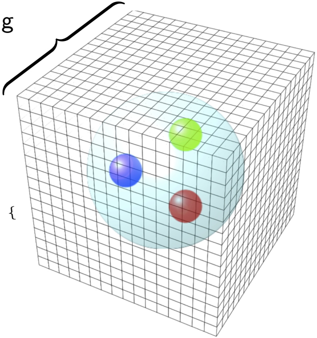


$\Rightarrow$ On the lattice: if there is UVFP, expect dramatic change in behaviour if the short-distance (bare) coupling is large enough.

Qualitative difference vs. lattice QCD:

in QCD, the coupling is large here

. . . but small here



At large N_f situation is opposite: coupling large at short distances, small at large distances
$\rightarrow$ must live with strong bare lattice coupling
$\rightarrow$ HEX smearing; mixed gauge action

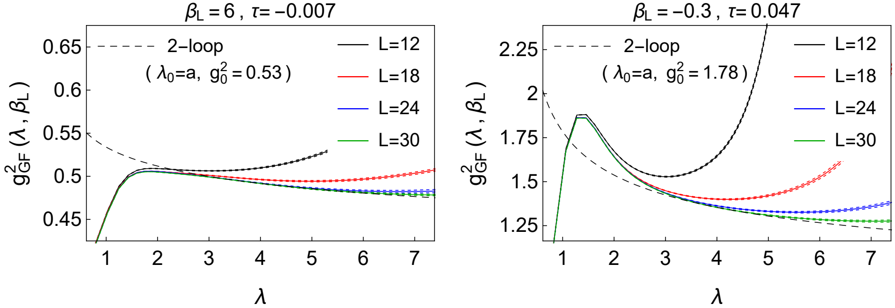
On the lattice gauge action (plaquette action) is parametrized with inverse bare coupling

$$\beta_L = \frac{4}{g_0^2}$$

Large coupling $\rightarrow$ small β_L.
Wilson fermions make the effective coupling weaker [DeGrand,Hasenfratz].
$\rightarrow$ compensate at large N_f by making bare $beta_L$ smaller, even negative!

Gradient flow coupling

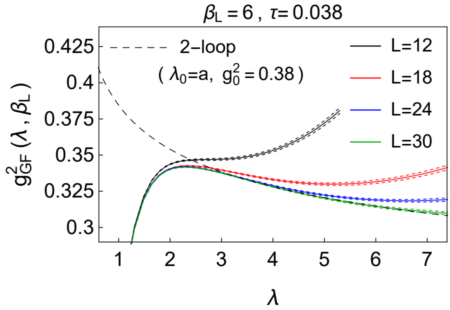
$N_f = 24$ gradient flow coupling vs. distance λ at weak (left) and strong (right) bare lattice coupling.

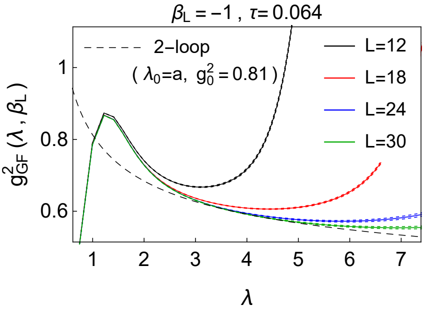


- Lattice volume $V = L^4$
- As $V \to \infty$, $g^2_{\mathrm{GF}} \to g^2_{\mathrm{pert.}}$. Very large finite volume effects at small L
- At small λ gradient flow coupling does not make sense – min distance $\lambda_{\min} \sim 3$.

Gradient flow coupling

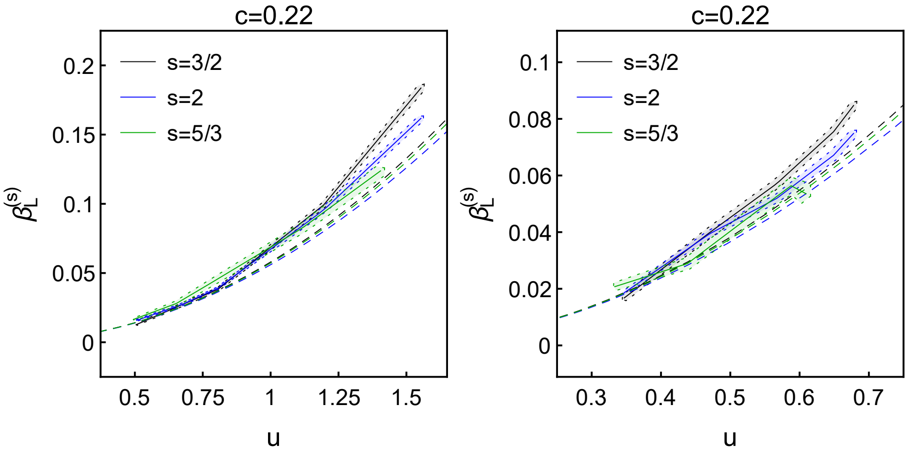
Same at $N_f = 48$:





- Note: measured gradient flow coupling is very small even at strong bare lattice coupling.
- Can be explained by very rapid evolution at small λ: Landau pole?

Discrete beta function



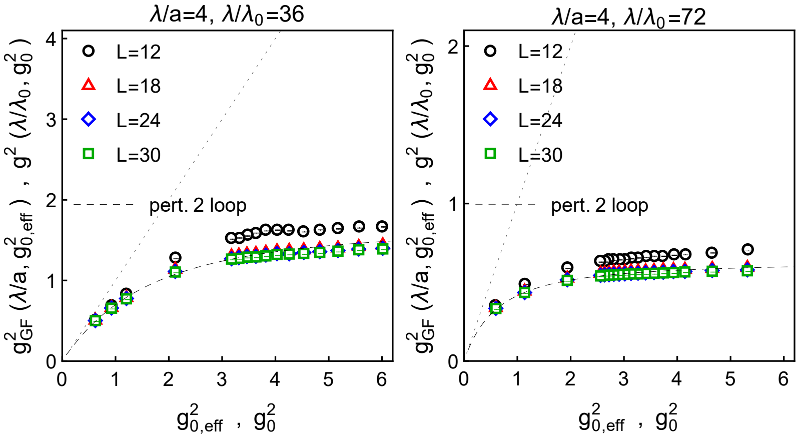
$N_f = 24$ (left) and $N_f = 48$ (right) discrete beta-functions, compared with perturbation theory.

Measured using $c = 0.22L$, and at $s = L_1/L_2 = 18/12, 24/12$ and $30/18$

Effective "bare" lattice coupling

- Using gradient flow g^2_{GF} we cannot measure coupling at very small distance
- We can define an effective UV-scale (1 lattice unit, "plaquette scale") coupling as follows:
  - measure plaquette (the most UV quantity)
  - simulate pure gauge theory, and find bare coupling $g^2_{0,\mathrm{gauge}}$ which gives the same plaquette as the measurement above.
  - define the effective coupling $g^2_{0,\mathrm{eff}} = g^2_{0,gauge}$.

Effective "bare" lattice coupling



- x-axis: effective UV coupling; y-axis: GF coupling at length scale $\lambda = 4a$.
- Matches 2-loop beta-function very well, despite different scheme
- *Flattening out: consistent with Landau pole*

Conclusions

- In these initial studies, behaviour compatible with a Landau pole both at $N_f = 24$ and 48 $\rightarrow$ standard picture.
- Nevertheless:
  - We cannot "prove" the absence of the UV fixed point. (It would be easier to demonstrate its existence.)
  - Coupling strong at short distances, weak at large distances: this is not an application to which lattice methods have been developed and tuned!
  - Ambiguities in defining the coupling at very small (in lattice units) distances, but the effective plaquette coupling seems to work
  - Larger lattice scale (short distance) effective couplings required
- Further development: optimize the lattice action and measurements?
- Experiment with other theories, for example with added scalars.