Searching for asymptotic safety in large-N_f gauge theory

Viljami Leino, Tobias Rindlisbacher, Kari Rummukainen, Francesco Sannino, Kimmo Tuominen

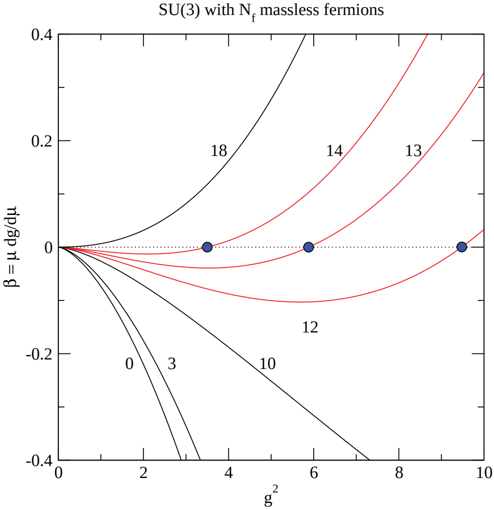
University of Helsinki and Helsinki Institute of Physics
CP3-Origins and DIAS, University of Southern Denmark
Technische Universität München

The standard picture:

Consider *2-loop perturbative* β-function of SU(N) + N_f fermions:

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

- Small N_f: $\beta_0 > 0$, $\beta_1 > 0$
  running coupling, confinement and χSB (QCD-like)
- Medium N_f: $\beta_0 > 0$, $\beta_1 < 0$
  IR fixed point, no χSB [Banks,Zaks]:
  conformal window
- Large N_f: $\beta_0 < 0$
  Asymptotic freedom lost
  - $\rightarrow$ Landau pole?
  - $\rightarrow$ Asymptotic safety?



What really happens at large N_f?

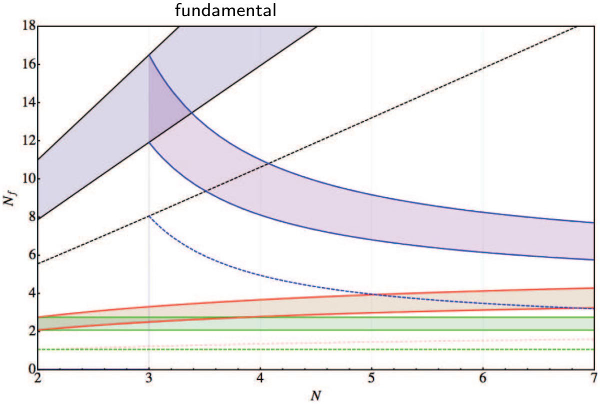
Consider SU(N) gauge with N_f (fundamental) fermions:

- Standard lore: as the asymptotic freedom is lost, theory has a Landau pole.
- However: $N_f \to \infty$ calculations suggest that there appears an UVFP (Asymptotic safety) [Antipin,Sannino 17], see also [Gracey 96]

In this talk: first attempts to study the behaviour on the lattice

- SU(2) gauge with $N_f = 24$ and 48 at $m_{\rm fermion} = 0$
- Measure the evolution of the coupling constant
- Use similar methods as used earlier within the conformal window

Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

2-index symmetric

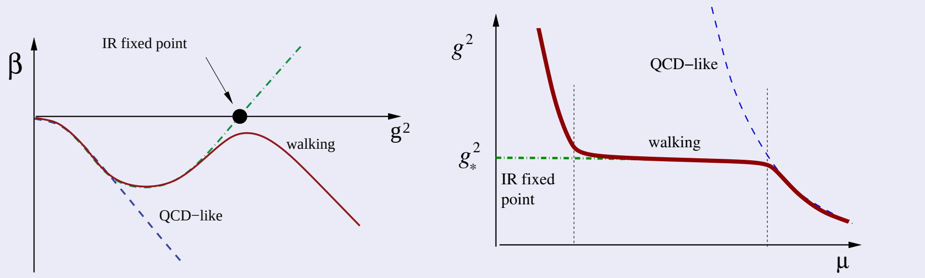
adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- *Building BSM models using higher reps: easier to satisfy EW constraints*
  [Sannino,Tuominen,Dietrich] $\rightarrow$ recent interest

Walking coupling

- Just below the conformal window β-function *may* get close to zero at finite coupling

$\Rightarrow$ The coupling evolves slowly, *walks*.



- Building blocks for strongly coupled BSM scenarios
- CP3-Origins very active!

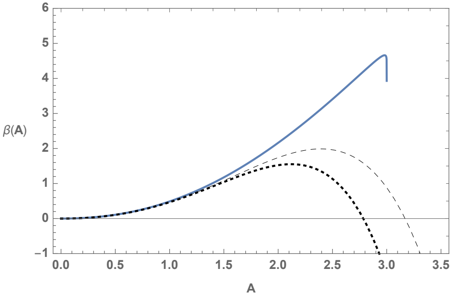
Large N_f:

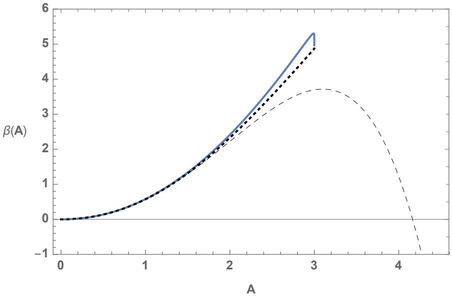
Let $A = N_f \frac{\alpha}{2\pi}$ (for fundamental fermions).
At large N_f:

$$\frac{3}{2}\frac{\beta(A)}{A} = 1 + \frac{H_1(A)}{N_f} + \frac{H_2(A)}{N_f^2} + \ldots$$

$H_1(A)$ has a logarithmic singularity at $A = 3$
$\Rightarrow$ β-function vanishes, UVFP.
[Antipin, Sannino 17; Gracey 95; Litim, Sannino 14]

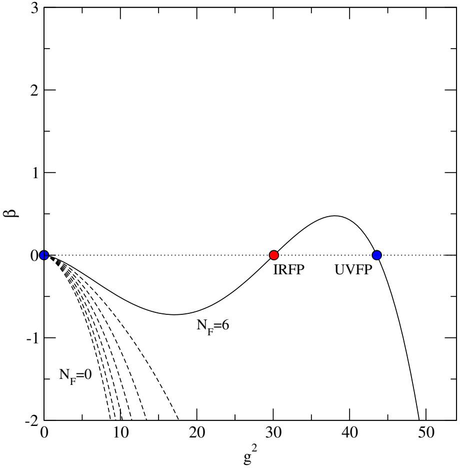
SU(2) with $N_f = 24$ (top) and 48 (bottom):
Large-N_f result compared with 2-loop and 5-loop $\overline{MS}$.





Cartoon of β-function

Consider 4-loop β-function for SU(2)+N_F fermions:



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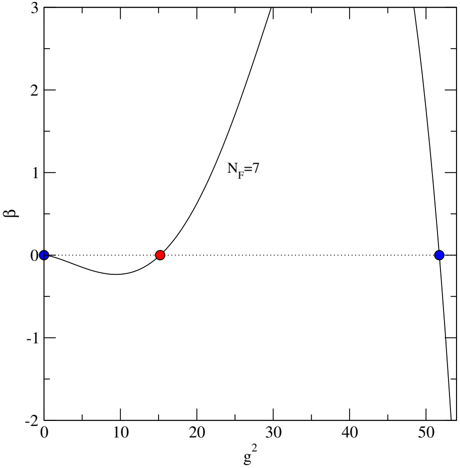

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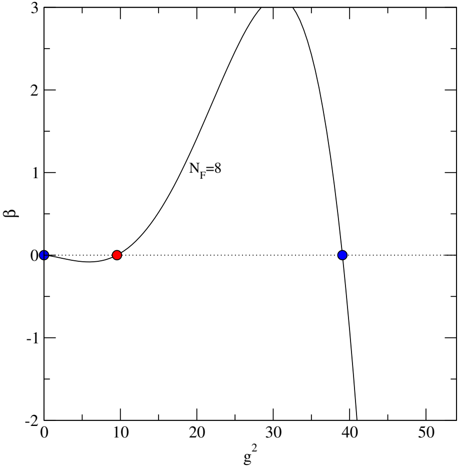

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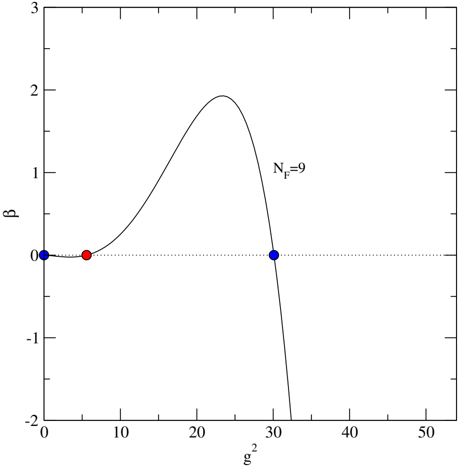

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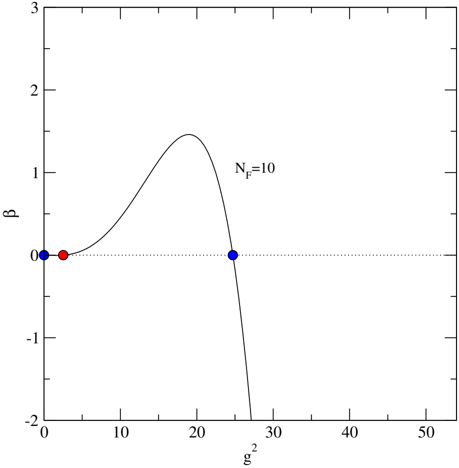

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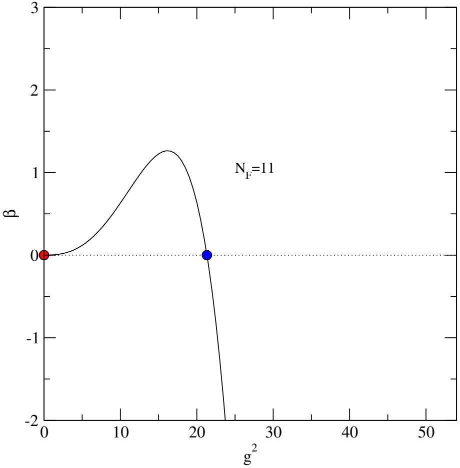

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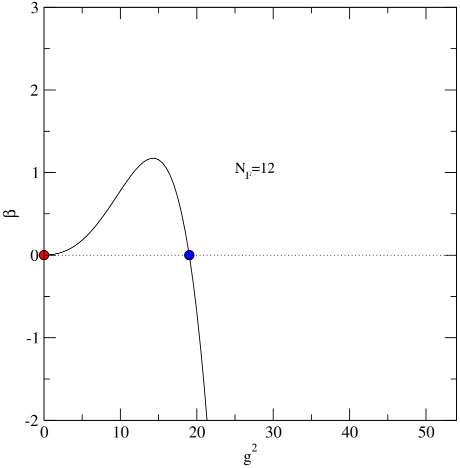

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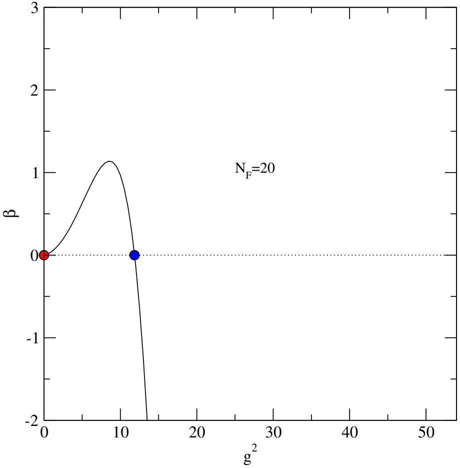

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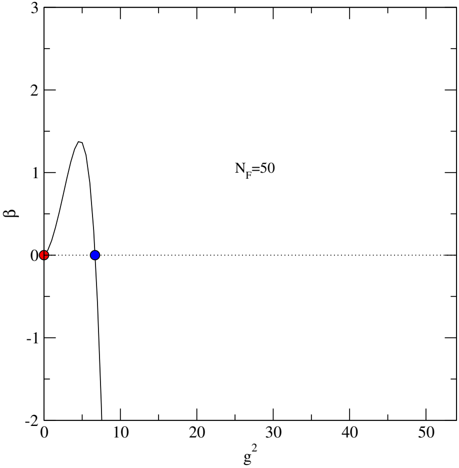
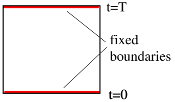


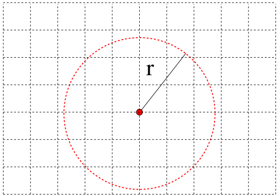
How to study the system on the lattice?

We use methods successfully used to study conformal window in $SU(2) + N_f = 6$ and 8:

1. Wilson-clover action with HEX smearing
2. Dirichlet boundary conditions in time:
   - Allows $m_{\rm fermion} = 0$
   - Tuned using axial Ward identity



3. Measure coupling through **gradient flow** [Fritzsch,Ramos]:
   - Cool
   $$\partial_t A_\mu = D_\nu F_{\nu\mu}$$
   smooths gauge over radius $r \approx \sqrt{8t}$.
   We use $\sqrt{8t} = cL$, with $c = 0.3$ (+ other values).
   - Define the gradient flow coupling as
   $$g^2_{\rm GF} = \frac{t^2}{\mathcal{N}} \langle E(t) \rangle$$
   where $E = -\frac{1}{4}\langle F_{\mu\nu} F_{\mu\nu} \rangle$.



How to study the system on the lattice?

4 Step scaling function:

$$\Sigma(u, s, L/a) = g^2_{\rm GF}(g_0^2, sL/a)\Big|_{g^2_{\rm GF}(g_0^2, L/a)=u} \tag{1}$$

$$\sigma(u, s) = \lim_{a/L\to 0} \Sigma(u, s, L/a), \tag{2}$$

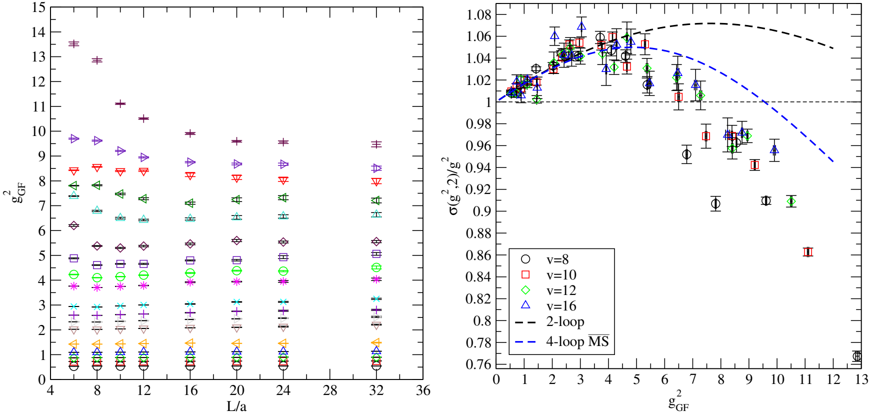
Step scaling tells us how much the coupling evolves as length scale is increased by a constant factor s.

System size is increased by the same factor: finite volume effects cancel

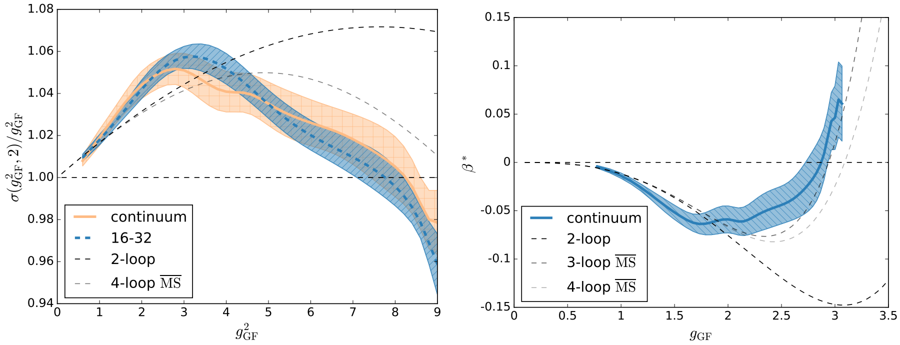
Example: what happens at $N_f = 8$?

[Leino et al. 17]

Measured gradient flow coupling and step scaling:



$N_f = 8$ Interpolation to continuum

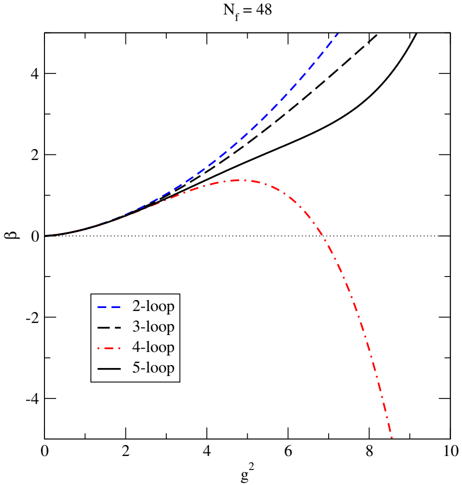


IRFP at $g^2_{\mathrm{GF}} \approx 8$

Works well! Let us now try the same method for large N_f

What to expect at large N_f?

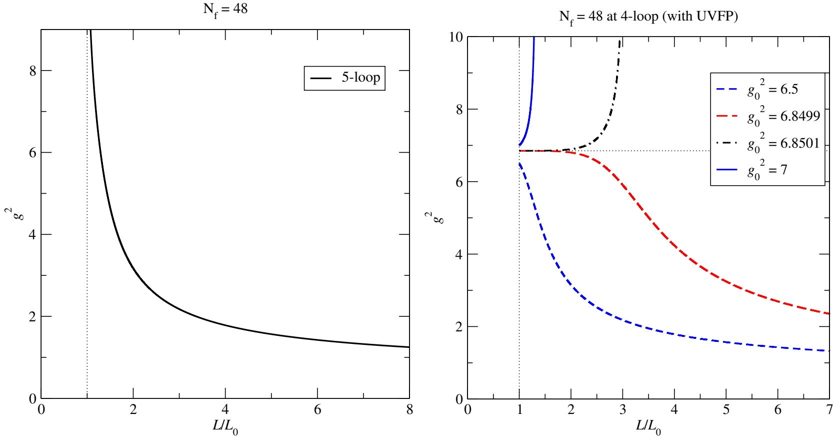
Perturbative $\overline{MS}$ β-function for $N_f = 48$:



At 4 loops, there appears an UVFP, at 5 loops Landau pole.
We can take these as "toy models" for UVFP and Landau pole

What to expect at large N_f?

Integrate the 5- and 4-loop β-functions to obtain evolution as a function of length scale L:

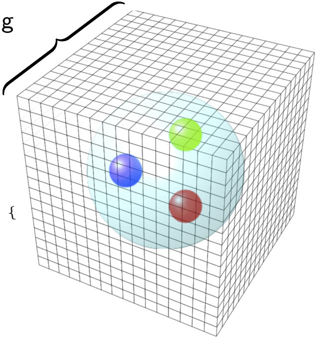


$\Rightarrow$ On the lattice: if there is UVFP, expect dramatic change in behaviour if the short-distance (bare) coupling is large enough.

Big difference vs. lattice QCD:

in QCD, the coupling is large here

. . . but small here



At large N_f situation is opposite: coupling large at short distances, small at large distances
→ must live with strong bare lattice coupling
→ HEX smearing; mixed gauge action

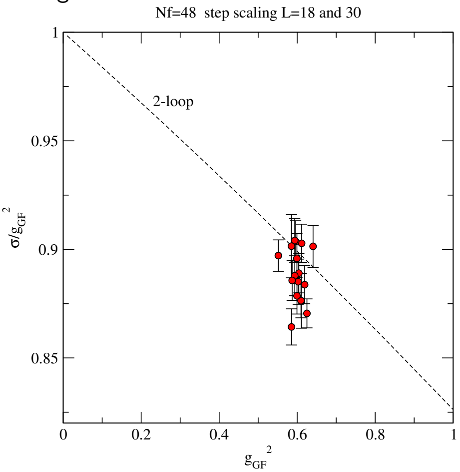
On the lattice gauge action (plaquette action) is parametrized with inverse bare coupling

$$\beta_L = \frac{4}{g_0^2}$$

Large coupling → small β_L.
Wilson fermions make the effective coupling weaker [DeGrand,Hasenfratz].
→ compensate at large N_f by making bare $beta_L$ smaller, even negative!

Step scaling at $N_f = 48$

$N_f = 48$ $c = 0.3$ step scaling



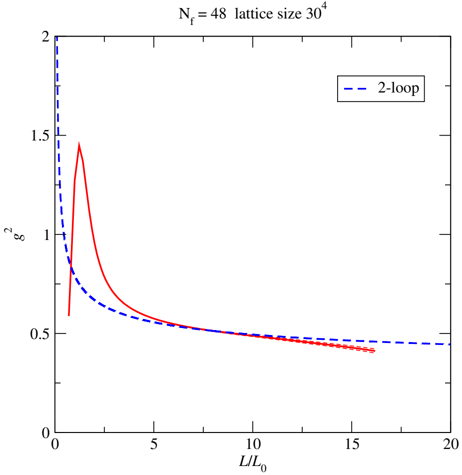
Dramatic difference to $N_f = 8$! All $\beta_L = -1 \ldots +1$ points cluster together at very weak coupling

Step scaling at $N_f = 48$

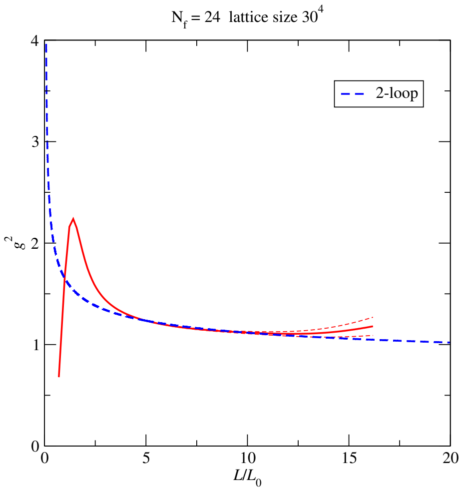
Looking at the gradient flow coupling as a function of the "smoothing radius" $L = \sqrt{8t}$ we get the idea of what is going on:

- at small L the lattice artifacts (irrelevant lattice operators) have not yet been removed by the gradient flow
- at large L $g^2_{\rm GF}$ becomes small rapidly
- can match with perturbation theory there
- $g^2_{\rm GF}$ in practice independent of the bare gauge coupling (β_L)

$\Rightarrow$ Looks like a Landau pole, but we cannot be sure because the effective coupling is small



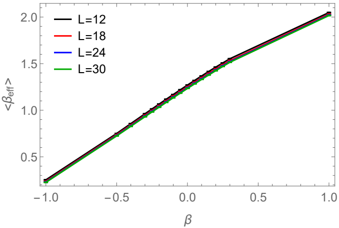
Step scaling at $N_f = 24$

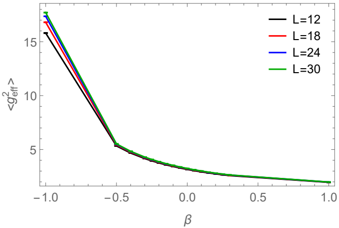


At $N_f = 24$ the situation is similar

Effective "bare" lattice coupling

We can also obtain a measure of the effective lattice $\beta_{L,\mathrm{eff}} = 4/g^2_{0,\mathrm{eff}}$ by measuring the plaquette and "inverting" its value to pure gauge β_L, i.e. comparing to pure gauge simulation which gives the same plaquette:





- At $\beta_L = -1$ the effective bare coupling is $g^2_{0,\mathrm{eff}} \sim 15$
- Location of the UVFP in $N_f \to \infty$ calculation: $g^2_{\mathrm{FP}} \approx \frac{24\pi^2}{N_f} \approx 4.9$

$\Rightarrow$ in the right ballpark (but numbers are not comparable, different scheme)

Conclusions

- In these initial studies, behaviour compatible with a Landau pole both at $N_f = 24$ and 48.
- However:
  - Coupling strong at short distances, weak at large distances: this is not an application to which lattice methods have been developed and tuned!
  - coupling cannot be reliably defined at very small (in lattice units) distances
  - Larger lattice scale (short distance) effective couplings required
- UVFP cannot be excluded
- Further development: optimize the lattice action and measurements?