Walking through the conformal window: SU(2) with 2–8 fermions

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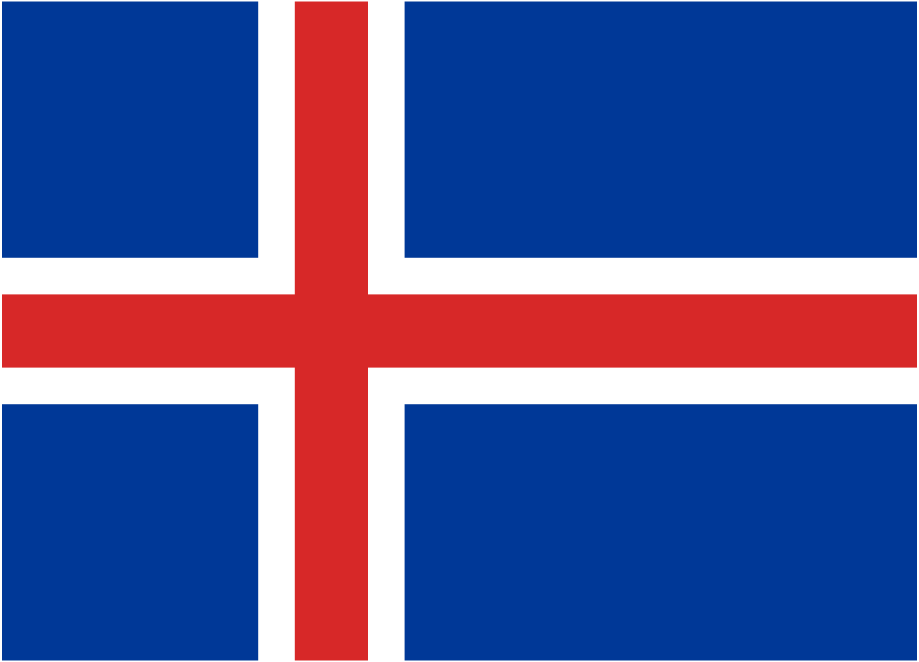
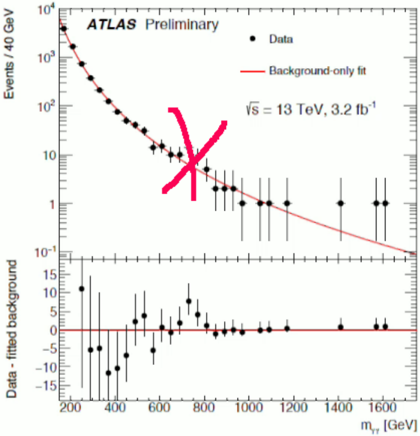


The “Bump”

- Bad(?) news(?): the “bump” has ceased to be
  . . . or is it just resting?
- **Bumpxit?**



Introduction: Conformal Window

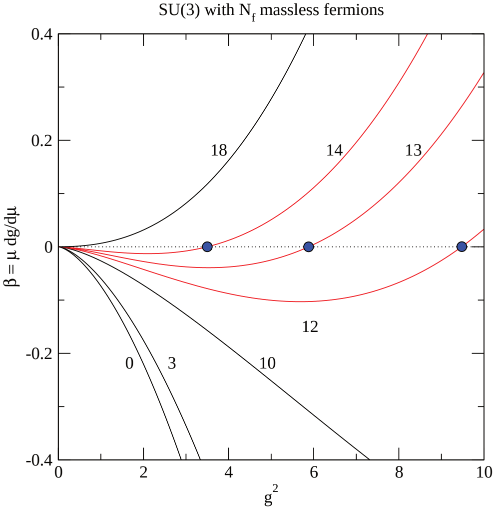
Consider *2-loop perturbative* β-function

$$\beta(g) = \mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

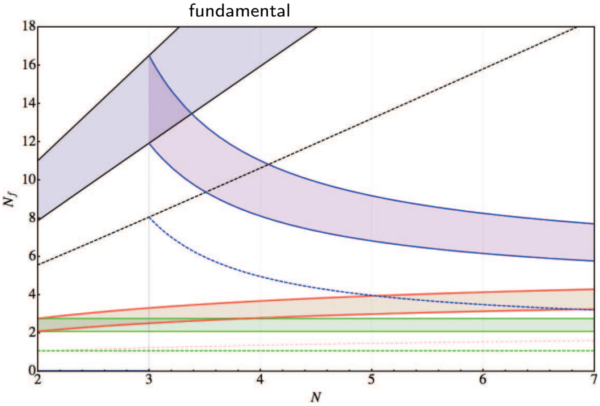
Generically 3 different behaviours:

- Small N_f: $\beta_0 > 0$, $\beta_1 > 0$ running coupling, confinement and χSB (QCD-like)
- Medium N_f: $\beta_0 > 0$, $\beta_1 < 0$ IR fixed point, no χSB [Banks,Zaks]
- Large N_f: $\beta_0 < 0$ Asymptotic freedom lost

Conformal window: range of N_f where IRFP exists



Conformal window in SU(N) gauge



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

2-index symmetric

adjoint

- Upper edge of band: asymptotic freedom lost
- Lower edge of band: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- *In higher reps it is easier to satisfy EW constraints [Sannino,Tuominen,Dietrich]* → recent interest

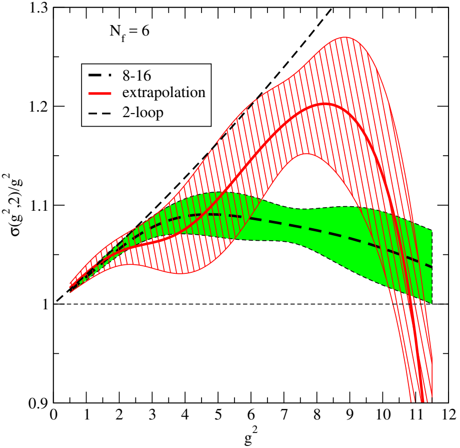
SU(2) + N_f fundamental fermions

- Here: SU(2) gauge + $N_f = 2, 4, 6, 8$ fundamental fermions
- $N_f = 11$ asymptotic freedom lost
- What we expect:
  - $N_f = 2$ "QCD-like" , $N_f = 4$ a bit less so (χSB)
  - $N_f = 8$ within conformal window
  - $N_f = 6$ borderline, possibly within CW
- Previous studies at $N_f = 6$ inconclusive: [Karavirta et al. 2011] ($N_f = 2, 6, 10$), [Appelquist et al. 2014]

We use

- Gradient flow finite volume step scaling [Fritsch and Ramos]: measure the evolution of the coupling and γ (at $N_f = 6, 8$)
- Spectrum at $N_f = 2 \ldots 8$
- Use HEX-smeared Wilson-clover action w. mixed smeared/non-smeared gauge action.

$N_f = 6$ step scaling by Karavirta et al. 2011



- Schrödinger functional, background field method

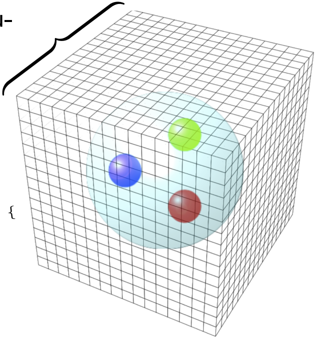
⇒ Noisy, prevents large lattices

- Non-pert improved Wilson-clover action
- Now: HEX smeared W-c action, GF step scaling

QCD vs. (almost) conformal – why simulations are difficult?

in QCD, the coupling is large here

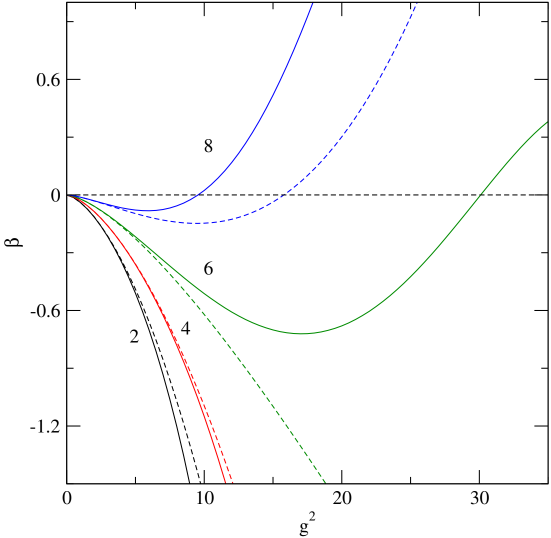
. . . but small here



In (almost) conformal theories, the coupling is ∼ equal everywhere!
→ must live with strong lattice coupling
→ HEX smearing; mixed gauge action

Perturbative β-function

$\overline{\text{MS}}$ β-function [Larin,Vermaseren] at $N_f = 2, 4, 6, 8$ at 2 (dashed) and 4 (solid) loops:



Evolution of the coupling at $N_f = 8$

- Fixed, trivial "Schrödinger functional" boundaries (no background field)
- Tune to vanishing fermion mass using axial Ward identity (on 24^4)
- Run Wilson flow time t to scale [Fritsch, Ramos]

$$\mu^{-1} = cL = \sqrt{8t}.$$

  We use $c = 0.4$ (+ other values).
- Define

$$g^2_{\mathrm{GF}} = \frac{t^2}{\mathcal{N}} \langle E(t + \tau_0 a^2) \rangle$$

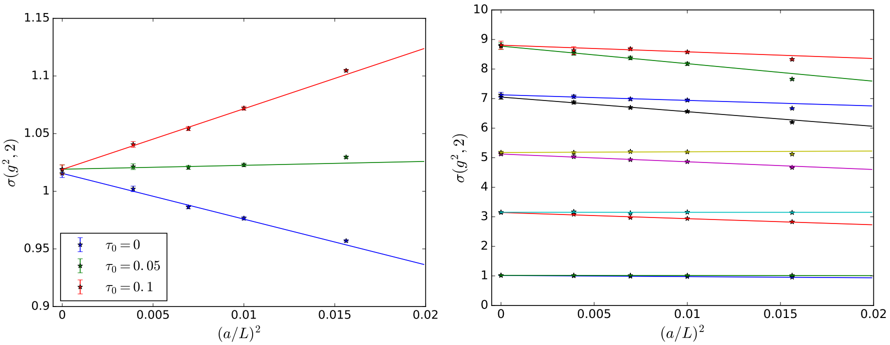
  where τ_0 is a tunable correction [Cheng et al.]
- Step scaling function ($s = 2$):

$$\Sigma(u, s, L/a) = g^2_{\mathrm{GF}}(g_0^2, sL/a)\big|_{g^2_{\mathrm{GF}}(g_0^2, L/a) = u} \quad (1)$$

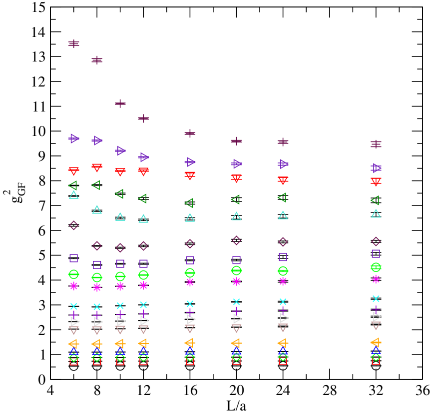
$$\sigma(u, s) = \lim_{a/L \to 0} \Sigma(u, s, L/a), \quad (2)$$

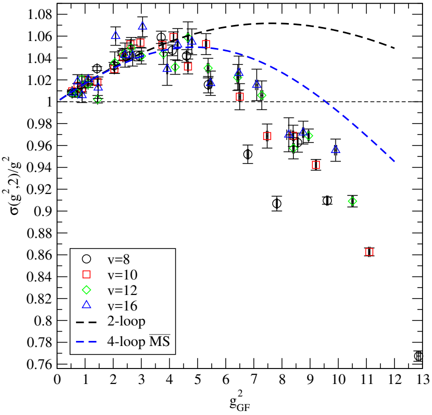
- Use rational interpolation for $g_{\mathrm{GF}}(g_0^2, L/a)$

τ_0 correction

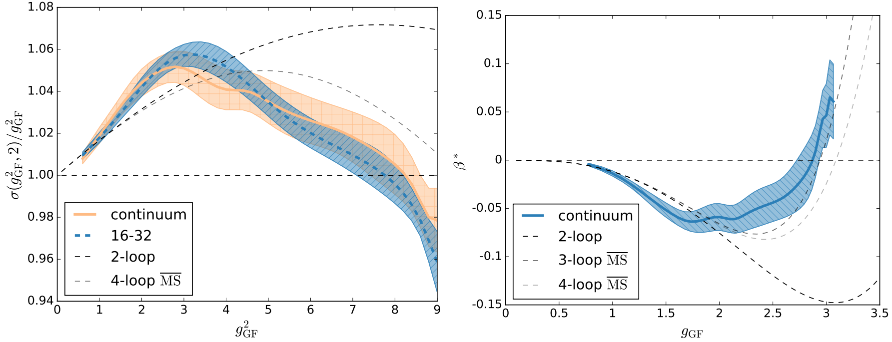


$N_f = 8$ raw coupling & step scaling



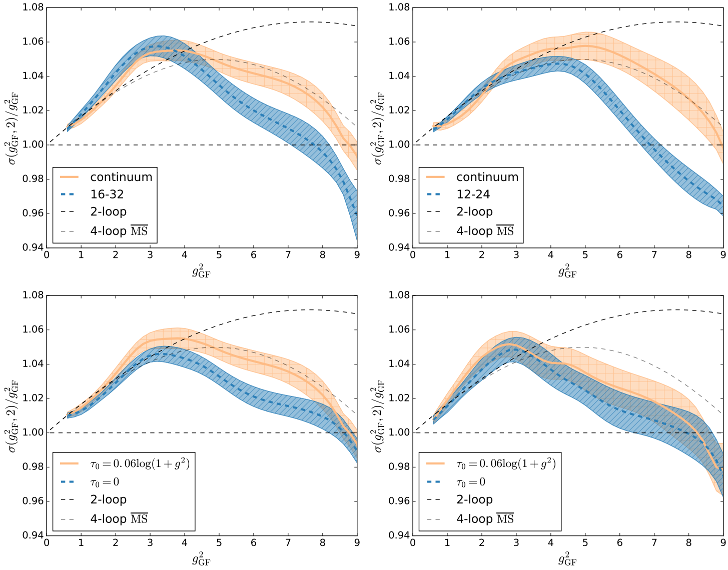


$N_f = 8$ Interpolation to continuum



IRFP at $g_{\rm GF} \approx 8$

$N_f = 8$ sensitivity to parameter choices



$N_f = 8$ mass anomalous exponent γ

Use 2 methods to determine γ:

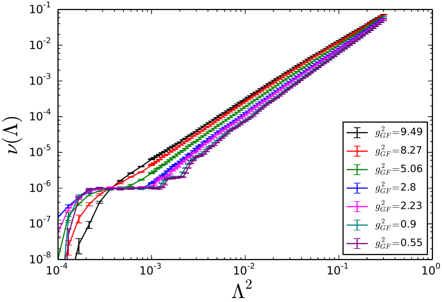
- Mass step scaling (Ward identities) [Luscher,Weisz]
- Dirac mode number density [Patella]

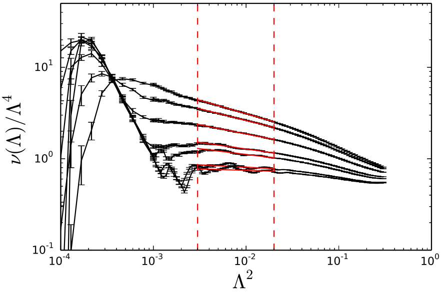
Both using the same configs than used for the coupling

$N_f = 8$ mode number density

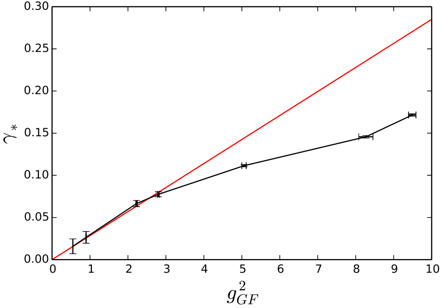
Slope of the mode number density determines the exponent γ:

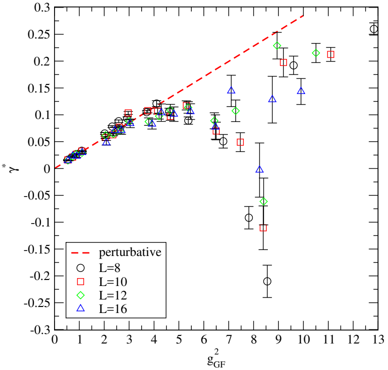
$$\nu(\Lambda) \propto \Lambda^{4/(1+\gamma)}$$





$N_f = 8$ anomalous exponent γ

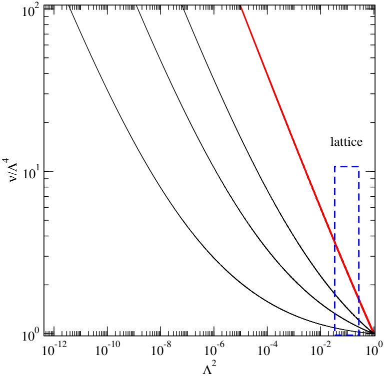




- mode number much more stable than mass step scaling
- At IRFP $g_{\rm GF} \approx 8 \Rightarrow \gamma^* \approx 0.15$ (preliminary)
- NOTE: we need to know the location of the IRFP in order to determine γ^*

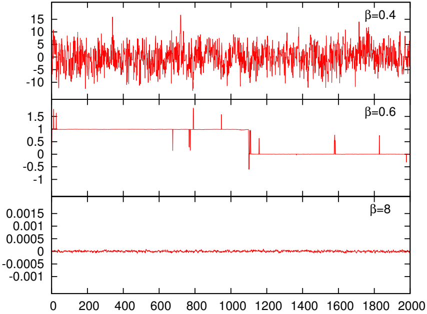
Why lattice has difficult time seeing universal γ^*?

- Evolution is slow, and lattice has finite range of scales.
- To illustrate: take perturbative $\beta(g^2)$ and $\gamma(g^2)$, and integrate ν:



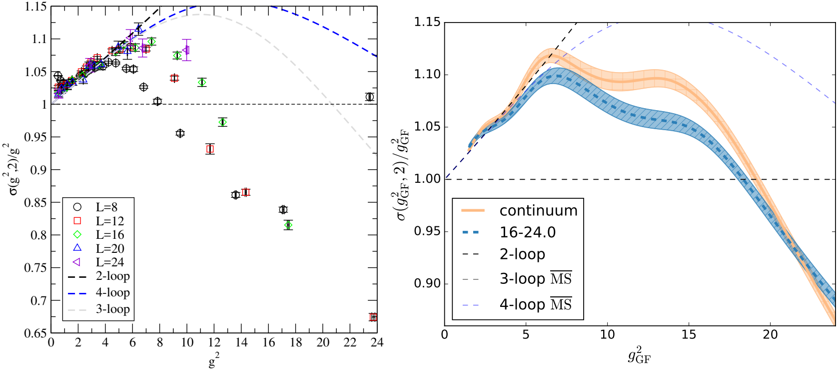
- To reach universal behaviour "early" we should choose parameters so that we're alreay close to the IRFP.

$N_f = 8$ topology (still on "SF" lattices)



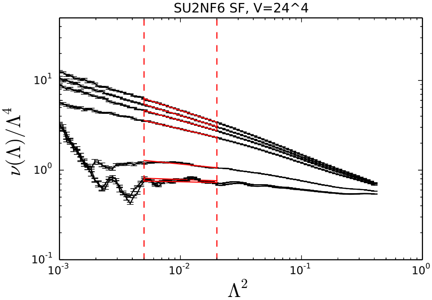
Topology frozen at small (bare) coupling, becomes "liberated" at strongest couplings – threshold effects?

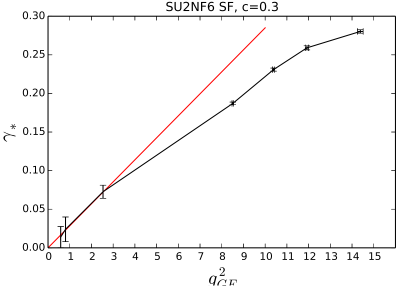
$N_f = 6$ step scaling (PRELIMINARY)



Step scaling with $s = 3/2$, $c = 0.3$, $\tau_0 = 0.05$

$N_f = 6$ γ (PRELIMINARY)

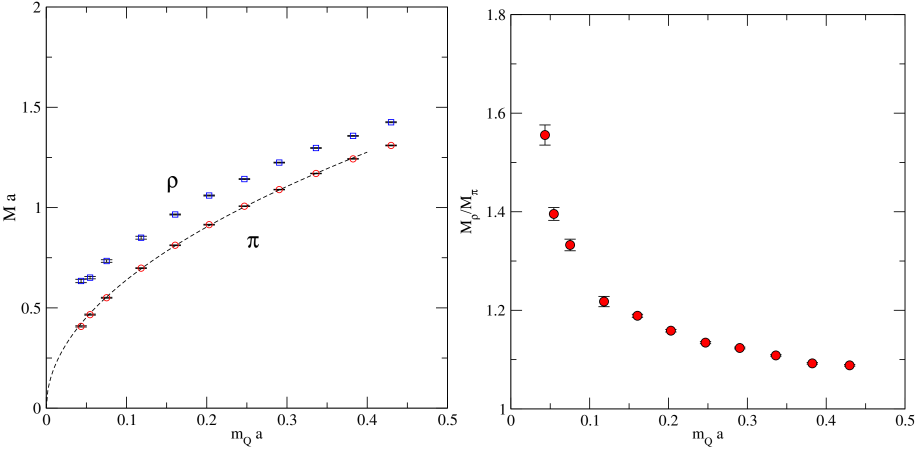




Mass spectra

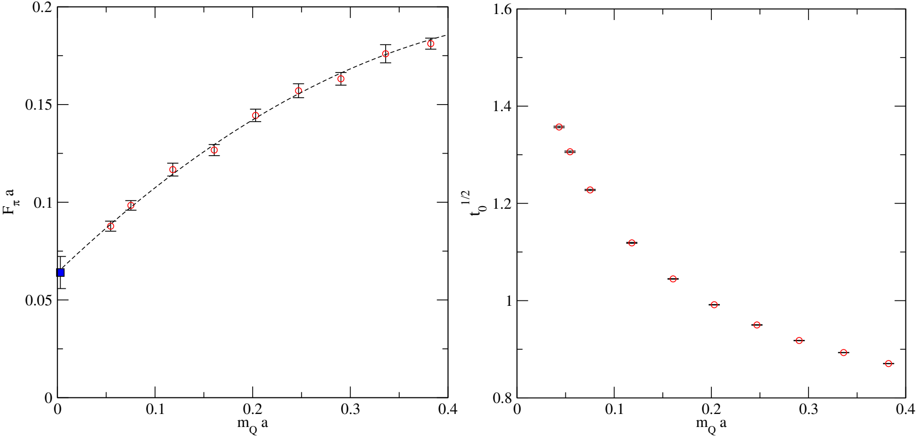
$N_f = 2$ masses

$24^3 \times 48$, $\beta_G = 1.0$

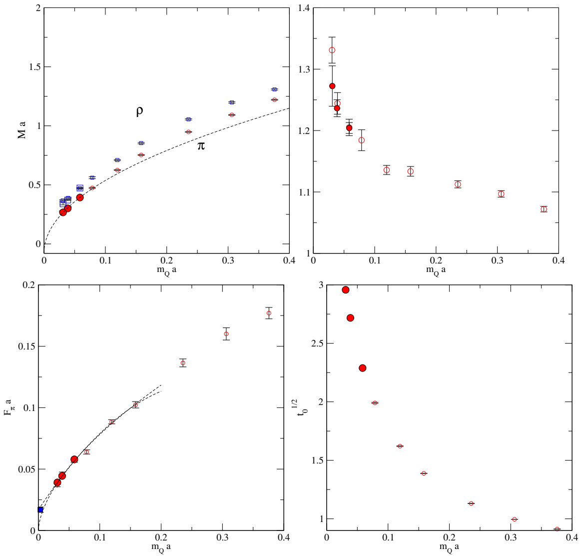


$N_f = 2$ masses

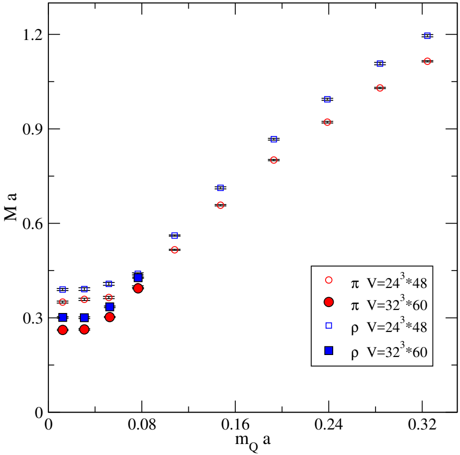
$24^3 \times 48$, $\beta_G = 1.0$



$N_f = 4$ $\beta_G = 0.8$ $24^3 \times 48$ and $32^3 \times 60$

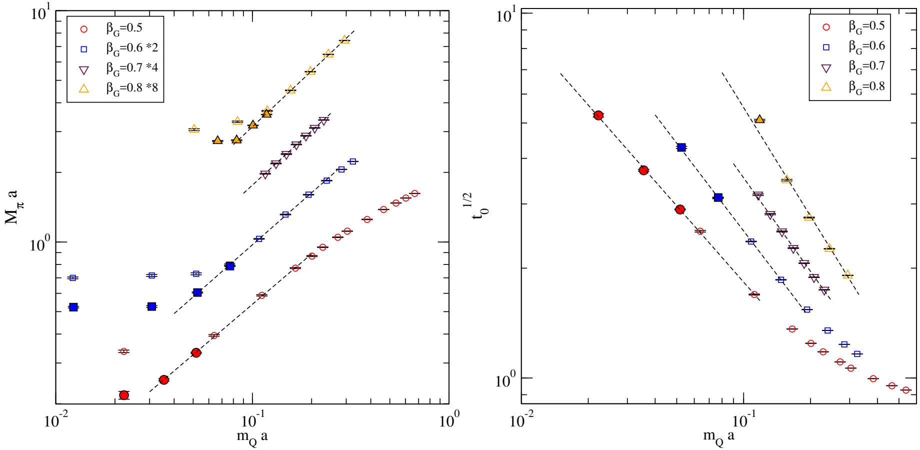


$N_f = 6$ masses



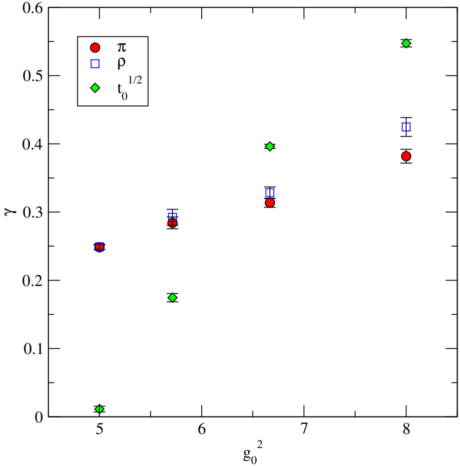
1.2
0.9
0.6
0.3
0
M a
0
0.08
0.16
0.24
0.32
m_Q a
π V=24^3*48
π V=32^3*60
ρ V=24^3*48
ρ V=32^3*60

$N_f = 6$ masses

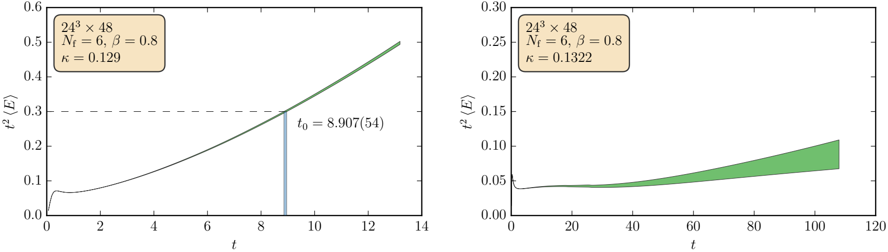

βG=0.5
βG=0.6 *2
βG=0.7 *4
βG=0.8 *8
Mπ a
mQ a
10^-2
10^-1
10^0
10^1
βG=0.5
βG=0.6
βG=0.7
βG=0.8
t0^1/2
mQ a

$N_f = 6$ γ from spectrum?

Universality at IRFP: $M \propto m_Q^{1/(1+\gamma)}$, $\sqrt{t_0} \propto m_Q^{-1/(1+\gamma)}$

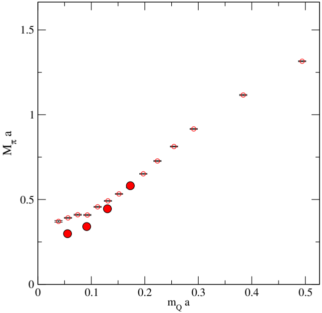


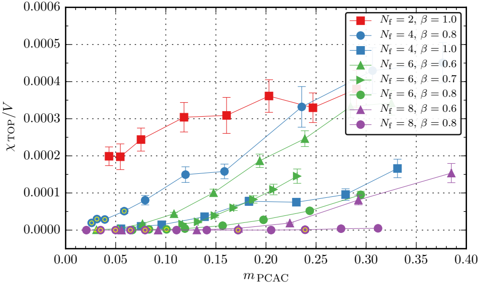
$N_f = 6$ gradient flow



The flow runs out of the lattice at small m_Q (at $N_f = 6, 8$)

$N_f = 8$ spectrum?





- $N_f = 8$ spectrum shows very strong finite size effects already at moderate $m_Q a$
- Topology completely freezes at moderate m_Q (other N_f still OK)
- Mass measurements unreliable

Conclusions

- Iceland wins Euro cup
- Results consistent with expected behaviour: $N_f \leq 4$ χSB; $N_f \geq 6$ IRFP
- Finite volume GF step scaling works at strong coupling
- With IRFP, relying on the universality of γ^* may be asking too much from the lattice: not enough range

$\Rightarrow$ result may depend on simulation parameters