THE UNIVERSALITY CLASS OF THE ELECTROWEAK THEORY

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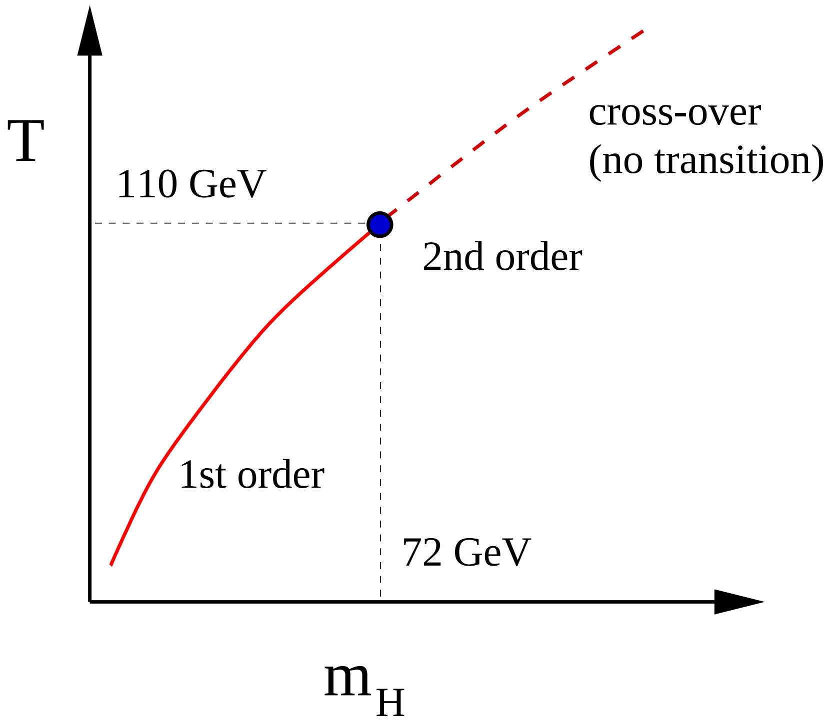
- The 1st order electroweak phase transition turns into a regular cross-over at $m_H \sim 72\,\mathrm{GeV}$



- The universality class at the endpoint?

Effective action: SU(2) gauge + Higgs in 3D:

$$L_3 = \frac{1}{4} F^a_{ij} F^a_{ij} + (D_i\phi)^\dagger (D_i\phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2$$

- Dimensions: $[\phi] = \text{GeV}^{1/2}$, $[g_3^2] = [\lambda_3] = \text{GeV}$
- Theory is fixed by

$$g_3^2 \qquad x \equiv \frac{\lambda_3}{g_3^2} \qquad y \equiv \frac{m_3^2(g_3^2)}{g_3^4}$$

On the lattice:

- $\Phi = a_0 \mathbf{1} + i a_i \sigma_i$, $\Phi^2 \equiv \frac{1}{2} \text{Tr}\, \Phi^\dagger \Phi$

$$\begin{aligned} S &= \beta_G \sum_\Box (1 - \tfrac{1}{2} \text{Tr}\, U_\Box) - \beta_H \sum_{x,i} \tfrac{1}{2} \text{Tr}\, \phi_x^\dagger U_{x,i} \phi_{x+i} \\ &\quad + \sum_x \Phi^2 + \beta_R \sum_x (\Phi^2 - 1)^2 \\ &= S_{\text{Gauge}} + S_{\text{Hopping}} + S_{\phi^2} + S_{(\phi^2-1)^2} \end{aligned}$$

- $\beta_G = 4/a g_3^2$, where a is the lattice spacing

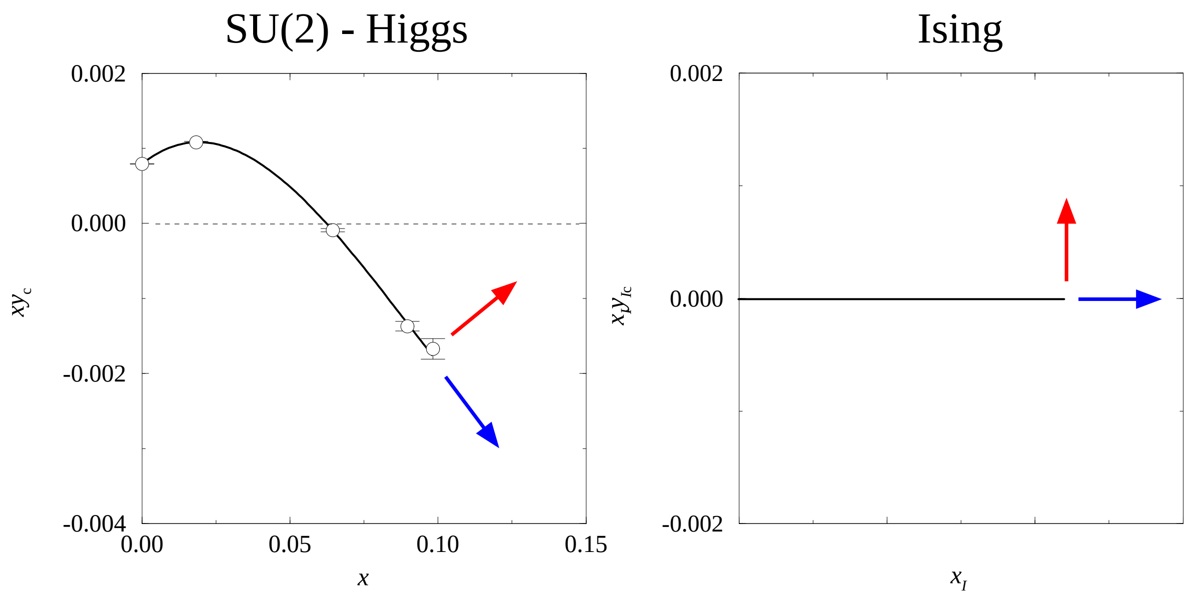
What kind of universal behaviour we can expect?

- Only 1 light exitation near the endpoint: *scalar* $|\Phi|$
- Φ has $SU(2)_{gauge} \otimes SU(2)_{isospin}$ symmetry: *unbroken.*

$\mapsto$ *scalar-type universality*: ϕ^4/Ising, mean field, multicritical, ?

In order to study the universality quantitatively, we:

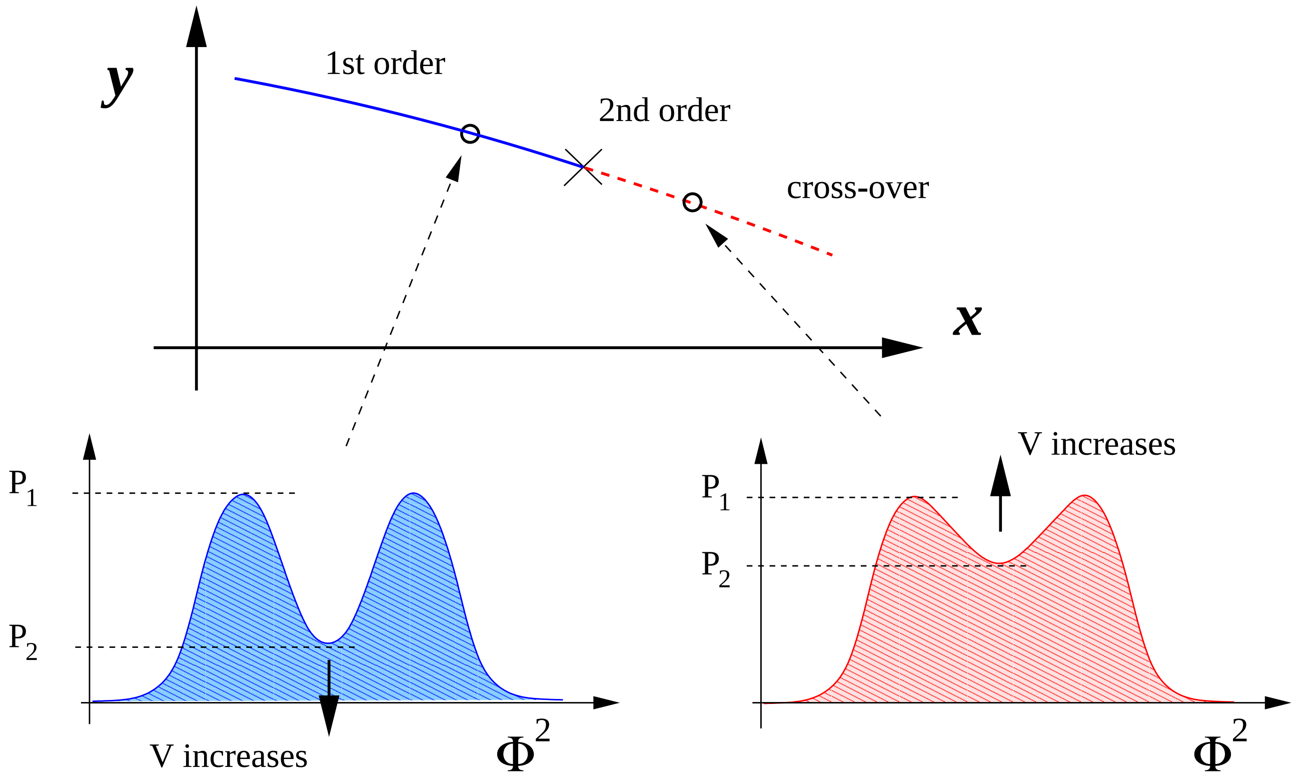
1. Locate the $V \to \infty$ critical point $(\beta_H, \beta_R)_{crit.}$ at a fixed lattice spacing (fixed β_G).
2. Determine the critical observables (M-like and E-like directions).



3. Finite-size scaling (FSS) analysis $\longrightarrow$ critical indices.
4. Higher moments: skewness of E.

Locating the critical point:

- Intersection of Binder cumulants – works, but cumbersome: 2-parameter search (x, y).
- We use the following method:
  - For any $x \sim x_{\text{crit.}}$, find *equal weight* $y_{\text{eq}}(x)$
  - Find x_R (and $y_{\text{eq}}(x_R)$) so that the max/min probability ratio $P_1/P_2 = R$, a fixed value
  - When $V \to \infty$, $(x_R, y_{\text{eq}}(x_R)) \to (x, y)_{\text{crit.}}$



- Ising model at the critical point: $R \approx 2.17$

Analysis at the critical point:

1. Use a large number of operators O_i

   We use up to 6 ops:

$$\begin{aligned} O_1 &= S_{\text{Gauge}} \\ O_2 &= S_{\text{Hopping}} \\ O_3 &= S_{\phi^2} \\ O_4 &= S_{(\phi^2-1)^2} \\ O_5 &= \sum_x |\Phi_x| \\ O_6 &= \sum_{x,i} \frac{1}{2}\mathrm{Tr}\, V_x^\dagger U_{x,i} V_{x+i} \\ & \quad V = \Phi/|\Phi| \end{aligned}$$

2. Calculate $M_{ij} = \langle (O_i - \langle O_i \rangle)\, (O_j - \langle O_j \rangle) \rangle$

3. Diagonalize $M_{ij} \longrightarrow \lambda_k;\ V_k = c_{ki} O_i$

4. Probability distributions $P(V_i)$, $P(V_i, V_j)$

5. Volume dependence: $\boxed{\lambda_k \propto L^{3+x}}$

   $x = 0$ "regular"

   $x \neq 0$ "critical"

   specific heat:

$$\chi_E = \langle (E - \langle E \rangle)^2 \rangle / L^3 \propto L^{\alpha/\nu}$$

   magnetic susceptibility:

$$\chi_M = \langle (M - \langle M \rangle)^2 \rangle / L^3 \propto L^{\gamma/\nu}$$

Eigenvalues λ_k and eigenvector projections to operators O_i at the critical point (volume 64^3)

	λ	S_{Gauge}	$S_{\text{Hopp.}}$	S_{ϕ^2}	$S_{(\phi^2-1)^2}$	O_5	O_6
4 operators:							
M	1.28×10^{10}	0.051	0.725	-0.685	-0.018	–	–
r_1	8.51×10^{5}	0.996	0.008	0.083	0.005	–	–
r_2	2.59×10^{5}	-0.066	0.687	0.722	0.018	–	–
E	1.75×10^{3}	-0.003	0.0004	-0.0262	0.999	–	–
6 operators:							
M	1.33×10^{10}	0.050	0.713	-0.674	-0.018	-0.164	-0.085
r_1	8.52×10^{5}	0.995	0.010	0.087	0.005	0.008	-0.037
r_2	2.81×10^{5}	-0.078	0.655	0.687	0.026	0.136	-0.271
E	1.32×10^{5}	0.024	0.233	0.033	-0.105	0.450	0.855
r_3	4.05×10^{3}	1×10^{-5}	-0.091	-0.241	-0.217	0.836	-0.433
r_4	73	-2×10^{-5}	9×10^{-5}	-0.081	0.970	0.229	0.002

Conclusions:

- SU(2)-Higgs shows a *3D Ising universal behaviour:*
  – critical indices
  – joint probability distributions $P(V_i, V_j)$
  – exitation spectrum

- $x_{crit} = 0.11336(25)$ at $\beta_G = 5$
  $x_{crit} = 0.0983(15)$ continuum
  $\mapsto m_{H,crit} = 72(2)\,\mathrm{GeV}$ in the Standard Model

- The eigenvalue analysis of a *large enough* fluctuation matrix $\langle O_i O_j \rangle$ is a powerful tool in analyzing the critical behaviour.